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# Detailed results of the comparison of basis functions used in interval-based optimization process

Sébastien Lengagne

August 27, 2024

## 1 Introduction

In this paper, we assess the results of the method proposed in [1] by addressing Optimization Problems (OP) to find the posture of a robot that optimally fits robot constraints and desired end-effector position such as :

$$\begin{aligned} & \text{Find} && q \in \mathbb{Q} \subset \mathbb{R}^n \\ & \text{such as} && \min_q \mathcal{F}(q) \\ & \text{with } \forall j \in \{1, \dots, m\} && \mathcal{G}_j(q) \in [\underline{g}_j, \bar{g}_j] \end{aligned} \tag{1}$$

Where  $n$  is called the number of degrees of freedom.

## 2 Detail of the basis functions

The different basis functions presented in [1] are more deeply presented here. Table 1 to 8 represents the coefficient of the matrix  $\mathbf{B}$  and the coefficient of the matrix  $\mathbf{B}^{-1}$ . Eventually, Table 9 shows the plot for the different functions from order 2 to 6.

We consider basis functions of the difference types :

- Bernstein,
- MinVariance,
- B-Splines,
- MinNorm,
- MinVolume
- Approximation of MinVolume (ApproxMinVo),
- Recursive,
- Recursive2.

## 3 Bisection mode

The goal of this document is to evaluate the efficient of the evaluation process in the case of optimization. We do not focus on the bisection process that will be study in future works. Here, we consider the results of two different bisection process :

- **MinFirst** : Cut the largest interval and store in a FILO (First In Last Out) stack the higher part and then the lower part. So the lower part will be the next to be evaluated.
- **MaxFirst** : Cut the largest interval and store in a FILO (First In Last Out) stack the lower part and then the higher part. So the higher part will be the next to be evaluated.

## 4 Presentation of the Planar robot problem

First, we assess our method on multi degrees-of-freedom 2D-robots, and considering possible constraints on the end effector, the Center of Mass position and maximum torque:

## 4.1 End effector position

We define the end effector by recursively computing :

$$i \in 1, \dots, n \quad T_i = T_{i-1} \times \begin{bmatrix} \cos(q_i) & -\sin(q_i) & l \\ \sin(q_i) & \cos(q_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

With the number of a link  $l = 2/n$  and  $T_0 = I_{3 \times 3}$  and the end-effector position  $\{P_x, P_y\}$  and orientation  $\theta$  are defined as:

$$T_n = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & P_x \\ \sin(\theta) & \cos(\theta) & P_y \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

## 4.2 Considering several targets

We assess our method on nine different target positions considering shape as a square of size 0.2 by 0.2, as a circle target with a radius of 0.1 and as a ring target with an external radius of 0.15 and an interval radius of 0.05. The targets are localized at three different positions :

- problem 1,4,7 :  $x = 0.2, y = 1.7$
- problem 2,5,8 :  $x = 0.4, y = 1.4$
- problem 3,6,9 :  $x = 0.6, y = 1.1$

We defined the following constraints :

$$\text{square:} \quad \begin{cases} P_x \in [x - 0.1; x + 0.1] \\ P_y \in [y - 0.1; y + 0.1] \end{cases} \quad (4)$$

$$\text{circle:} \quad (P_x - x)^2 + (P_y - y)^2 \in [0; 0.01] \quad (5)$$

$$\text{ring:} \quad (P_x - x)^2 + (P_y - y)^2 \in [0.05^2; 0.15^2] \quad (6)$$

## 4.3 Center of Mass

To assess the performance of the evaluation for more complex functions we consider the Center of Mass (COM) of the robot. We consider that all the links have the same length  $l = 2/n$  and same weight 1 and the same Center Of Mass located in the middle of each links. Thus the x-coordinate of the COM of each link  $c_i$  can be computed through :

$$c_i = [1 \quad 0 \quad 0] T_i \times \begin{bmatrix} l/2 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

and the global COM as :

$$COM = \frac{1}{n} \sum_{i=1}^n c_i \quad (8)$$

Finally we consider that the COM of the robot must remain in  $[0.1, 0.1]$  on the horizontal plane.

## 4.4 Cost function

We consider two different cost functions to minimize. The first one is the sum of joint position square and the second is the sum of torque square:

$$\text{problem 1 to 9 position :} \quad \sum_{i=1}^n q_i^2 \quad (9)$$

$$\text{problem 11 to 19 torque :} \quad \sum_{i=1}^n \Gamma_i^2 \quad (10)$$

$$(11)$$

## 4.5 Results of the Planar robot problem

We assess our contribution with 18 different constraint satisfaction problems, with the two bisection processes (MinFirst and MaxFirst) such as presented in Table 11.

## 4.6 Results of the Bisection process

The results are summarized in the tables 12 to 47 and summarized in table 48. For all the cases, it appears that using basis functions reduce the number of iterations regarding to the state-of-the-art interval bisection process. This phenomenon is all the more important as the number of degrees of freedom increases and as the precision decreases. Regarding the computation time the BSplines basis function is the worst cases (even worse than interval bisection process). Excluding the preparation phase the other basis functions allows the optimization process to be faster than interval bisection. Taking into account the preparation phase, the basis function (except BSplines) are faster for degrees of freedom bigger than 4.

Regarding the global performances the Bernstein function seems to be the best solution, very close to the MinVariance (for computation time and number of iterations). Then the MinNo, the Recursive and the Reclusve2 produce quite good performances and the MinVo and ApproxMinVo are the less effective methods whereas BSplines is the worst one.

## 5 3D robot

### 5.1 Presentation fo the robot

In this section we consider the same kind of problem as presented in Equation 1 but with 3D-robots. To assess the effectiveness of our method on more complex systems, we consider the following problem:

$$\begin{aligned}
 &\text{Find} && q \in \mathbb{Q} \subset \mathbb{R}^n \\
 &\text{such as} && \min_q F(q) \\
 &\text{with} && x \in x_d + [-0.01; 0.01] \\
 &&& y \in y_d + [-0.01; 0.01] \\
 &&& z \in z_d + [-0.01; 0.01] \\
 &\forall j \in \{1, \dots, n\} && \Gamma_j \in [-\alpha \bar{\Gamma}_j : \alpha \bar{\Gamma}_j]
 \end{aligned} \tag{12}$$

We present the results for two different 3D robots (with  $n = 4$  and  $n = 6$  degrees of freedom) which derive from the KUKA LWR as presented on Figure 1. The 4-dof robot is obtained by considering the joint number 3 and 5 equal to zero.

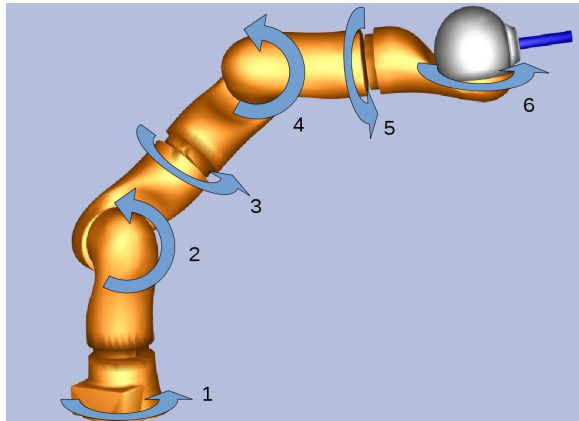


Figure 1: The 3D robot : KUKA LWR

The solution of Problem 12 is the static posture, i.e. the joint position, that minimizes the cost function  $F(q)$  and ensures that the end effector position  $\{x, y, z\}$  is within a 0.02 meter width box centered on a desired position  $\{x_d, y_d, z_d\}$ , and checks the joint torque  $\Gamma_i$  are within the joint limits  $[-\bar{\Gamma}_j : \bar{\Gamma}_j]$  considering different coefficient  $\alpha$ . Theses function can be obtained by using the forward kinematic model and the inverse dynamics model as presented in [2].

### 5.2 Constraint evaluation

First, Table 49 and 50 presents the results of the constraint and cost function computation considering several inputs to compare the effectiveness of the different basis function on the evaluation performances.

Figure 2 represents the diameter of the sum of square torque regarding the diameter of the input  $[q]$ . It shows that for large boxes (diameter of 2), the interval evaluation is quite similar using interval analysis or basis function based evaluation. For smaller intervals, the basis-functions produce tighter evaluation than interval analysis, except for the worst case of B-Splines.

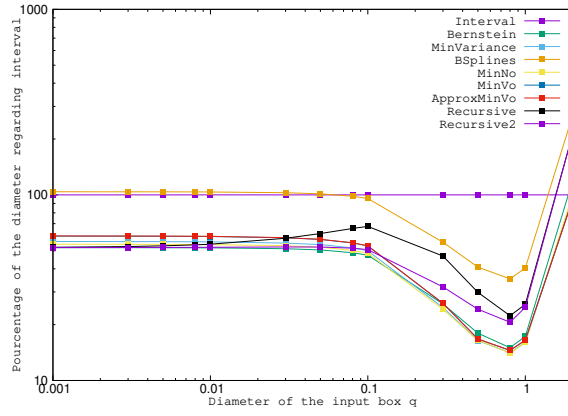


Figure 2: Representation of the diameter of the evaluation using basis function regarding interval analysis.

### 5.3 Optimization Results of the 3D robot problem

We assess our contribution with 12 different constraint satisfaction problems, with the two bisection processes (MinFirst and MaxFirst) such as presented in Table 51, with  $n$  the number of degrees of freedom. Some computation with 6-dof robot and sum of square torques are still in progress with huge computation time (more than 60 days), hence we do not present the results here.

### 5.4 Results of the Bisection process

We can note that the preparation time is quite important: around 100s for 4-dof-robot and around 400s for the 6-dof-robot. This will greatly impact the performance for fast resolution, but can be neglected for more complex problem that are solved in several hours or days.

As for the 2D-cases, we notice that the number of iterations of basis-function based method is lower than state-of-the-art interval analysis. This point emphasizes that the evaluation produce tighter boxes that was already showed in Section 5.2.

Regarding the performances of the different basis functions, it is clear that the BSplines are the worst case. The other basis function produce similar results in terms of reducing the number of bisection process or the computation even if we can notice that the Bernstein functions seems too be the best choice.

## 6 Result summary

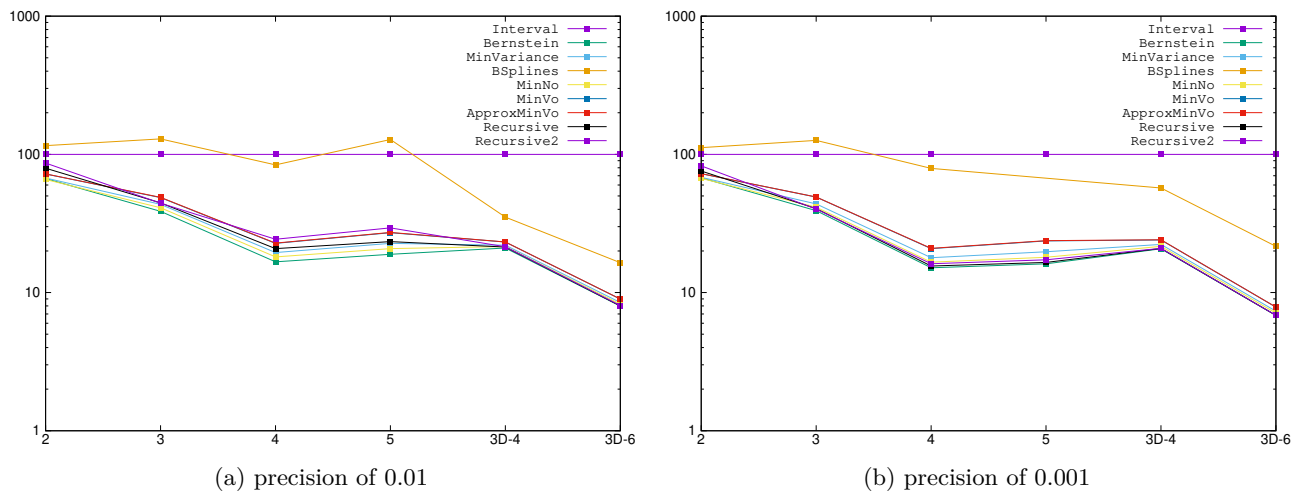


Figure 3: Comparison of the number of iterations

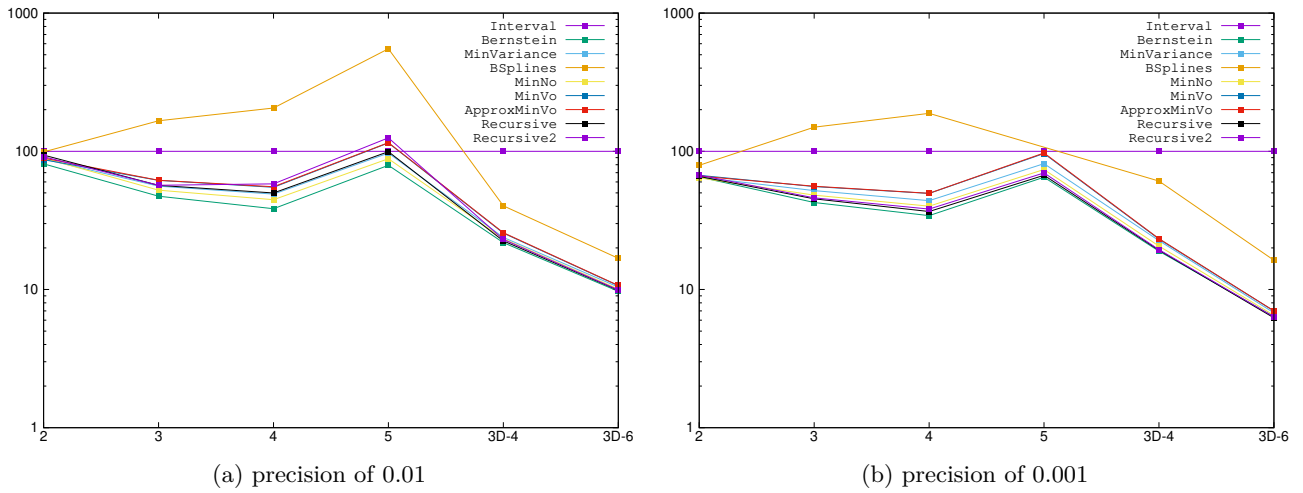


Figure 4: Comparison of the computation time.

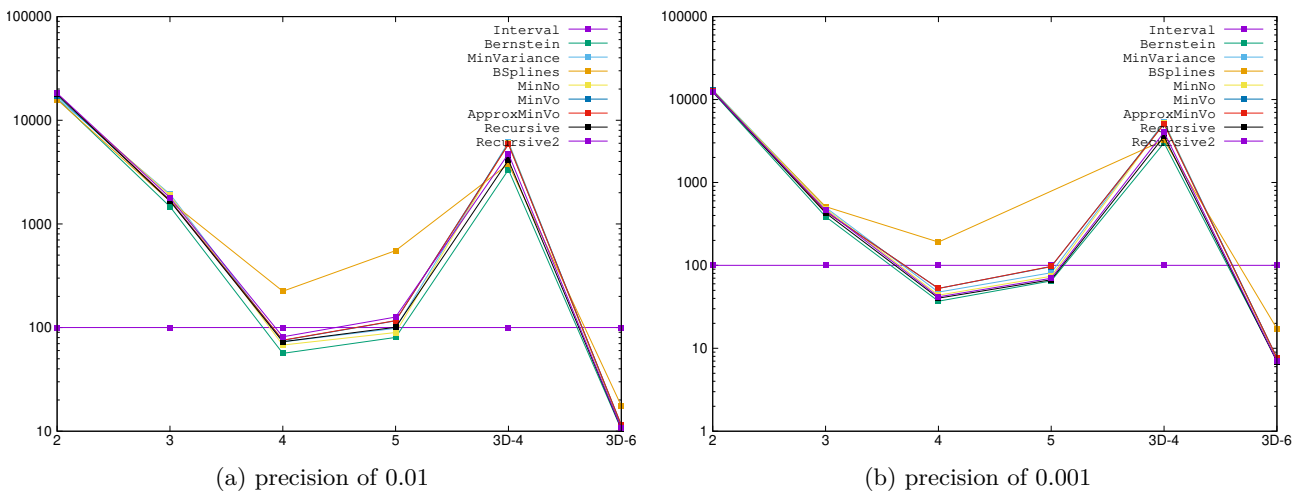


Figure 5: Comparison of the total computation time (including preparation phase).

## Acknowledgment

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## References

- [1] Sébastien Lengagne. Comparison of the basis-functions for interval-based optimization. submitted to International Journal of Approximate Reasoning, -.
- [2] Bruno Siciliano and Oussama Khatib. Springer Handbook of Robotics. Springer-Verlag, Berlin, Heidelberg, 2007.

| matrice               | order | value |           |           |           |           |           |           |           |  |  |  |  |  |  |
|-----------------------|-------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|--|--|--|--|--|--|
| <b>B</b>              | 1     |       |           | 0.500000  | 0.500000  |           |           |           |           |  |  |  |  |  |  |
|                       |       |       |           | -0.500000 | 0.500000  |           |           |           |           |  |  |  |  |  |  |
|                       | 2     |       |           | 0.250000  | 0.500000  | 0.250000  |           |           |           |  |  |  |  |  |  |
|                       |       |       |           | -0.500000 | 0.000000  | 0.500000  |           |           |           |  |  |  |  |  |  |
|                       |       |       |           | 0.250000  | -0.500000 | 0.250000  |           |           |           |  |  |  |  |  |  |
|                       | 3     |       |           | 0.125000  | 0.375000  | 0.375000  | 0.125000  |           |           |  |  |  |  |  |  |
|                       |       |       | -0.375000 | -0.375000 | 0.375000  | 0.375000  |           |           |           |  |  |  |  |  |  |
|                       |       |       | 0.375000  | -0.375000 | -0.375000 | 0.375000  |           |           |           |  |  |  |  |  |  |
|                       |       |       | -0.125000 | 0.375000  | -0.375000 | 0.125000  |           |           |           |  |  |  |  |  |  |
| 4                     |       |       | 0.062500  | 0.250000  | 0.375000  | 0.250000  | 0.062500  |           |           |  |  |  |  |  |  |
|                       |       |       | -0.250000 | -0.500000 | 0.000000  | 0.500000  | 0.250000  |           |           |  |  |  |  |  |  |
|                       |       |       | 0.375000  | 0.000000  | -0.750000 | 0.000000  | 0.375000  |           |           |  |  |  |  |  |  |
|                       |       |       | -0.250000 | 0.500000  | 0.000000  | -0.500000 | 0.250000  |           |           |  |  |  |  |  |  |
|                       |       |       | 0.062500  | -0.250000 | 0.375000  | -0.250000 | 0.062500  |           |           |  |  |  |  |  |  |
| 5                     |       |       | 0.031250  | 0.156250  | 0.312500  | 0.312500  | 0.156250  | 0.031250  |           |  |  |  |  |  |  |
|                       |       |       | -0.156250 | -0.468750 | -0.312500 | 0.312500  | 0.468750  | 0.156250  |           |  |  |  |  |  |  |
|                       |       |       | 0.312500  | 0.312500  | -0.625000 | -0.625000 | 0.312500  | 0.312500  |           |  |  |  |  |  |  |
|                       |       |       | -0.312500 | 0.312500  | 0.625000  | -0.625000 | -0.312500 | 0.312500  |           |  |  |  |  |  |  |
|                       |       |       | 0.156250  | -0.468750 | 0.312500  | 0.312500  | -0.468750 | 0.156250  |           |  |  |  |  |  |  |
|                       |       |       | -0.031250 | 0.156250  | -0.312500 | 0.312500  | -0.156250 | 0.031250  |           |  |  |  |  |  |  |
| 6                     |       |       | 0.015625  | 0.093750  | 0.234375  | 0.312500  | 0.234375  | 0.093750  | 0.015625  |  |  |  |  |  |  |
|                       |       |       | -0.093750 | -0.375000 | -0.468750 | 0.000000  | 0.468750  | 0.375000  | 0.093750  |  |  |  |  |  |  |
|                       |       |       | 0.234375  | 0.468750  | -0.234375 | -0.937500 | -0.234375 | 0.468750  | 0.234375  |  |  |  |  |  |  |
|                       |       |       | -0.312500 | 0.000000  | 0.937500  | 0.000000  | -0.937500 | 0.000000  | 0.312500  |  |  |  |  |  |  |
|                       |       |       | 0.234375  | -0.468750 | -0.234375 | 0.937500  | -0.234375 | -0.468750 | 0.234375  |  |  |  |  |  |  |
|                       |       |       | -0.093750 | 0.375000  | -0.468750 | 0.000000  | 0.468750  | -0.375000 | 0.093750  |  |  |  |  |  |  |
|                       |       |       | 0.015625  | -0.093750 | 0.234375  | -0.312500 | 0.234375  | -0.093750 | 0.015625  |  |  |  |  |  |  |
| <b>B<sup>-1</sup></b> | 1     |       |           | 1.000000  | -1.000000 |           |           |           |           |  |  |  |  |  |  |
|                       |       |       |           | 1.000000  | 1.000000  |           |           |           |           |  |  |  |  |  |  |
|                       | 2     |       |           | 1.000000  | -1.000000 | 1.000000  |           |           |           |  |  |  |  |  |  |
|                       |       |       |           | 1.000000  | 0.000000  | -1.000000 |           |           |           |  |  |  |  |  |  |
|                       |       |       |           | 1.000000  | 1.000000  | 1.000000  |           |           |           |  |  |  |  |  |  |
|                       | 3     |       |           | 1.000000  | -1.000000 | 1.000000  | -1.000000 |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.333333 | -0.333333 | 1.000000  |           |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.333333  | -0.333333 | -1.000000 |           |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  |           |           |           |  |  |  |  |  |  |
| 4                     |       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000  |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.500000 | -0.000000 | 0.500000  | -1.000000 |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.000000 | -0.333333 | -0.000000 | 1.000000  |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.500000  | 0.000000  | -0.500000 | -1.000000 |           |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000  |           |           |  |  |  |  |  |  |
| 5                     |       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000  | -1.000000 |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.600000 | 0.200000  | 0.200000  | -0.600000 | 1.000000  |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.200000 | -0.200000 | 0.200000  | 0.200000  | -1.000000 |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.200000  | -0.200000 | -0.200000 | 0.200000  | 1.000000  |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.600000  | 0.200000  | -0.200000 | -0.600000 | -1.000000 |           |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000  |           |  |  |  |  |  |  |
| 6                     |       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000  |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.666667 | 0.333333  | -0.000000 | -0.333333 | 0.666667  | -1.000000 |  |  |  |  |  |  |
|                       |       |       | 1.000000  | -0.333333 | -0.066667 | 0.200000  | -0.066667 | -0.333333 | 1.000000  |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.000000  | -0.200000 | -0.000000 | 0.200000  | 0.000000  | -1.000000 |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.333333  | -0.066667 | -0.200000 | -0.066667 | 0.333333  | 1.000000  |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 0.666667  | 0.333333  | 0.000000  | -0.333333 | -0.666667 | -1.000000 |  |  |  |  |  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000  |  |  |  |  |  |  |

Table 1: Values of the matrices **B** and **B<sup>-1</sup>** for the Bernstein basis functions from order 1 to 6.

| matrice               | order | value |           |           |           |           |           |
|-----------------------|-------|-------|-----------|-----------|-----------|-----------|-----------|
| <b>B</b>              | 1     |       |           | 0.500000  | 0.500000  |           |           |
|                       |       |       |           | -0.500000 | 0.500000  |           |           |
|                       | 2     |       |           | 0.155869  | 0.688262  | 0.155869  |           |
|                       |       |       |           | -0.463203 | 0.000000  | 0.463203  |           |
|                       |       |       |           | 0.344131  | -0.688262 | 0.344131  |           |
|                       | 3     |       |           | 0.019243  | 0.480757  | 0.480757  | 0.019243  |
|                       |       |       | -0.175647 | -0.592928 | 0.592928  | 0.175647  |           |
|                       |       |       | 0.474214  | -0.474214 | -0.474214 | 0.474214  |           |
|                       |       |       | -0.317810 | 0.599471  | -0.599471 | 0.317810  |           |
| 4                     |       |       | 0.000358  | 0.223129  | 0.553025  | 0.223129  | 0.000358  |
|                       |       |       | 0.017719  | -0.754489 | 0.000000  | 0.754489  | -0.017719 |
|                       |       |       | 0.198449  | 0.414677  | -1.226252 | 0.414677  | 0.198449  |
|                       |       |       | -0.510994 | 0.754489  | 0.000000  | -0.754489 | 0.510994  |
|                       |       |       | 0.297927  | -0.637806 | 0.679758  | -0.637806 | 0.297927  |
| 5                     |       |       | 0.001914  | 0.054005  | 0.444081  | 0.444081  | 0.054005  |
|                       |       |       | 0.020330  | -0.413584 | -0.682843 | 0.682843  | 0.413584  |
|                       |       |       | -0.000905 | 0.916831  | -0.915926 | -0.915926 | 0.916831  |
|                       |       |       | -0.272851 | -0.227279 | 1.473726  | -1.473726 | 0.227279  |
|                       |       |       | 0.496226  | -0.970088 | 0.473862  | 0.473862  | -0.970088 |
|                       |       |       | -0.244714 | 0.641611  | -0.792900 | 0.792900  | -0.641611 |
|                       |       |       | 0.002525  | 0.009926  | 0.212042  | 0.551016  | 0.212042  |
|                       |       |       | 0.005469  | -0.147132 | -0.828820 | 0.000000  | 0.828820  |
|                       |       |       | -0.051546 | 0.691264  | 0.368931  | -2.017299 | 0.368931  |
|                       |       |       | -0.013728 | -1.008610 | 1.734068  | 0.000000  | -1.734068 |
|                       |       |       | 0.343248  | -0.088754 | -1.475367 | 2.441746  | -1.475367 |
|                       |       |       | -0.489013 | 1.155742  | -0.906977 | 0.000000  | 0.906977  |
|                       |       |       | 0.203216  | -0.612436 | 0.896951  | -0.975463 | 0.896951  |
|                       |       |       |           |           |           |           |           |
| <b>B<sup>-1</sup></b> | 1     |       |           | 1.000000  | -1.000000 |           |           |
|                       |       |       |           | 1.000000  | 1.000000  |           |           |
|                       | 2     |       |           | 1.000000  | -1.079440 | 1.000000  |           |
|                       |       |       |           | 1.000000  | 0.000000  | -0.452934 |           |
|                       |       |       |           | 1.000000  | 1.079440  | 1.000000  |           |
|                       | 3     |       |           | 1.000000  | -1.020433 | 1.013797  | -1.009295 |
|                       |       |       | 1.000000  | -0.540983 | -0.040579 | 0.298991  |           |
|                       |       |       | 1.000000  | 0.540983  | -0.040579 | -0.298991 |           |
|                       |       |       | 1.000000  | 1.020433  | 1.013797  | 1.009295  |           |
| 4                     |       |       | 1.000000  | -1.013632 | 1.008328  | -1.013632 | 1.005414  |
|                       |       |       | 1.000000  | -0.686505 | 0.328883  | -0.023804 | -0.220271 |
|                       |       |       | 1.000000  | -0.000000 | -0.266696 | 0.000000  | 0.176443  |
|                       |       |       | 1.000000  | 0.686505  | 0.328883  | 0.023804  | -0.220271 |
|                       |       |       | 1.000000  | 1.013632  | 1.008328  | 1.013632  | 1.005414  |
| 5                     |       |       | 1.000000  | -1.005471 | 1.005090  | -1.006392 | 1.005581  |
|                       |       |       | 1.000000  | -0.758010 | 0.483305  | -0.240705 | -0.002976 |
|                       |       |       | 1.000000  | -0.303057 | -0.063107 | 0.115827  | -0.003972 |
|                       |       |       | 1.000000  | 0.303057  | -0.063107 | -0.115827 | -0.003972 |
|                       |       |       | 1.000000  | 0.758010  | 0.483305  | 0.240705  | -0.002976 |
|                       |       |       | 1.000000  | 1.005471  | 1.005090  | 1.006392  | 1.005581  |
| 6                     |       |       | 1.000000  | -1.003855 | 1.004594  | -1.005696 | 1.005546  |
|                       |       |       | 1.000000  | -0.792904 | 0.578203  | -0.378065 | 0.182506  |
|                       |       |       | 1.000000  | -0.469135 | 0.106350  | 0.060478  | -0.078809 |
|                       |       |       | 1.000000  | 0.000000  | -0.111888 | 0.000000  | 0.044865  |
|                       |       |       | 1.000000  | 0.469135  | 0.106350  | -0.060478 | -0.078809 |
|                       |       |       | 1.000000  | 0.792904  | 0.578203  | 0.378065  | 0.182506  |
|                       |       |       | 1.000000  | 1.003855  | 1.004594  | 1.005696  | 1.005546  |

Table 2: Values of the matrices **B** and **B<sup>-1</sup>** for the MinVariance basis functions from order 1 to 6.



| matrice               | order | value |           |           |           |             |            |              |              |
|-----------------------|-------|-------|-----------|-----------|-----------|-------------|------------|--------------|--------------|
| <b>B</b>              | 1     |       |           | 0.500000  | 0.500000  |             |            |              |              |
|                       |       |       |           | -0.500000 | 0.500000  |             |            |              |              |
|                       | 2     |       |           | 0.125000  | 0.750000  | 0.125000    |            |              |              |
|                       |       |       |           | -0.250000 | 0.000000  | 0.250000    |            |              |              |
|                       |       |       |           | 0.125000  | -0.250000 | 0.125000    |            |              |              |
|                       | 3     |       |           | 0.020833  | 0.479167  | 0.479167    | 0.020833   |              |              |
|                       |       |       | -0.062500 | -0.312500 | 0.312500  | 0.062500    |            |              |              |
|                       |       |       | 0.062500  | -0.062500 | -0.062500 | 0.062500    |            |              |              |
|                       |       |       | -0.020833 | 0.062500  | -0.062500 | 0.020833    |            |              |              |
| 4                     |       |       | 0.002604  | 0.197917  | 0.598958  | 0.197917    | 0.002604   |              |              |
|                       |       |       | -0.010417 | -0.229167 | 0.000000  | 0.229167    | 0.010417   |              |              |
|                       |       |       | 0.015625  | 0.062500  | -0.156250 | 0.062500    | 0.015625   |              |              |
|                       |       |       | -0.010417 | 0.020833  | 0.000000  | -0.020833   | 0.010417   |              |              |
|                       |       |       | 0.002604  | -0.010417 | 0.015625  | -0.010417   | 0.002604   |              |              |
| 5                     |       |       | 0.000260  | 0.061719  | 0.438021  | 0.438021    | 0.061719   | 0.000260     |              |
|                       |       |       | -0.001302 | -0.097656 | -0.200521 | 0.200521    | 0.097656   | 0.001302     |              |
|                       |       |       | 0.002604  | 0.054688  | -0.057292 | -0.057292   | 0.054688   | 0.002604     |              |
|                       |       |       | -0.002604 | -0.007812 | 0.036458  | -0.036458   | 0.007812   | 0.002604     |              |
|                       |       |       | 0.001302  | -0.003906 | 0.002604  | 0.002604    | -0.003906  | 0.001302     |              |
|                       |       |       | -0.000260 | 0.001302  | -0.002604 | 0.002604    | -0.001302  | 0.000260     |              |
| 6                     |       |       | 0.000022  | 0.015668  | 0.228798  | 0.511024    | 0.228798   | 0.015668     | 0.000022     |
|                       |       |       | -0.000130 | -0.030729 | -0.188151 | 0.000000    | 0.188151   | 0.030729     | 0.000130     |
|                       |       |       | 0.000326  | 0.024089  | 0.025716  | -0.100260   | 0.025716   | 0.024089     | 0.000326     |
|                       |       |       | -0.000434 | -0.008681 | 0.018663  | 0.000000    | -0.018663  | 0.008681     | 0.000434     |
|                       |       |       | 0.000326  | 0.000651  | -0.005534 | 0.009115    | -0.005534  | 0.000651     | 0.000326     |
|                       |       |       | -0.000130 | 0.000521  | -0.000651 | -0.000000   | 0.000651   | -0.000521    | 0.000130     |
|                       |       |       | 0.000022  | -0.000130 | 0.000326  | -0.000434   | 0.000326   | -0.000130    | 0.000022     |
| <b>B<sup>-1</sup></b> | 1     |       |           | 1.000000  | -1.000000 |             |            |              |              |
|                       |       |       |           | 1.000000  | 1.000000  |             |            |              |              |
|                       | 2     |       |           | 1.000000  | -2.000000 | 3.000000    |            |              |              |
|                       |       |       |           | 1.000000  | 0.000000  | -1.000000   |            |              |              |
|                       |       |       |           | 1.000000  | 2.000000  | 3.000000    |            |              |              |
|                       | 3     |       |           | 1.000000  | -3.000000 | 7.666667    | -15.000000 |              |              |
|                       |       |       | 1.000000  | -1.000000 | -0.333333 | 3.000000    |            |              |              |
|                       |       |       | 1.000000  | 1.000000  | -0.333333 | -3.000000   |            |              |              |
|                       |       |       | 1.000000  | 3.000000  | 7.666667  | 15.000000   |            |              |              |
| 4                     |       |       | 1.000000  | -4.000000 | 14.333333 | -44.000000  | 105.000000 |              |              |
|                       |       |       | 1.000000  | -2.000000 | 2.333333  | 2.000000    | -15.000000 |              |              |
|                       |       |       | 1.000000  | 0.000000  | -1.666667 | 0.000000    | 9.000000   |              |              |
|                       |       |       | 1.000000  | 2.000000  | 2.333333  | -2.000000   | -15.000000 |              |              |
|                       |       |       | 1.000000  | 4.000000  | 14.333333 | 44.000000   | 105.000000 |              |              |
| 5                     |       |       | 1.000000  | -5.000000 | 23.000000 | -95.000000  | 337.800000 | -945.000000  |              |
|                       |       |       | 1.000000  | -3.000000 | 7.000000  | -9.000000   | -14.200000 | 105.000000   |              |
|                       |       |       | 1.000000  | -1.000000 | -1.000000 | 5.000000    | 1.800000   | -45.000000   |              |
|                       |       |       | 1.000000  | 1.000000  | -1.000000 | -5.000000   | 1.800000   | 45.000000    |              |
|                       |       |       | 1.000000  | 3.000000  | 7.000000  | 9.000000    | -14.200000 | -105.000000  |              |
|                       |       |       | 1.000000  | 5.000000  | 23.000000 | 95.000000   | 337.800000 | 945.000000   |              |
| 6                     |       |       | 1.000000  | -6.000000 | 33.666667 | -174.000000 | 809.266667 | -3254.000000 | 10395.000000 |
|                       |       |       | 1.000000  | -4.000000 | 13.666667 | -36.000000  | 49.266667  | 124.000000   | -945.000000  |
|                       |       |       | 1.000000  | -2.000000 | 1.666667  | 6.000000    | -22.733333 | -18.000000   | 315.000000   |
|                       |       |       | 1.000000  | 0.000000  | -2.333333 | 0.000000    | 17.266667  | 0.000000     | -225.000000  |
|                       |       |       | 1.000000  | 2.000000  | 1.666667  | -6.000000   | -22.733333 | 18.000000    | 315.000000   |
|                       |       |       | 1.000000  | 4.000000  | 13.666667 | 36.000000   | 49.266667  | -124.000000  | -945.000000  |
|                       |       |       | 1.000000  | 6.000000  | 33.666667 | 174.000000  | 809.266667 | 3254.000000  | 10395.000000 |

Table 3: Values of the matrices **B** and **B<sup>-1</sup>** for the BSplines basis functions from order 1 to 6.

| matrice               | order  | value  |   |  |   |  |  |  |  |
|-----------------------|--|--|---|--|---|--|--|--|--|
| <b>B</b>              | 1  | 0.500000 0.500000  |   |  |   | -0.500000 0.500000                                       |  |  |  |
|                       | 2  | 0.178806 0.642387 0.178806                               |   | -0.479297 0.000000 0.479297                              |   | 0.321194 -0.642387 0.321194                              |  |  |  |
|                       | 3  | 0.028323 0.471677 0.471677 0.028323                      |   | -0.208384 -0.572950 0.572950 0.208384                    |   | 0.466241 -0.466241 -0.466241 0.466241                    |  | -0.286180 0.578386 -0.578386 0.286180                    |  |
|                       | 4  | 0.001808 0.232346 0.531691 0.232346 0.001808             |   | -0.040769 -0.719547 0.000000 0.719547 0.040769           |   | 0.270116 0.324740 -1.189712 0.324740 0.270116            |  | -0.454129 0.719547 0.000000 -0.719547 0.454129           |  |
|                       |  | 0.224324 -0.557086 0.665525 -0.557086 0.224324           |   |  |   |  |  |  |  |
|                       | 5  | 0.000548 0.087040 0.412412 0.412412 0.087040 0.000548    |   | -0.013185 -0.484330 -0.611879 0.611879 0.484330 0.013185 |   | 0.104674 0.774368 -0.879042 -0.879042 0.774368 0.104674  |  | -0.313820 0.006407 1.345160 -1.345160 -0.006407 0.313820 |  |
|                       | 0.390724 -0.861075 0.470351 0.470351 -0.861075 0.390724            |  | -0.168943 0.478256 -0.737001 0.737001 -0.478256 0.168943          |  |   |  |  |  |  |
| 6                     | 0.000246 0.039541 0.234007 0.452411 0.234007 0.039541 0.000246     |  | -0.004996 -0.286016 -0.769529 0.000000 0.769529 0.286016 0.004996 |  | 0.040290 0.715108 0.132833 -1.776461 0.132833 0.715108 0.040290   |  | -0.162385 -0.572706 1.643712 0.000000 -1.643712 0.572706 0.162385  |  |  |
|                       | 0.336335 -0.398220 -1.084518 2.292808 -1.084518 -0.398220 0.336335 |  | -0.328442 0.858721 -0.877742 0.000000 0.877742 -0.858721 0.328442 |  | 0.119119 -0.356428 0.721688 -0.968758 0.721688 -0.356428 0.119119 |  |  |  |  |
| <b>B<sup>-1</sup></b> | 1  | 1.000000 -1.000000                                       |   |  |   | 1.000000 1.000000  |  |  |  |
|                       | 2  | 1.000000 -1.043194 1.000000                              |   | 1.000000 0.000000 -0.556693                              |   | 1.000000 1.043194 1.000000                               |  |  |  |
|                       | 3  | 1.000000 -1.016520 1.011659 -1.006966                    |   | 1.000000 -0.502965 -0.060748 0.366237                    |   | 1.000000 0.502965 -0.060748 -0.366237                    |  | 1.000000 1.016520 1.011659 1.006966                      |  |
|                       | 4  | 1.000000 -1.010308 1.009359 -1.010308 1.005456           |   | 1.000000 -0.637638 0.264341 0.057244 -0.326362           |   | 1.000000 -0.000000 -0.237895 -0.000000 0.278399          |  | 1.000000 0.637638 0.264341 -0.057244 -0.326362           |  |
|                       |  | 1.000000 1.010308 1.009359 1.010308 1.005456             |   |  |   |  |  |  |  |
|                       | 5  | 1.000000 -1.006914 1.008556 -1.009297 1.008081 -1.006181 |   | 1.000000 -0.712459 0.409683 -0.145518 -0.111157 0.325908 |   | 1.000000 -0.231515 -0.087805 0.136932 0.022119 -0.236290 |  | 1.000000 0.231515 -0.087805 -0.136932 0.022119 0.236290  |  |
|                       | 1.000000 0.712459 0.409683 0.145518 -0.111157 -0.325908            |  | 1.000000 1.006914 1.008556 1.009297 1.008081 1.006181             |  |   |  |  |  |  |
| 6                     | 1.000000 -1.005823 1.007968 -1.009204 1.008783 -1.008078 1.006174  |  | 1.000000 -0.755246 0.509137 -0.274875 0.056328 0.147388 -0.333314 |  | 1.000000 -0.362511 0.014527 0.108716 -0.086189 -0.048237 0.236376 |  | 1.000000 -0.000000 -0.105121 0.000000 0.078218 -0.000000 -0.187359 |  |  |
|                       | 1.000000 0.362511 0.014527 -0.108716 -0.086189 0.048237 0.236376   |  | 1.000000 0.755246 0.509137 0.274875 0.056328 -0.147388 -0.333314  |  | 1.000000 1.005823 1.007968 1.009204 1.008783 1.008078 1.006174    |  |  |  |  |

Table 4: Values of the matrices **B** and **B<sup>-1</sup>** for the MinNorm basis functions from order 1 to 6.

| matrice               | order | value     |           |           |           |           |           |           |           |  |  |
|-----------------------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|--|--|
| <b>B</b>              | 1     |           |           |           |           | 0.500000  | 0.500000  |           |           |  |  |
|                       |       |           |           |           |           | -0.500000 | 0.500000  |           |           |  |  |
|                       | 2     |           |           |           |           | 0.125000  | 0.750000  | 0.125000  |           |  |  |
|                       |       |           |           |           |           | -0.433013 | 0.000000  | 0.433013  |           |  |  |
|                       |       |           |           |           |           | 0.375000  | -0.750000 | 0.375000  |           |  |  |
|                       | 3     |           |           |           |           | 0.000410  | 0.499590  | 0.499590  | 0.000410  |  |  |
|                       |       |           |           |           | -0.026982 | -0.792093 | 0.792093  | 0.026982  |           |  |  |
|                       |       |           |           |           | 0.456775  | -0.456775 | -0.456775 | 0.456775  |           |  |  |
|                       |       |           |           |           | -0.430204 | 0.834907  | -0.834907 | 0.430204  |           |  |  |
| 4                     |       |           |           |           | 0.030227  | 0.148259  | 0.643029  | 0.148259  | 0.030227  |  |  |
|                       |       |           |           |           | 0.138091  | -0.810775 | 0.000000  | 0.810775  | -0.138091 |  |  |
|                       |       |           |           |           | -0.094352 | 0.960202  | -1.731700 | 0.960202  | -0.094352 |  |  |
|                       |       |           |           |           | -0.575789 | 0.810775  | 0.000000  | -0.810775 | 0.575789  |  |  |
|                       |       |           |           |           | 0.525520  | -1.108461 | 1.165882  | -1.108461 | 0.525520  |  |  |
| 5                     |       |           | 0.045894  | 0.002895  | 0.451210  | 0.451210  | 0.002895  | 0.045894  |           |  |  |
|                       |       |           | -0.036501 | -0.120977 | -1.064403 | 1.064403  | 0.120977  | 0.036501  |           |  |  |
|                       |       |           | -0.377284 | 1.333003  | -0.955718 | -0.955718 | 1.333003  | -0.377284 |           |  |  |
|                       |       |           | 0.330192  | -1.365550 | 2.776543  | -2.776543 | 1.365550  | -0.330192 |           |  |  |
|                       |       |           | 0.776885  | -1.319318 | 0.542433  | 0.542433  | -1.319318 | 0.776885  |           |  |  |
|                       |       |           | -0.739186 | 1.503106  | -1.750066 | 1.750066  | -1.503106 | 0.739186  |           |  |  |
| 6                     |       | 0.018328  | 0.031209  | 0.152009  | 0.596907  | 0.152009  | 0.031209  | 0.018328  |           |  |  |
|                       |       | -0.136709 | 0.243263  | -1.080915 | 0.000000  | 1.080915  | -0.243263 | 0.136709  |           |  |  |
|                       |       | 0.105859  | -0.084538 | 1.580291  | -3.203224 | 1.580291  | -0.084538 | 0.105859  |           |  |  |
|                       |       | 0.834817  | -2.298575 | 2.468184  | 0.000000  | -2.468184 | 2.298575  | -0.834817 |           |  |  |
|                       |       | -0.736850 | 2.281168  | -4.269993 | 5.451350  | -4.269993 | 2.281168  | -0.736850 |           |  |  |
|                       |       | -1.134019 | 2.055312  | -1.408577 | 0.000000  | 1.408577  | -2.055312 | 1.134019  |           |  |  |
|                       |       | 1.060626  | -2.227839 | 2.589730  | -2.845034 | 2.589730  | -2.227839 | 1.060626  |           |  |  |
| <b>B<sup>-1</sup></b> | 1     |           |           |           |           | 1.000000  | -1.000000 |           |           |  |  |
|                       |       |           |           |           |           | 1.000000  | 1.000000  |           |           |  |  |
|                       | 2     |           |           |           |           | 1.000000  | -1.154701 | 1.000000  |           |  |  |
|                       |       |           |           |           |           | 1.000000  | 0.000000  | -0.333333 |           |  |  |
|                       |       |           |           |           |           | 1.000000  | 1.154701  | 1.000000  |           |  |  |
|                       | 3     |           |           |           |           | 1.000000  | -1.149096 | 1.093732  | -1.090170 |  |  |
|                       |       |           |           |           | 1.000000  | -0.592096 | -0.000898 | 0.037136  |           |  |  |
|                       |       |           |           |           | 1.000000  | 0.592096  | -0.000898 | -0.037136 |           |  |  |
|                       |       |           |           |           | 1.000000  | 1.149096  | 1.093732  | 1.090170  |           |  |  |
| 4                     |       |           |           |           | 1.000000  | -1.142338 | 1.106977  | -1.142338 | 1.092669  |  |  |
|                       |       |           |           |           | 1.000000  | -0.811256 | 0.378336  | -0.194562 | 0.010409  |  |  |
|                       |       |           |           |           | 1.000000  | -0.000000 | -0.278531 | -0.000000 | -0.107525 |  |  |
|                       |       |           |           |           | 1.000000  | 0.811256  | 0.378336  | 0.194562  | 0.010409  |  |  |
|                       |       |           |           |           | 1.000000  | 1.142338  | 1.106977  | 1.142338  | 1.092669  |  |  |
| 5                     |       |           | 1.000000  | -1.129437 | 1.109754  | -1.147162 | 1.123459  | -1.133082 |           |  |  |
|                       |       |           | 1.000000  | -0.933700 | 0.605476  | -0.457764 | 0.234967  | -0.158376 |           |  |  |
|                       |       |           | 1.000000  | -0.324894 | -0.116762 | 0.091367  | -0.115779 | 0.056856  |           |  |  |
|                       |       |           | 1.000000  | 0.324894  | -0.116762 | -0.091367 | -0.115779 | -0.056856 |           |  |  |
|                       |       |           | 1.000000  | 0.933700  | 0.605476  | 0.457764  | 0.234967  | 0.158376  |           |  |  |
|                       |       |           | 1.000000  | 1.129437  | 1.109754  | 1.147162  | 1.123459  | 1.133082  |           |  |  |
| 6                     |       | 1.000000  | -1.120437 | 1.105897  | -1.147175 | 1.129137  | -1.150340 | 1.128210  |           |  |  |
|                       |       | 1.000000  | -0.990939 | 0.751557  | -0.630814 | 0.445470  | -0.344918 | 0.217191  |           |  |  |
|                       |       | 1.000000  | -0.543877 | 0.088407  | 0.003123  | -0.111657 | 0.067865  | -0.103675 |           |  |  |
|                       |       | 1.000000  | 0.000000  | -0.191530 | 0.000000  | -0.059053 | 0.000000  | -0.039190 |           |  |  |
|                       |       | 1.000000  | 0.543877  | 0.088407  | -0.003123 | -0.111657 | -0.067865 | -0.103675 |           |  |  |
|                       |       | 1.000000  | 0.990939  | 0.751557  | 0.630814  | 0.445470  | 0.344918  | 0.217191  |           |  |  |
|                       |       | 1.000000  | 1.120437  | 1.105897  | 1.147175  | 1.129137  | 1.150340  | 1.128210  |           |  |  |

Table 5: Values of the matrices **B** and **B<sup>-1</sup>** for the MinVolume basis functions from order 1 to 6.

| matrice               | order | value |           |           |           |           |           |
|-----------------------|-------|-------|-----------|-----------|-----------|-----------|-----------|
| <b>B</b>              | 1     |       |           | 0.500000  | 0.500000  |           |           |
|                       |       |       |           | -0.500000 | 0.500000  |           |           |
|                       | 2     |       |           | 0.124377  | 0.751246  | 0.124377  |           |
|                       |       |       |           | -0.432291 | 0.000000  | 0.432291  |           |
|                       |       |       |           | 0.375623  | -0.751246 | 0.375623  |           |
|                       | 3     |       |           | 0.000649  | 0.499351  | 0.499351  | 0.000649  |
|                       |       |       | -0.033676 | -0.803260 | 0.803260  | 0.033676  |           |
|                       |       |       | 0.453110  | -0.453110 | -0.453110 | 0.453110  |           |
|                       |       |       | -0.420083 | 0.849501  | -0.849501 | 0.420083  |           |
| 4                     |       |       | 0.035652  | 0.146138  | 0.636420  | 0.146138  | 0.035652  |
|                       |       |       | 0.149222  | -0.808622 | 0.000000  | 0.808622  | -0.149222 |
|                       |       |       | -0.125047 | 0.972447  | -1.694798 | 0.972447  | -0.125047 |
|                       |       |       | -0.588448 | 0.808622  | 0.000000  | -0.808622 | 0.588448  |
|                       |       |       | 0.554424  | -1.118584 | 1.128320  | -1.118584 | 0.554424  |
| 5                     |       |       | 0.046170  | 0.000790  | 0.453040  | 0.453040  | 0.000790  |
|                       |       |       | -0.030551 | -0.064281 | -1.047189 | 1.047189  | 0.064281  |
|                       |       |       | -0.376843 | 1.346724  | -0.969881 | -0.969881 | 1.346724  |
|                       |       |       | 0.299901  | -1.536987 | 2.717358  | -2.717358 | 1.536987  |
|                       |       |       | 0.773030  | -1.326775 | 0.553745  | 0.553745  | -1.326775 |
|                       |       |       | -0.711707 | 1.622007  | -1.707074 | 1.707074  | -1.622007 |
| 6                     |       |       | 0.018148  | 0.034239  | 0.143952  | 0.607323  | 0.143952  |
|                       |       |       | -0.137219 | 0.252439  | -1.058872 | 0.000000  | 1.058872  |
|                       |       |       | 0.112414  | -0.122731 | 1.632940  | -3.245245 | 1.632940  |
|                       |       |       | 0.836717  | -2.293984 | 2.375355  | 0.000000  | -2.375355 |
|                       |       |       | -0.765241 | 2.327842  | -4.313797 | 5.502391  | -4.313797 |
|                       |       |       | -1.138248 | 2.041545  | -1.331193 | 0.000000  | 1.331193  |
|                       |       |       | 1.088642  | -2.239350 | 2.582942  | -2.864469 | 2.582942  |
| <b>B<sup>-1</sup></b> | 1     |       |           | 1.000000  | -1.000000 |           |           |
|                       |       |       |           | 1.000000  | 1.000000  |           |           |
|                       | 2     |       |           | 1.000000  | -1.156628 | 1.000000  |           |
|                       |       |       |           | 1.000000  | 0.000000  | -0.331122 |           |
|                       |       |       |           | 1.000000  | 1.156628  | 1.000000  |           |
|                       | 3     |       |           | 1.000000  | -1.160380 | 1.102051  | -1.097218 |
|                       |       |       | 1.000000  | -0.573815 | -0.001433 | 0.046001  |           |
|                       |       |       | 1.000000  | 0.573815  | -0.001433 | -0.046001 |           |
|                       |       |       | 1.000000  | 1.160380  | 1.102051  | 1.097218  |           |
| 4                     |       |       | 1.000000  | -1.138365 | 1.097973  | -1.138365 | 1.085173  |
|                       |       |       | 1.000000  | -0.828408 | 0.391487  | -0.210072 | 0.023992  |
|                       |       |       | 1.000000  | -0.000000 | -0.302808 | 0.000000  | -0.132602 |
|                       |       |       | 1.000000  | 0.828408  | 0.391487  | 0.210072  | 0.023992  |
|                       |       |       | 1.000000  | 1.138365  | 1.097973  | 1.138365  | 1.085173  |
| 5                     |       |       | 1.000000  | -1.140533 | 1.111718  | -1.156170 | 1.129029  |
|                       |       |       | 1.000000  | -0.909198 | 0.600007  | -0.443176 | 0.232770  |
|                       |       |       | 1.000000  | -0.388384 | -0.114344 | 0.060935  | -0.115468 |
|                       |       |       | 1.000000  | 0.388384  | -0.114344 | -0.060935 | -0.115468 |
|                       |       |       | 1.000000  | 0.909198  | 0.600007  | 0.443176  | 0.232770  |
|                       |       |       | 1.000000  | 1.140533  | 1.111718  | 1.156170  | 1.129029  |
| 6                     |       |       | 1.000000  | -1.120708 | 1.091898  | -1.139451 | 1.112410  |
|                       |       |       | 1.000000  | -0.992297 | 0.764313  | -0.638222 | 0.458383  |
|                       |       |       | 1.000000  | -0.563536 | 0.093807  | -0.004494 | -0.108458 |
|                       |       |       | 1.000000  | 0.000000  | -0.195905 | 0.000000  | -0.066752 |
|                       |       |       | 1.000000  | 0.563536  | 0.093807  | 0.004494  | -0.108458 |
|                       |       |       | 1.000000  | 0.992297  | 0.764313  | 0.638222  | 0.458383  |
|                       |       |       | 1.000000  | 1.120708  | 1.091898  | 1.139451  | 1.112410  |

Table 6: Values of the matrices **B** and **B<sup>-1</sup>** for the ApproxMinVo basis functions from order 1 to 6.

| matrice               | order | value |           |           |           |           |            |            |             |  |
|-----------------------|-------|-------|-----------|-----------|-----------|-----------|------------|------------|-------------|--|
| <b>B</b>              | 1     |       |           |           |           |           | 0.500000   | 0.500000   |             |  |
|                       |       |       |           |           |           |           | -0.500000  | 0.500000   |             |  |
|                       | 2     |       |           |           | 0.500000  | 0.250000  | 0.250000   |            |             |  |
|                       |       |       |           |           | -0.500000 | 0.000000  | 0.500000   |            |             |  |
|                       |       |       |           |           | 0.000000  | -0.250000 | 0.250000   |            |             |  |
|                       | 3     |       |           | 0.500000  | 0.250000  | 0.125000  | 0.125000   |            |             |  |
|                       |       |       | -0.500000 | 0.000000  | 0.125000  | 0.375000  |            |            |             |  |
|                       |       |       | 0.000000  | -0.250000 | -0.125000 | 0.375000  |            |            |             |  |
|                       |       |       | 0.000000  | 0.000000  | -0.125000 | 0.125000  |            |            |             |  |
| 4                     |       |       | 0.500000  | 0.250000  | 0.125000  | 0.062500  | 0.062500   |            |             |  |
|                       |       |       | -0.500000 | 0.000000  | 0.125000  | 0.125000  | 0.250000   |            |             |  |
|                       |       |       | 0.000000  | -0.250000 | -0.125000 | 0.000000  | 0.375000   |            |             |  |
|                       |       |       | 0.000000  | 0.000000  | -0.125000 | -0.125000 | 0.250000   |            |             |  |
|                       |       |       | 0.000000  | 0.000000  | 0.000000  | -0.062500 | 0.062500   |            |             |  |
| 5                     |       |       | 0.500000  | 0.250000  | 0.125000  | 0.062500  | 0.031250   | 0.031250   |             |  |
|                       |       |       | -0.500000 | 0.000000  | 0.125000  | 0.125000  | 0.093750   | 0.156250   |             |  |
|                       |       |       | 0.000000  | -0.250000 | -0.125000 | 0.000000  | 0.062500   | 0.312500   |             |  |
|                       |       |       | 0.000000  | 0.000000  | -0.125000 | -0.125000 | -0.062500  | 0.312500   |             |  |
|                       |       |       | 0.000000  | 0.000000  | 0.000000  | -0.062500 | -0.093750  | 0.156250   |             |  |
|                       |       |       | 0.000000  | 0.000000  | 0.000000  | 0.000000  | -0.031250  | 0.031250   |             |  |
| 6                     |       |       | 0.500000  | 0.250000  | 0.125000  | 0.062500  | 0.031250   | 0.015625   | 0.015625    |  |
|                       |       |       | -0.500000 | 0.000000  | 0.125000  | 0.125000  | 0.093750   | 0.062500   | 0.093750    |  |
|                       |       |       | 0.000000  | -0.250000 | -0.125000 | 0.000000  | 0.062500   | 0.078125   | 0.234375    |  |
|                       |       |       | 0.000000  | 0.000000  | -0.125000 | -0.125000 | -0.062500  | 0.000000   | 0.312500    |  |
|                       |       |       | 0.000000  | 0.000000  | 0.000000  | -0.062500 | -0.093750  | -0.078125  | 0.234375    |  |
|                       |       |       | 0.000000  | 0.000000  | 0.000000  | 0.000000  | -0.031250  | -0.062500  | 0.093750    |  |
|                       |       |       | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 0.000000   | -0.015625  | 0.015625    |  |
| <b>B<sup>-1</sup></b> | 1     |       |           |           |           |           | 1.000000   | -1.000000  |             |  |
|                       |       |       |           |           |           |           | 1.000000   | 1.000000   |             |  |
|                       | 2     |       |           |           | 1.000000  | -1.000000 | 1.000000   |            |             |  |
|                       |       |       |           |           | 1.000000  | 1.000000  | -3.000000  |            |             |  |
|                       |       |       |           |           | 1.000000  | 1.000000  | 1.000000   |            |             |  |
|                       | 3     |       |           | 1.000000  | -1.000000 | 1.000000  | -1.000000  |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | -3.000000 | 5.000000  |            |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | -7.000000 |            |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  |            |            |             |  |
| 4                     |       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | -3.000000 | 5.000000  | -7.000000  |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | -7.000000 | 17.000000  |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | -15.000000 |            |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   |            |             |  |
| 5                     |       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | -1.000000  |             |  |
|                       |       |       | 1.000000  | 1.000000  | -3.000000 | 5.000000  | -7.000000  | 9.000000   |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | -7.000000 | 17.000000  | -31.000000 |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | -15.000000 | 49.000000  |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | -31.000000 |             |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | 1.000000   |             |  |
| 6                     |       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | -1.000000  | 1.000000    |  |
|                       |       |       | 1.000000  | 1.000000  | -3.000000 | 5.000000  | -7.000000  | 9.000000   | -11.000000  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | -7.000000 | 17.000000  | -31.000000 | 49.000000   |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | -15.000000 | 49.000000  | -111.000000 |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | -31.000000 | 129.000000  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | 1.000000   | -63.000000  |  |
|                       |       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | 1.000000   | 1.000000    |  |

Table 7: Values of the matrices **B** and **B<sup>-1</sup>** for the Recursive basis functions from order 1 to 6.

| matrice               | order | value     |           |           |           |            |            |             |  |
|-----------------------|-------|-----------|-----------|-----------|-----------|------------|------------|-------------|--|
| <b>B</b>              | 1     |           |           | 0.500000  | 0.500000  |            |            |             |  |
|                       |       |           |           | -0.500000 | 0.500000  |            |            |             |  |
|                       | 2     |           |           | 0.250000  | 0.250000  | 0.500000   |            |             |  |
|                       |       |           |           | -0.500000 | 0.000000  | 0.500000   |            |             |  |
|                       |       |           |           | 0.250000  | -0.250000 | 0.000000   |            |             |  |
|                       | 3     |           |           | 0.125000  | 0.125000  | 0.250000   | 0.500000   |             |  |
|                       |       |           | -0.375000 | -0.125000 | 0.000000  | 0.500000   |            |             |  |
|                       |       |           | 0.375000  | -0.125000 | -0.250000 | 0.000000   |            |             |  |
|                       |       |           | -0.125000 | 0.125000  | 0.000000  | 0.000000   |            |             |  |
| 4                     |       |           | 0.062500  | 0.062500  | 0.125000  | 0.250000   | 0.500000   |             |  |
|                       |       |           | -0.250000 | -0.125000 | -0.125000 | 0.000000   | 0.500000   |             |  |
|                       |       |           | 0.375000  | 0.000000  | -0.125000 | -0.250000  | 0.000000   |             |  |
|                       |       |           | -0.250000 | 0.125000  | 0.125000  | 0.000000   | 0.000000   |             |  |
|                       |       |           | 0.062500  | -0.062500 | 0.000000  | 0.000000   | 0.000000   |             |  |
| 5                     |       | 0.031250  | 0.031250  | 0.062500  | 0.125000  | 0.250000   | 0.500000   |             |  |
|                       |       | -0.156250 | -0.093750 | -0.125000 | -0.125000 | 0.000000   | 0.500000   |             |  |
|                       |       | 0.312500  | 0.062500  | 0.000000  | -0.125000 | -0.250000  | 0.000000   |             |  |
|                       |       | -0.312500 | 0.062500  | 0.125000  | 0.125000  | 0.000000   | 0.000000   |             |  |
|                       |       | 0.156250  | -0.093750 | -0.062500 | 0.000000  | 0.000000   | 0.000000   |             |  |
|                       |       | -0.031250 | 0.031250  | 0.000000  | 0.000000  | 0.000000   | 0.000000   |             |  |
| 6                     |       | 0.015625  | 0.015625  | 0.031250  | 0.062500  | 0.125000   | 0.250000   | 0.500000    |  |
|                       |       | -0.093750 | -0.062500 | -0.093750 | -0.125000 | -0.125000  | 0.000000   | 0.500000    |  |
|                       |       | 0.234375  | 0.078125  | 0.062500  | 0.000000  | -0.125000  | -0.250000  | 0.000000    |  |
|                       |       | -0.312500 | 0.000000  | 0.062500  | 0.125000  | 0.125000   | 0.000000   | 0.000000    |  |
|                       |       | 0.234375  | -0.078125 | -0.093750 | -0.062500 | 0.000000   | 0.000000   | 0.000000    |  |
|                       |       | -0.093750 | 0.062500  | 0.031250  | 0.000000  | 0.000000   | 0.000000   | 0.000000    |  |
|                       |       | 0.015625  | -0.015625 | 0.000000  | 0.000000  | 0.000000   | 0.000000   | 0.000000    |  |
| <b>B<sup>-1</sup></b> | 1     |           |           | 1.000000  | -1.000000 |            |            |             |  |
|                       |       |           |           | 1.000000  | 1.000000  |            |            |             |  |
|                       | 2     |           |           | 1.000000  | -1.000000 | 1.000000   |            |             |  |
|                       |       |           |           | 1.000000  | -1.000000 | -3.000000  |            |             |  |
|                       |       |           |           | 1.000000  | 1.000000  | 1.000000   |            |             |  |
|                       | 3     |           |           | 1.000000  | -1.000000 | 1.000000   | -1.000000  |             |  |
|                       |       |           | 1.000000  | -1.000000 | 1.000000  | 7.000000   |            |             |  |
|                       |       |           | 1.000000  | -1.000000 | -3.000000 | -5.000000  |            |             |  |
|                       |       |           | 1.000000  | 1.000000  | 1.000000  | 1.000000   |            |             |  |
| 4                     |       |           | 1.000000  | -1.000000 | 1.000000  | -1.000000  | 1.000000   |             |  |
|                       |       |           | 1.000000  | -1.000000 | 1.000000  | -1.000000  | -15.000000 |             |  |
|                       |       |           | 1.000000  | -1.000000 | 1.000000  | 7.000000   | 17.000000  |             |  |
|                       |       |           | 1.000000  | -1.000000 | -3.000000 | -5.000000  | -7.000000  |             |  |
|                       |       |           | 1.000000  | 1.000000  | 1.000000  | 1.000000   | 1.000000   |             |  |
| 5                     |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | -1.000000  |             |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | 31.000000  |             |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | -15.000000 | -49.000000 |             |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | 7.000000  | 17.000000  | 31.000000  |             |  |
|                       |       | 1.000000  | -1.000000 | -3.000000 | -5.000000 | -7.000000  | -9.000000  |             |  |
|                       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | 1.000000   |             |  |
| 6                     |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | -1.000000  | 1.000000    |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | -1.000000  | -63.000000  |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | 1.000000   | 31.000000  | 129.000000  |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | -1.000000 | -15.000000 | -49.000000 | -111.000000 |  |
|                       |       | 1.000000  | -1.000000 | 1.000000  | 7.000000  | 17.000000  | 31.000000  | 49.000000   |  |
|                       |       | 1.000000  | -1.000000 | -3.000000 | -5.000000 | -7.000000  | -9.000000  | -11.000000  |  |
|                       |       | 1.000000  | 1.000000  | 1.000000  | 1.000000  | 1.000000   | 1.000000   | 1.000000    |  |

Table 8: Values of the matrices **B** and **B<sup>-1</sup>** for the Recursive2 basis functions from order 1 to 6.

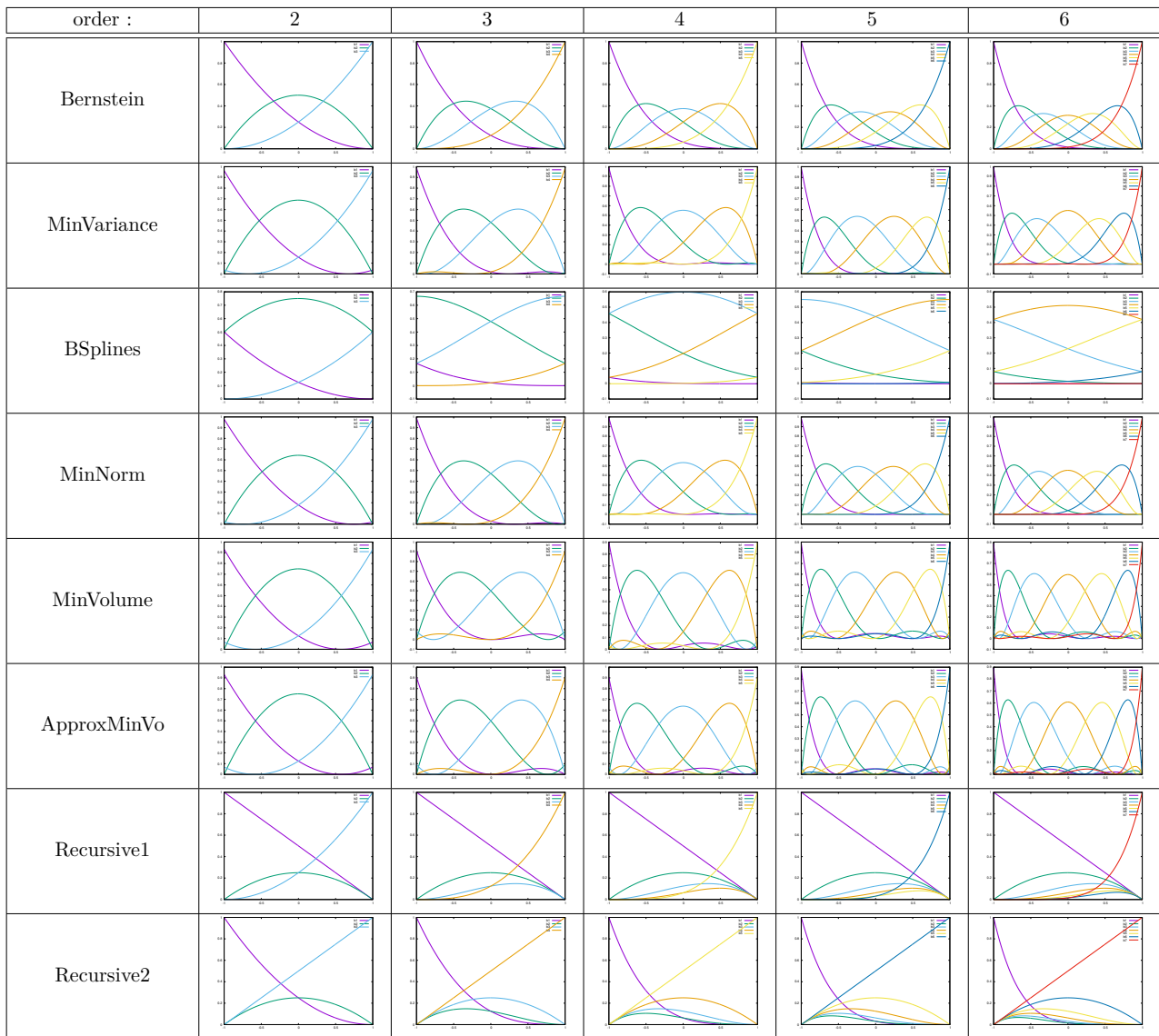


Table 9: Representation of the basis functions for the order 2 to 6

| order | type        | MinVolume<br>$-\det(\mathbf{B})^2$ | MinNorm<br>$\ \mathbf{B}^{-1}\ ^2$ | MinVariance<br>$\ \mathbf{B}^{-1} - \text{Average}(\mathbf{B}^{-1})\ ^2$ |
|-------|-------------|------------------------------------|------------------------------------|--|
| 1     | Bernstein   | -0.25                              | 4                                  | 2  |
|       | MinVariance | -0.25                              | 4                                  | <b>2</b>   |
|       | BSplines    | -0.25                              | 4                                  | 2  |
|       | MinNorm     | -0.25                              | <b>4</b>                           | 2  |
|       | MinVolume   | <b>-0.25</b>                       | 4                                  | 2  |
|       | ApproxMinVo | -0.25                              | 4                                  | 2  |
|       | Recursive   | -0.25                              | 4                                  | 2  |
|       | Recursive2  | -0.25                              | 4                                  | 2  |
| 2     | Bernstein   | -0.0625                            | 8                                  | 4.66667  |
|       | MinVariance | -0.101637                          | 7.53553                            | <b>3.73773</b>   |
|       | BSplines    | -0.00390625                        | 30                                 | 18.6667  |
|       | MinNorm     | -0.094799                          | <b>7.48641</b>                     | 3.79204  |
|       | MinVolume   | <b>-0.105469</b>                   | 7.77778                            | 3.85185  |
|       | ApproxMinVo | -0.105467                          | 7.78522                            | 3.85684  |
|       | Recursive   | -0.015625                          | 17                                 | 13.3333  |
|       | Recursive2  | -0.015625                          | 17                                 | 13.3333  |
| 3     | Bernstein   | -0.0197754                         | 12.4444                            | 8  |
|       | MinVariance | -0.0776098                         | 10.9429                            | <b>5.99574</b>   |
|       | BSplines    | -1.69542e-06                       | 609.778                            | 552  |
|       | MinNorm     | -0.0703759                         | <b>10.9231</b>                     | 6.01885  |
|       | MinVolume   | <b>-0.110146</b>                   | 12.1142                            | 6.91991  |
|       | ApproxMinVo | -0.110036                          | 12.1925                            | 6.98117  |
|       | Recursive   | -0.000244141                       | 96                                 | 90   |
|       | Recursive2  | -0.000244141                       | 96                                 | 90   |
| 4     | Bernstein   | -0.00878906                        | 17.1111                            | 11.3556  |
|       | MinVariance | -0.138639                          | 14.5243                            | <b>7.75466</b>   |
|       | BSplines    | -1.14978e-11                       | 26930.6                            | 19580.8  |
|       | MinNorm     | -0.101192                          | <b>14.449</b>                      | 7.84653  |
|       | MinVolume   | <b>-0.322429</b>                   | 16.826                             | 9.49568  |
|       | ApproxMinVo | -0.321755                          | 16.8275                            | 9.52516  |
|       | Recursive   | -9.53674e-07                       | 665                                | 656  |
|       | Recursive2  | -9.53674e-07                       | 665                                | 656  |
| 5     | Bernstein   | -0.00582077                        | 21.92                              | 15.0133  |
|       | MinVariance | -0.477859                          | 18.1917                            | <b>10.1726</b>   |
|       | BSplines    | -7.79745e-19                       | 2.06027e+06                        | 1.98912e+06  |
|       | MinNorm     | -0.255511                          | <b>18.0599</b>                     | 10.3167  |
|       | MinVolume   | <b>-2.88564</b>                    | 22.0832                            | 13.3504  |
|       | ApproxMinVo | -2.86456                           | 22.1827                            | 13.446   |
|       | Recursive   | -9.31323e-10                       | 5076                               | 5063.33  |
|       | Recursive2  | -9.31323e-10                       | 5076                               | 5063.33  |
| 6     | Bernstein   | -0.00596719                        | 26.8444                            | 18.6438  |
|       | MinVariance | -3.65267                           | 22.0119                            | <b>12.2725</b>   |
|       | BSplines    | -3.67221e-28                       | 2.40738e+08                        | 1.87088e+08  |
|       | MinNorm     | -1.19714                           | <b>21.7376</b>                     | 12.5152  |
|       | MinVolume   | <b>-82.8592</b>                    | 27.6542                            | 16.6707  |
|       | ApproxMinVo | -82.3749                           | 27.5176                            | 16.5483  |
|       | Recursive   | -2.27374e-13                       | 40537                              | 40520  |
|       | Recursive2  | -2.27374e-13                       | 40537                              | 40520  |

Table 10: Values of the different criteria from the basis function from order 1 to 6.



| problem | center of the target | shape of the target | criteria            |
|---------|----------------------|---------------------|---------------------|
| 1       | {0.2; 1.7}           | square              | $\sum_i q_i^2$      |
| 2       | {0.4; 1.4}           | square              | $\sum_i q_i^2$      |
| 3       | {0.6; 1.1}           | square              | $\sum_i q_i^2$      |
| 4       | {0.2; 1.7}           | circle              | $\sum_i q_i^2$      |
| 5       | {0.4; 1.4}           | circle              | $\sum_i q_i^2$      |
| 6       | {0.6; 1.1}           | circle              | $\sum_i q_i^2$      |
| 7       | {0.2; 1.7}           | ring                | $\sum_i q_i^2$      |
| 8       | {0.4; 1.4}           | ring                | $\sum_i q_i^2$      |
| 9       | {0.6; 1.1}           | ring                | $\sum_i q_i^2$      |
| 11      | {0.2; 1.7}           | square              | $\sum_i \Gamma_i^2$ |
| 12      | {0.4; 1.4}           | square              | $\sum_i \Gamma_i^2$ |
| 13      | {0.6; 1.1}           | square              | $\sum_i \Gamma_i^2$ |
| 14      | {0.2; 1.7}           | circle              | $\sum_i \Gamma_i^2$ |
| 15      | {0.4; 1.4}           | circle              | $\sum_i \Gamma_i^2$ |
| 16      | {0.6; 1.1}           | circle              | $\sum_i \Gamma_i^2$ |
| 17      | {0.2; 1.7}           | ring                | $\sum_i \Gamma_i^2$ |
| 18      | {0.4; 1.4}           | ring                | $\sum_i \Gamma_i^2$ |
| 19      | {0.6; 1.1}           | ring                | $\sum_i \Gamma_i^2$ |

Table 11: Problem list and their specificities.







| precision | ndof | solver          | prep_time       | comput_time | total_time | (D-H:M:S.ms) | nb_iter     | time_per_iter | criteria  | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------|------|-----------------|-----------------|-------------|------------|--------------|-------------|---------------|-----------|-----------------|----------------|-------------|-------|
| 0.001     | 2    | Interval        | 30.99e-6        | 16.64e-3    | 16.67e-3   | 0.017        | 2383        | 6.981e-6      | 1.0755    | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 764.4e-3        | 6.178e-3    | 770.6e-3   | 0.771        | 817         | 7.562e-6      | 1.0716    | 37.14           | 4623.5         | 34.28       |       |
|           |      | MinVariance     | 842.4e-3        | 5.851e-3    | 848.2e-3   | 0.848        | 877         | 6.672e-6      | 1.0717    | 35.17           | 5089.2         | 36.80       |       |
|           |      | BSplines        | 863.2e-3        | 15.61e-3    | 878.9e-3   | 0.879        | 2039        | 7.657e-6      | 1.0735    | 93.85           | 5273.0         | 85.56       |       |
|           |      | MinNorm         | 954.0e-3        | 7.705e-3    | 961.8e-3   | 0.962        | 853         | 9.033e-6      | 1.0717    | 46.31           | 5770.4         | 35.80       |       |
|           |      | MinVolume       | 920.7e-3        | 7.322e-3    | 928.0e-3   | 0.928        | 955         | 7.667e-6      | 1.0718    | 44.01           | 5567.9         | 40.08       |       |
|           |      | ApproxMinVolume | 893.4e-3        | 6.409e-3    | 899.8e-3   | 0.900        | 957         | 6.697e-6      | 1.0718    | 38.52           | 5398.6         | 40.16       |       |
|           |      | Recursive       | 873.4e-3        | 7.287e-3    | 880.7e-3   | 0.881        | 1033        | 7.054e-6      | 1.0716    | 43.80           | 5283.9         | 43.35       |       |
|           |      | Recursive2      | 860.3e-3        | 7.059e-3    | 867.4e-3   | 0.867        | 1033        | 6.833e-6      | 1.0716    | 42.43           | 5204.0         | 43.35       |       |
|           | 3    | Interval        | 23.13e-6        | 4.811       | 4.811      | 4.811        | 711195      | 6.764e-6      | 741.67e-3 | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 2.985           | 2.027       | 5.012      | 5.012        | 193081      | 10.50e-6      | 738.33e-3 | 42.14           | 104.2          | 27.15       |       |
|           |      | MinVariance     | 4.365           | 2.589       | 6.954      | 6.954        | 223843      | 11.56e-6      | 738.62e-3 | 53.81           | 144.5          | 31.47       |       |
|           |      | BSplines        | 3.130           | 8.459       | 11.59      | 11.589       | 775983      | 10.90e-6      | 742.24e-3 | 175.8           | 240.9          | 109.1       |       |
|           |      | MinNorm         | 4.151           | 2.365       | 6.516      | 6.516        | 208239      | 11.36e-6      | 738.47e-3 | 49.17           | 135.5          | 29.28       |       |
|           |      | MinVolume       | 3.412           | 2.716       | 6.128      | 6.128        | 255471      | 10.63e-6      | 738.94e-3 | 56.46           | 127.4          | 35.92       |       |
|           |      | ApproxMinVolume | 3.387           | 2.728       | 6.116      | 6.116        | 256251      | 10.65e-6      | 738.94e-3 | 56.72           | 127.1          | 36.03       |       |
|           |      | Recursive       | 3.437           | 2.278       | 5.716      | 5.716        | 209177      | 10.89e-6      | 738.33e-3 | 47.36           | 118.8          | 29.41       |       |
|           |      | Recursive2      | 3.664           | 2.224       | 5.888      | 5.888        | 203103      | 10.95e-6      | 738.33e-3 | 46.23           | 122.4          | 28.56       |       |
|           | 4    | Interval        | 29.09e-6        | 1060        | 1060       | 17.40        | 127570319   | 8.315e-6      | 621.32e-3 | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 3.980           | 372.9       | 376.9      | 06.16        | 21728921    | 17.16e-6      | 617.74e-3 | 35.15           | 35.53          | 17.03       |       |
|           |      | MinVariance     | 5.156           | 449.7       | 454.8      | 07.34        | 26186913    | 17.17e-6      | 618.08e-3 | 42.39           | 42.88          | 20.53       |       |
|           |      | BSplines        | 3.946           | 2073        | 2077       | 34.37        | 123119939   | 16.84e-6      | 621.91e-3 | 195.5           | 195.9          | 96.51       |       |
|           |      | MinNorm         | 5.077           | 405.0       | 410.1      | 06.50        | 24001845    | 16.87e-6      | 617.91e-3 | 38.18           | 38.66          | 18.81       |       |
|           |      | MinVolume       | 4.473           | 537.7       | 542.2      | 09.02        | 30954991    | 17.37e-6      | 618.38e-3 | 50.70           | 51.12          | 24.27       |       |
|           |      | ApproxMinVolume | 4.623           | 523.6       | 528.2      | 08.48        | 31078695    | 16.85e-6      | 618.38e-3 | 49.36           | 49.80          | 24.36       |       |
|           |      | Recursive       | 4.062           | 396.9       | 400.9      | 06.40        | 23405821    | 16.96e-6      | 617.74e-3 | 37.42           | 37.80          | 18.35       |       |
|           |      | Recursive2      | 4.028           | 449.1       | 453.1      | 07.33        | 25972355    | 17.29e-6      | 617.74e-3 | 42.34           | 42.72          | 20.36       |       |
|           | 5    | Interval        | 27.00e-6        | 279920      | 279920     | 3-05:45:20   | 28828576484 | 9.710e-6      | 529.85e-3 | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 59.44           | 184551      | 184551     | 2-03:15:51   | 4449324341  | 41.48e-6      | 526.69e-3 | 65.93           | 65.93          | 15.43       |       |
|           |      | MinVariance     | 65.23           | 234787      | 234787     | 2-17:13:07   | 5621872305  | 41.76e-6      | 527.05e-3 | 83.88           | 83.88          | 19.50       |       |
|           |      | BSplines        | 56.03           | 1525850     | 1525850    | 17-15:50:50  | 37268701884 | 40.94e-6      | 531.29e-3 | 545.1           | 545.1          | 129.3       |       |
|           |      | MinNorm         | 66.44           | 215741      | 215741     | 2-11:55:41   | 50736282993 | 42.52e-6      | 526.90e-3 | 77.07           | 77.07          | 17.60       |       |
|           |      | MinVolume       | 68.35           | 296081      | 296081     | 3-10:14:41   | 6915560218  | 42.81e-6      | 527.38e-3 | 105.8           | 105.8          | 23.99       |       |
|           |      | ApproxMinVolume | 65.48           | 288926      | 288927     | 3-08:15:27   | 6949340844  | 41.58e-6      | 527.39e-3 | 103.2           | 103.2          | 24.11       |       |
|           |      | Recursive       | 60.67           | 197947      | 197947     | 2-06:59:07   | 4766507865  | 41.53e-6      | 526.69e-3 | 70.72           | 70.72          | 16.53       |       |
|           |      | Recursive2      | 61.72           | 220353      | 220353     | 2-13:12:33   | 5287820395  | 41.67e-6      | 526.69e-3 | 78.72           | 78.72          | 18.34       |       |
|           | 0.01 | 2               | Interval        | 34.09e-6    | 9.222e-3   | 9.256e-3     | 0.009       | 899           | 10.26e-6  | 1.1306          | 100.0          | 100.0       | 100.0 |
|           |      |                 | Bernstein       | 1.066       | 4.494e-3   | 1.070        | 1.070       | 361           | 12.45e-6  | 1.0927          | 48.73          | 11560.2     | 40.16 |
|           |      |                 | MinVariance     | 1.145       | 4.969e-3   | 1.150        | 1.150       | 385           | 12.91e-6  | 1.0933          | 53.88          | 12422.6     | 42.83 |
|           |      |                 | BSplines        | 1.066       | 7.612e-3   | 1.073        | 1.073       | 913           | 8.337e-6  | 1.1115          | 82.54          | 11595.7     | 101.6 |
|           |      |                 | MinNorm         | 1.124       | 4.219e-3   | 1.128        | 1.128       | 375           | 11.25e-6  | 1.0930          | 45.75          | 12186.5     | 41.71 |
|           |      |                 | MinVolume       | 1.108       | 4.839e-3   | 1.113        | 1.113       | 425           | 11.39e-6  | 1.0940          | 52.47          | 12027.7     | 47.27 |
|           |      |                 | ApproxMinVolume | 1.088       | 5.074e-3   | 1.093        | 1.093       | 425           | 11.94e-6  | 1.0940          | 55.02          | 11812.5     | 47.27 |
|           |      |                 | Recursive       | 1.066       | 5.791e-3   | 1.071        | 1.071       | 559           | 10.36e-6  | 1.0927          | 62.79          | 11574.8     | 62.18 |
|           |      |                 | Recursive2      | 1.106       | 7.776e-3   | 1.114        | 1.114       | 577           | 13.48e-6  | 1.0927          | 84.32          | 12036.7     | 64.18 |
| 3         |      | Interval        | 33.14e-6        | 762.0e-3    | 762.0e-3   | 0.762        | 106721      | 7.140e-6      | 794.69e-3 | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 2.911           | 342.0e-3    | 3.253      | 3.253        | 30803       | 11.10e-6      | 766.06e-3 | 44.89           | 426.9          | 28.86       |       |
|           |      | MinVariance     | 4.297           | 434.5e-3    | 4.731      | 4.731        | 35577       | 12.21e-6      | 768.90e-3 | 57.03           | 620.9          | 33.34       |       |
|           |      | BSplines        | 3.210           | 1.439       | 4.650      | 4.650        | 130221      | 11.05e-6      | 797.06e-3 | 188.9           | 610.2          | 122.0       |       |
|           |      | MinNorm         | 4.267           | 393.1e-3    | 4.660      | 4.660        | 33395       | 11.77e-6      | 768.21e-3 | 51.60           | 611.6          | 31.29       |       |
|           |      | MinVolume       | 3.654           | 487.5e-3    | 4.141      | 4.141        | 41623       | 11.71e-6      | 770.54e-3 | 63.98           | 543.4          | 39.00       |       |
|           |      | ApproxMinVolume | 3.627           | 497.5e-3    | 4.125      | 4.125        | 41803       | 11.90e-6      | 770.68e-3 | 65.29           | 541.3          | 39.17       |       |
|           |      | Recursive       | 3.513           | 499.7e-3    | 4.013      | 4.013        | 42361       | 11.80e-6      | 766.06e-3 | 65.58           | 526.7          | 39.69       |       |
|           |      | Recursive2      | 3.741           | 500.4e-3    | 4.241      | 4.241        | 39681       | 12.61e-6      | 766.06e-3 | 65.67           | 556.6          | 37.18       |       |
| 4         |      | Interval        | 22.17e-6        | 67.97       | 67.97      | 01.07        | 7790067     | 8.725e-6      | 678.03e-3 | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 3.987           | 23.07       | 27.06      | 27.058       | 1369727     | 16.84e-6      | 647.64e-3 | 33.95           | 39.81          | 17.58       |       |
|           |      | MinVariance     | 5.235           | 27.03       | 32.27      | 32.269       | 1577077     | 17.14e-6      | 649.64e-3 | 39.78           | 47.48          | 20.24       |       |
|           |      | BSplines        | 4.078           | 152.7       | 156.8      | 02:36        | 9051187     | 16.87e-6      | 678.47e-3 | 224.7           | 230.7          | 116.2       |       |
|           |      | MinNorm         | 5.032           | 24.99       | 30.02      | 30.019       | 1448057     | 17.26e-6      | 648.77e-3 | 36.76           | 44.17          | 18.59       |       |
|           |      | MinVolume       | 4.597           | 33.28       | 37.88      | 37.876       | 1913031     | 17.40e-6      | 651.67e-3 | 48.96           | 55.73          | 24.56       |       |
|           |      | ApproxMinVolume | 4.735           | 33.69       | 38.43      | 38.428       | 1921565     | 17.53e-6      | 651.69e-3 | 49.57           | 56.54          | 24.67       |       |
|           |      | Recursive       | 4.172           | 43.24       | 47.41      | 47.408       | 2509615     | 17.23e-6      | 647.64e-3 | 63.62           | 69.75          | 32.22       |       |
|           |      | Recursive2      | 4.083           | 59.19       | 63.27      | 01:03        | 3398545     | 17.42e-6      | 647.64e-3 | 87.08           | 93.09          | 43.63       |       |
| 5         |      | Interval        | 24.08e-6        | 6106        | 6106       | 01:41:46     | 653254743   | 9.348e-6      | 587.12e-3 | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 54.65           | 4499        | 4553       | 01:15:53     | 106824699   | 42.12e-6      | 559.75e-3 | 73.68           | 74.57          | 16.35       |       |
|           |      | MinVariance     | 60.05           | 5599        | 5660       | 01:34:20     | 131642925   | 42.54e-6      | 562.22e-3 | 91.71           | 92.69          | 20.15       |       |
|           |      | BSplines        | 55.14           | 49263       | 49319      | 13:41:59     | 1145719257  | 43.00e-6      | 595.66e-3 | 806.8           | 807.7          | 175.4       |       |
|           |      | MinNorm         | 59.58           | 4969        | 5029       | 01:23:49     | 116862765   | 42.53e-6      | 561.16e-3 | 81.38           | 82.36          | 17.89       |       |
|           |      | MinVolume       | 57.21           | 7161        | 7218       | 02:00:18     | 167099243   | 42.86e-6      | 564.56e-3 | 117.3           | 118.2          | 25.58       |       |
|           |      | ApproxMinVolume | 57.63           | 7515        | 7573       | 02:06:13     | 168217769   | 44.68e-6      | 564.77e-3 | 123.1           | 124.0          | 25.75       |       |
|           |      | Recursive       | 55.21           | 8476        | 8531       | 02:22:11     | 203131201   | 41.73e-6      | 559.75e-3 | 138.8           | 139.7          | 31.10       |       |
|           |      | Recursive2      | 55.43           | 12781       | 12837      | 03:33:57     | 301335357   | 42.42e-6      | 559.75e-3 | 209.3           | 210.2          | 46.13       |       |

Table 15: Bissection of problem 2D with problem: 4, type: MinFirst

| precision       | ndof            | solver          | prep_time | comput_time | total_time  | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------------|-----------|-------------|-------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | 2               | Interval        | 26.94e-6  | 18.85e-3    | 18.88e-3    | 0.019        | 2661     | 7.085e-6      | 2.7566   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 847.4e-3  | 7.550e-3    | 854.9e-3    | 0.855        | 1229     | 6.143e-6      | 2.7533   | 40.04           | 4528.1         | 46.19       |
|                 |                 | MinVariance     | 948.1e-3  | 8.439e-3    | 956.6e-3    | 0.957        | 1351     | 6.247e-6      | 2.7535   | 44.76           | 5066.3         | 50.77       |
|                 |                 | BSplines        | 863.5e-3  | 15.00e-3    | 878.6e-3    | 0.879        | 3251     | 4.614e-6      | 2.7564   | 79.55           | 4653.1         | 122.2       |
|                 |                 | MinNorm         | 925.9e-3  | 8.227e-3    | 934.2e-3    | 0.934        | 1295     | 6.353e-6      | 2.7534   | 43.64           | 4947.6         | 48.67       |
|                 |                 | MinVolume       | 907.5e-3  | 9.963e-3    | 917.4e-3    | 0.917        | 1851     | 5.382e-6      | 2.7536   | 52.84           | 4859.0         | 69.56       |
|                 |                 | ApproxMinVolume | 900.4e-3  | 9.286e-3    | 909.7e-3    | 0.910        | 1857     | 5.001e-6      | 2.7536   | 49.25           | 4817.9         | 69.79       |
|                 |                 | Recursive       | 857.2e-3  | 8.117e-3    | 865.3e-3    | 0.865        | 1377     | 5.895e-6      | 2.7533   | 43.05           | 4583.1         | 51.75       |
|                 |                 | Recursive2      | 858.7e-3  | 8.092e-3    | 866.8e-3    | 0.867        | 1351     | 5.990e-6      | 2.7533   | 42.92           | 4590.8         | 50.77       |
|                 |                 | Interval        | 27.89e-6  | 3.576       | 3.576       | 3.576        | 493957   | 7.240e-6      | 2.4706   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 3.025     | 1.149       | 4.174       | 4.174        | 115307   | 9.961e-6      | 2.4646   | 32.12           | 116.7          | 23.34       |
|                 |                 | MinVariance     | 4.468     | 1.433       | 5.901       | 5.901        | 133685   | 10.72e-6      | 2.4649   | 40.07           | 165.0          | 27.06       |
|                 | BSplines        | 3.221           | 4.467     | 7.688       | 7.688       | 439985       | 10.15e-6 | 2.4685        | 124.9    | 215.0           | 89.07          |             |
|                 | MinNorm         | 4.232           | 1.339     | 5.570       | 5.570       | 125473       | 10.67e-6 | 2.4647        | 37.43    | 155.8           | 25.40          |             |
|                 | MinVolume       | 3.620           | 1.543     | 5.163       | 5.163       | 151441       | 10.19e-6 | 2.4652        | 43.15    | 144.4           | 30.66          |             |
|                 | ApproxMinVolume | 3.601           | 1.571     | 5.172       | 5.172       | 153139       | 10.26e-6 | 2.4652        | 43.92    | 144.6           | 31.00          |             |
|                 | Recursive       | 3.498           | 1.276     | 4.773       | 4.773       | 123205       | 10.35e-6 | 2.4646        | 35.67    | 133.5           | 24.94          |             |
|                 | Recursive2      | 3.725           | 1.310     | 5.034       | 5.034       | 129811       | 10.66e-6 | 2.4646        | 36.62    | 140.8           | 24.86          |             |
|                 | Interval        | 27.89e-6        | 1506      | 1506        | 25.06       | 189085251    | 7.968e-6 | 1.4003        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 4.009           | 707.1     | 711.2       | 11.51       | 41367965     | 17.09e-6 | 1.3955        | 46.93    | 47.20           | 21.88          |             |
|                 | MinVariance     | 5.156           | 865.4     | 870.5       | 14.30       | 50285639     | 17.21e-6 | 1.3959        | 57.43    | 57.78           | 26.59          |             |
|                 | BSplines        | 3.960           | 4073      | 4077        | 01:07:57    | 236383013    | 17.23e-6 | 1.4017        | 270.4    | 270.6           | 125.0          |             |
|                 | MinNorm         | 5.080           | 797.4     | 802.5       | 13:22       | 46103895     | 17.30e-6 | 1.3957        | 52.92    | 53.26           | 24.38          |             |
|                 | MinVolume       | 4.497           | 1029      | 1033        | 17:13       | 59826583     | 17.21e-6 | 1.3964        | 68.32    | 68.62           | 31.64          |             |
|                 | ApproxMinVolume | 4.653           | 1053      | 1057        | 17:37       | 60059399     | 17.54e-6 | 1.3964        | 69.90    | 70.21           | 31.76          |             |
|                 | Recursive       | 4.082           | 717.4     | 721.5       | 12:01       | 42155919     | 17.02e-6 | 1.3955        | 47.62    | 47.89           | 22.29          |             |
|                 | Recursive2      | 4.030           | 791.0     | 795.0       | 13:15       | 45781889     | 17.28e-6 | 1.3955        | 52.50    | 52.77           | 24.21          |             |
|                 | Interval        | 26.00e-6        | 666585    | 666585      | 7-17:09:45  | 68829667707  | 9.685e-6 | 1.1860        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 60.23           | 542708    | 542708      | 6-06:45:08  | 13295526250  | 40.82e-6 | 1.1815        | 81.42    | 81.42           | 19.32          |             |
|                 | MinVariance     | 71.97           | 691783    | 691783      | 8-00:09:43  | 16640937217  | 41.57e-6 | 1.1821        | 103.8    | 103.8           | 24.18          |             |
|                 | BSplines        | 64.29           | 4257390   | 4257390     | 49-06:36:30 | 104557258199 | 40.72e-6 | 1.1886        | 638.7    | 638.7           | 151.9          |             |
|                 | MinNorm         | 71.34           | 626854    | 626854      | 7-06:07:34  | 15059663173  | 41.62e-6 | 1.1818        | 94.04    | 94.04           | 21.88          |             |
|                 | MinVolume       | 65.11           | 842860    | 842860      | 9-18:07:40  | 20385478051  | 41.35e-6 | 1.1826        | 126.4    | 126.4           | 29.62          |             |
|                 | ApproxMinVolume | 65.09           | 847141    | 847141      | 9-19:19:01  | 20481565927  | 41.36e-6 | 1.1826        | 127.1    | 127.1           | 29.76          |             |
|                 | Recursive       | 62.05           | 553289    | 553290      | 6-09:41:30  | 13624167152  | 40.61e-6 | 1.1815        | 83.00    | 83.00           | 19.79          |             |
|                 | Recursive2      | 59.29           | 611016    | 611016      | 7-01:43:36  | 14983085410  | 40.78e-6 | 1.1815        | 91.66    | 91.66           | 21.77          |             |
|                 | Interval        | 31.95e-6        | 6.617e-3  | 6.649e-3    | 0.007       | 661          | 10.01e-6 | 2.8082        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 1.086           | 5.092e-3  | 1.091       | 1.091       | 405          | 12.57e-6 | 2.7809        | 76.95    | 16414.9         | 61.27          |             |
|                 | MinVariance     | 1.175           | 4.338e-3  | 1.180       | 1.180       | 425          | 10.21e-6 | 2.7820        | 65.56    | 17741.4         | 64.30          |             |
|                 | BSplines        | 1.100           | 7.231e-3  | 1.107       | 1.107       | 773          | 9.354e-6 | 2.8057        | 109.3    | 16645.9         | 116.9          |             |
|                 | MinNorm         | 1.156           | 4.606e-3  | 1.161       | 1.161       | 409          | 11.26e-6 | 2.7815        | 69.61    | 17457.3         | 61.88          |             |
|                 | MinVolume       | 1.147           | 4.408e-3  | 1.151       | 1.151       | 469          | 9.399e-6 | 2.7830        | 66.62    | 17313.7         | 70.95          |             |
|                 | ApproxMinVolume | 1.142           | 5.727e-3  | 1.148       | 1.148       | 469          | 12.21e-6 | 2.7830        | 86.55    | 17264.6         | 70.95          |             |
|                 | Recursive       | 1.097           | 5.886e-3  | 1.103       | 1.103       | 523          | 11.25e-6 | 2.7809        | 88.95    | 16581.7         | 79.12          |             |
|                 | Recursive2      | 1.097           | 6.848e-3  | 1.104       | 1.104       | 513          | 13.35e-6 | 2.7809        | 103.5    | 16601.1         | 77.61          |             |
|                 | Interval        | 29.09e-6        | 503.9e-3  | 503.9e-3    | 0.504       | 69249        | 7.277e-6 | 2.5447        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 2.992           | 215.7e-3  | 3.208       | 3.208       | 19403        | 11.12e-6 | 2.4963        | 42.80    | 636.5           | 28.02          |             |
|                 | MinVariance     | 4.437           | 266.3e-3  | 4.703       | 4.703       | 22085        | 12.06e-6 | 2.4994        | 52.86    | 933.3           | 31.89          |             |
| BSplines        | 3.219           | 868.4e-3        | 4.087     | 4.087       | 76557       | 11.34e-6     | 2.5347   | 172.3         | 811.0    | 110.6           |                |             |
| MinNorm         | 4.267           | 248.5e-3        | 4.516     | 4.516       | 20789       | 11.95e-6     | 2.4977   | 49.31         | 896.0    | 30.02           |                |             |
| MinVolume       | 3.619           | 301.9e-3        | 3.921     | 3.921       | 25765       | 11.72e-6     | 2.5005   | 59.91         | 778.1    | 37.21           |                |             |
| ApproxMinVolume | 3.588           | 296.0e-3        | 3.884     | 3.884       | 25839       | 11.45e-6     | 2.5005   | 58.73         | 770.7    | 37.31           |                |             |
| Recursive       | 3.512           | 286.0e-3        | 3.798     | 3.798       | 25157       | 11.37e-6     | 2.4963   | 56.75         | 753.7    | 36.33           |                |             |
| Recursive2      | 3.714           | 316.1e-3        | 4.030     | 4.030       | 26221       | 12.06e-6     | 2.4963   | 62.73         | 799.7    | 37.86           |                |             |
| Interval        | 34.09e-6        | 95.20           | 95.20     | 01:35       | 10621131    | 8.963e-6     | 1.4800   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 4.083           | 37.65           | 41.74     | 41.735      | 2137275     | 17.62e-6     | 1.4400   | 39.55         | 43.84    | 20.12           |                |             |
| MinVariance     | 5.235           | 46.05           | 51.28     | 51.281      | 2563541     | 17.96e-6     | 1.4436   | 48.37         | 53.87    | 24.14           |                |             |
| BSplines        | 4.097           | 239.8           | 243.9     | 04:03       | 13596675    | 17.64e-6     | 1.4884   | 251.9         | 256.2    | 128.0           |                |             |
| MinNorm         | 5.067           | 40.68           | 45.75     | 45.750      | 2341173     | 17.38e-6     | 1.4422   | 42.73         | 48.06    | 22.04           |                |             |
| MinVolume       | 4.540           | 54.99           | 59.53     | 59.528      | 3113329     | 17.66e-6     | 1.4475   | 57.76         | 62.53    | 29.31           |                |             |
| ApproxMinVolume | 4.727           | 55.92           | 60.65     | 01:00       | 3128003     | 17.88e-6     | 1.4475   | 58.74         | 63.70    | 29.45           |                |             |
| Recursive       | 4.117           | 47.86           | 51.97     | 51.974      | 2762643     | 17.32e-6     | 1.4400   | 50.27         | 54.59    | 26.01           |                |             |
| Recursive2      | 4.150           | 72.52           | 76.67     | 01:16       | 4081563     | 17.77e-6     | 1.4400   | 76.18         | 80.54    | 38.43           |                |             |
| Interval        | 20.98e-6        | 12162           | 12162     | 03:22:42    | 1253315413  | 9.704e-6     | 1.2690   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 54.84           | 10305           | 10360     | 02:52:40    | 240612025   | 42.83e-6     | 1.2322   | 84.73         | 85.18    | 19.20           |                |             |
| MinVariance     | 59.96           | 13151           | 13211     | 03:40:11    | 301916707   | 43.56e-6     | 1.2361   | 108.1         | 108.6    | 24.09           |                |             |
| BSplines        | 53.41           | 93187           | 93240     | 1-01:54:00  | 2218459037  | 42.01e-6     | 1.2853   | 766.2         | 766.6    | 177.0           |                |             |
| MinNorm         | 59.29           | 11602           | 11661     | 03:14:21    | 271019129   | 42.81e-6     | 1.2342   | 95.40         | 95.88    | 21.62           |                |             |
| MinVolume       | 56.84           | 16282           | 16339     | 04:32:19    | 380170441   | 42.83e-6     | 1.2396   | 133.9         | 134.3    | 30.33           |                |             |
| ApproxMinVolume | 57.40           | 16292           | 16349     | 04:32:29    | 381984097   | 42.65e-6     | 1.2397   | 134.0         | 134.4    | 30.48           |                |             |
| Recursive       | 54.73           | 13669           | 13724     | 03:48:44    | 319837953   | 42.74e-6     | 1.2322   | 112.4         | 112.8    | 25.52           |                |             |
| Recursive2      | 55.35           | 22787           | 22843     | 06:20:43    | 538867919   | 42.29e-6     | 1.2322   | 187.4         | 187.8    | 43.00           |                |             |

Table 16: Bisection of problem 2D with problem: 5, type: MinFirst







| precision       | ndof            | solver          | prep_time | comput_time | total_time  | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------------|-----------|-------------|-------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | 2               | Interval        | 35.05e-6  | 21.99e-3    | 22.02e-3    | 0.022        | 2865     | 7.675e-6      | 2.4606   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 878.1e-3  | 7.775e-3    | 885.9e-3    | 0.886        | 1363     | 5.704e-6      | 2.4563   | 35.36           | 4022.6         | 47.57       |
|                 |                 | MinVariance     | 951.0e-3  | 8.664e-3    | 959.7e-3    | 0.960        | 1469     | 5.898e-6      | 2.4564   | 39.40           | 4357.6         | 51.27       |
|                 |                 | BSplines        | 868.8e-3  | 13.89e-3    | 882.7e-3    | 0.883        | 2817     | 4.930e-6      | 2.4598   | 63.15           | 4008.0         | 98.32       |
|                 |                 | MinNorm         | 941.1e-3  | 8.407e-3    | 949.6e-3    | 0.950        | 1415     | 5.941e-6      | 2.4564   | 38.23           | 4311.5         | 49.39       |
|                 |                 | MinVolume       | 911.1e-3  | 8.794e-3    | 919.9e-3    | 0.920        | 1623     | 5.418e-6      | 2.4571   | 39.99           | 4176.7         | 56.65       |
|                 |                 | ApproxMinVolume | 895.9e-3  | 8.709e-3    | 907.7e-3    | 0.908        | 1623     | 5.366e-6      | 2.4571   | 39.61           | 4121.2         | 56.65       |
|                 |                 | Recursive       | 873.7e-3  | 8.270e-3    | 881.9e-3    | 0.882        | 1467     | 5.637e-6      | 2.4563   | 37.61           | 4004.5         | 51.20       |
|                 |                 | Recursive2      | 878.8e-3  | 8.374e-3    | 887.2e-3    | 0.887        | 1479     | 5.662e-6      | 2.4563   | 38.08           | 4028.2         | 51.62       |
|                 |                 | Interval        | 32.90e-6  | 4.437       | 4.437       | 4.437        | 624303   | 7.108e-6      | 2.2083   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 3.050     | 1.389       | 4.439       | 4.439        | 135837   | 10.22e-6      | 2.2022   | 31.30           | 100.0          | 21.76       |
|                 |                 | MinVariance     | 4.476     | 1.774       | 6.251       | 6.251        | 158425   | 11.20e-6      | 2.2026   | 39.98           | 140.9          | 25.38       |
|                 |                 | BSplines        | 3.219     | 5.687       | 8.906       | 8.906        | 541987   | 10.49e-6      | 2.2060   | 128.2           | 200.7          | 86.81       |
|                 |                 | MinNorm         | 4.249     | 1.666       | 5.916       | 5.916        | 148295   | 11.24e-6      | 2.2024   | 37.55           | 133.3          | 23.75       |
|                 |                 | MinVolume       | 3.631     | 1.998       | 5.628       | 5.628        | 182409   | 10.95e-6      | 2.2028   | 45.02           | 126.8          | 29.22       |
|                 | ApproxMinVolume | 3.575           | 1.931     | 5.506       | 5.506       | 182933       | 10.55e-6 | 2.2028        | 43.51    | 124.1           | 29.30          |             |
|                 | Recursive       | 3.496           | 1.525     | 5.020       | 5.020       | 142967       | 10.66e-6 | 2.2022        | 34.36    | 113.1           | 22.90          |             |
|                 | Recursive2      | 3.721           | 1.557     | 5.278       | 5.278       | 142927       | 10.89e-6 | 2.2022        | 35.09    | 119.0           | 22.89          |             |
|                 | Interval        | 29.09e-6        | 1724      | 1724        | 28.44       | 213360751    | 8.081e-6 | 1.2199        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 3.937           | 831.2     | 835.1       | 13.55       | 50454883     | 16.47e-6 | 1.2155        | 48.21    | 48.44           | 23.65          |             |
|                 | MinVariance     | 5.187           | 1042      | 1047        | 17.27       | 61013369     | 17.09e-6 | 1.2160        | 60.47    | 60.77           | 28.60          |             |
|                 | BSplines        | 4.039           | 4407      | 4411        | 01:13.31    | 277497315    | 15.88e-6 | 1.2216        | 255.6    | 255.9           | 130.1          |             |
|                 | MinNorm         | 5.032           | 967.2     | 972.2       | 16.12       | 56014997     | 17.27e-6 | 1.2157        | 56.10    | 56.39           | 26.25          |             |
|                 | MinVolume       | 4.518           | 1232      | 1236        | 20.36       | 72210907     | 17.07e-6 | 1.2164        | 71.47    | 71.74           | 33.84          |             |
|                 | ApproxMinVolume | 4.615           | 1246      | 1250        | 20.50       | 72439199     | 17.20e-6 | 1.2164        | 72.29    | 72.55           | 33.95          |             |
|                 | Recursive       | 4.021           | 864.4     | 868.4       | 14.28       | 51056239     | 16.93e-6 | 1.2155        | 50.14    | 50.37           | 23.93          |             |
|                 | Recursive2      | 4.075           | 928.0     | 932.1       | 15.32       | 54123815     | 17.15e-6 | 1.2155        | 53.83    | 54.06           | 25.37          |             |
|                 | Interval        | 28.00e-6        | 756744    | 756744      | 8-18:12:24  | 77832279864  | 9.723e-6 | 1.0322        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 54.95           | 603329    | 603330      | 6-23:35:30  | 15121569445  | 39.90e-6 | 1.0281        | 79.73    | 79.73           | 19.43          |             |
|                 | MinVariance     | 59.79           | 776113    | 776113      | 8-23:35:13  | 19026489275  | 40.79e-6 | 1.0287        | 102.6    | 102.6           | 24.45          |             |
|                 | BSplines        | 54.96           | 4920300   | 4920300     | 56-22:45:00 | 121021741505 | 40.66e-6 | 1.0350        | 650.2    | 650.2           | 155.5          |             |
|                 | MinNorm         | 59.30           | 711755    | 711755      | 8-05:42:35  | 17136869562  | 41.53e-6 | 1.0284        | 94.05    | 94.05           | 22.02          |             |
|                 | MinVolume       | 57.06           | 959119    | 959119      | 11-02:25:19 | 23305249196  | 41.15e-6 | 1.0292        | 126.7    | 126.7           | 29.94          |             |
|                 | ApproxMinVolume | 57.50           | 976462    | 976463      | 11-07:14:23 | 23435498886  | 41.67e-6 | 1.0292        | 129.0    | 129.0           | 30.11          |             |
|                 | Recursive       | 55.27           | 637856    | 637856      | 7-09:10:56  | 15332653901  | 41.60e-6 | 1.0281        | 84.29    | 84.29           | 19.70          |             |
|                 | Recursive2      | 55.29           | 675568    | 675568      | 7-19:39:28  | 16431248947  | 41.11e-6 | 1.0281        | 89.27    | 89.27           | 21.11          |             |
|                 | Interval        | 29.09e-6        | 7.103e-3  | 7.132e-3    | 0.007       | 685          | 10.37e-6 | 2.5169        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 1.893           | 4.445e-3  | 1.897       | 1.897       | 337          | 13.19e-6 | 2.4865        | 62.58    | 26605.3         | 49.20          |             |
|                 | MinVariance     | 1.889           | 14.20e-3  | 1.903       | 1.903       | 343          | 41.41e-6 | 2.4875        | 200.0    | 26687.7         | 50.07          |             |
|                 | BSplines        | 1.892           | 13.75e-3  | 1.906       | 1.906       | 703          | 19.56e-6 | 2.5101        | 193.6    | 26725.0         | 102.6          |             |
|                 | MinNorm         | 2.166           | 6.107e-3  | 2.172       | 2.172       | 331          | 18.45e-6 | 2.4871        | 85.98    | 30453.0         | 48.32          |             |
|                 | MinVolume       | 2.167           | 11.74e-3  | 2.179       | 2.179       | 383          | 30.66e-6 | 2.4885        | 165.3    | 30546.9         | 55.91          |             |
|                 | ApproxMinVolume | 2.167           | 7.666e-3  | 2.175       | 2.175       | 383          | 20.02e-6 | 2.4885        | 107.9    | 30494.0         | 55.91          |             |
|                 | Recursive       | 2.167           | 6.831e-3  | 2.174       | 2.174       | 411          | 16.62e-6 | 2.4865        | 96.17    | 30476.1         | 60.00          |             |
|                 | Recursive2      | 2.166           | 4.908e-3  | 2.171       | 2.171       | 449          | 10.93e-6 | 2.4865        | 69.10    | 30442.0         | 65.55          |             |
| Interval        | 26.94e-6        | 657.8e-3        | 657.8e-3  | 0.658       | 98143       | 6.703e-6     | 2.2797   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 2.904           | 296.7e-3        | 3.201     | 3.201       | 26149       | 11.35e-6     | 2.2314   | 45.10         | 486.6    | 26.64           |                |             |
| MinVariance     | 4.409           | 357.4e-3        | 4.766     | 4.766       | 29819       | 11.98e-6     | 2.2340   | 54.33         | 724.5    | 30.38           |                |             |
| BSplines        | 3.196           | 1.159           | 4.356     | 4.356       | 100765      | 11.50e-6     | 2.2655   | 176.2         | 662.1    | 102.7           |                |             |
| MinNorm         | 4.253           | 343.8e-3        | 4.597     | 4.597       | 28013       | 12.27e-6     | 2.2334   | 52.27         | 698.8    | 28.54           |                |             |
| MinVolume       | 3.645           | 388.9e-3        | 4.034     | 4.034       | 33971       | 11.45e-6     | 2.2350   | 59.11         | 613.2    | 34.61           |                |             |
| ApproxMinVolume | 3.589           | 400.0e-3        | 3.989     | 3.989       | 34081       | 11.74e-6     | 2.2351   | 60.81         | 606.4    | 34.73           |                |             |
| Recursive       | 3.515           | 363.4e-3        | 3.878     | 3.878       | 31559       | 11.51e-6     | 2.2314   | 55.24         | 589.5    | 32.16           |                |             |
| Recursive2      | 3.744           | 379.0e-3        | 4.123     | 4.123       | 32767       | 11.57e-6     | 2.2314   | 57.62         | 626.7    | 33.39           |                |             |
| Interval        | 25.99e-6        | 93.35           | 93.35     | 01:33       | 11048427    | 8.449e-6     | 1.2952   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 4.059           | 43.40           | 47.46     | 47.462      | 2422819     | 17.91e-6     | 1.2587   | 46.49         | 50.84    | 21.93           |                |             |
| MinVariance     | 5.260           | 53.05           | 58.31     | 58.310      | 2939667     | 18.05e-6     | 1.2628   | 56.83         | 62.46    | 26.61           |                |             |
| BSplines        | 4.070           | 243.0           | 247.0     | 04:07       | 13938915    | 17.43e-6     | 1.3068   | 260.3         | 264.6    | 126.2           |                |             |
| MinNorm         | 5.107           | 47.16           | 52.27     | 52.269      | 2691281     | 17.52e-6     | 1.2610   | 50.52         | 55.99    | 24.36           |                |             |
| MinVolume       | 4.548           | 62.00           | 66.55     | 01:06       | 3517435     | 17.63e-6     | 1.2665   | 66.41         | 71.28    | 31.84           |                |             |
| ApproxMinVolume | 4.775           | 63.06           | 67.84     | 01:07       | 3530765     | 17.86e-6     | 1.2665   | 67.55         | 72.67    | 31.96           |                |             |
| Recursive       | 4.120           | 50.63           | 54.75     | 54.752      | 2868969     | 17.65e-6     | 1.2587   | 54.24         | 58.65    | 25.97           |                |             |
| Recursive2      | 4.191           | 69.51           | 73.70     | 01:13       | 3940107     | 17.64e-6     | 1.2587   | 74.46         | 78.95    | 35.66           |                |             |
| Interval        | 20.03e-6        | 13449           | 13449     | 03:44:09    | 1366693269  | 9.841e-6     | 1.1113   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 54.58           | 11390           | 11444     | 03:10:44    | 272881425   | 41.74e-6     | 1.0771   | 84.69         | 85.10    | 19.97           |                |             |
| MinVariance     | 60.09           | 14933           | 14993     | 04:09:53    | 345680561   | 43.20e-6     | 1.0810   | 111.0         | 111.5    | 25.29           |                |             |
| BSplines        | 55.34           | 103622          | 103677    | 1-04:47:57  | 2444878999  | 42.38e-6     | 1.1296   | 770.5         | 770.9    | 178.9           |                |             |
| MinNorm         | 59.25           | 13373           | 13432     | 03:43:52    | 309751173   | 43.17e-6     | 1.0789   | 99.44         | 99.88    | 22.66           |                |             |
| MinVolume       | 57.14           | 18399           | 18457     | 05:07:37    | 431370277   | 42.65e-6     | 1.0845   | 136.8         | 137.2    | 31.56           |                |             |
| ApproxMinVolume | 56.87           | 17898           | 17954     | 04:59:14    | 433301867   | 41.31e-6     | 1.0845   | 133.1         | 133.5    | 31.70           |                |             |
| Recursive       | 55.41           | 13724           | 13780     | 03:49:40    | 326904643   | 41.98e-6     | 1.0771   | 102.0         | 102.5    | 23.92           |                |             |
| Recursive2      | 55.55           | 21385           | 21441     | 05:57:21    | 504445255   | 42.39e-6     | 1.0771   | 159.0         | 159.4    | 36.91           |                |             |

Table 19: Bisection of problem 2D with problem: 8, type: MinFirst

| precision       | ndof            | solver          | prep_time       | comput_time | total_time  | (D-H:M:S.ms) | nb_iter   | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------------|-----------------|-----------------|-----------------|-------------|-------------|--------------|-----------|---------------|------------|-----------------|----------------|-------------|-------|
| 0.001           | 2               | Interval        | 30.04e-6        | 3.881e-3    | 3.911e-3    | 0.004        | 107       | 36.27e-6      | infeasible | 100.0           | 100.0          | 100.0       |       |
|                 |                 | Bernstein       | 846.8e-3        | 3.931e-3    | 850.7e-3    | 0.851        | 69        | 56.97e-6      | infeasible | 101.3           | 21752.4        | 64.49       |       |
|                 |                 | MinVariance     | 951.6e-3        | 3.598e-3    | 955.2e-3    | 0.955        | 63        | 57.11e-6      | infeasible | 92.71           | 24423.7        | 58.88       |       |
|                 |                 | BSplines        | 815.2e-3        | 3.912e-3    | 819.2e-3    | 0.819        | 113       | 34.62e-6      | infeasible | 100.8           | 20945.4        | 105.6       |       |
|                 |                 | MinNorm         | 876.2e-3        | 3.897e-3    | 880.1e-3    | 0.880        | 61        | 63.88e-6      | infeasible | 100.4           | 22502.8        | 57.01       |       |
|                 |                 | MinVolume       | 809.4e-3        | 3.393e-3    | 812.8e-3    | 0.813        | 69        | 49.17e-6      | infeasible | 87.42           | 20782.5        | 64.49       |       |
|                 |                 | ApproxMinVolume | 810.7e-3        | 3.030e-3    | 813.8e-3    | 0.814        | 69        | 43.91e-6      | infeasible | 78.07           | 20807.3        | 64.49       |       |
|                 |                 | Recursive       | 874.4e-3        | 3.224e-3    | 877.7e-3    | 0.878        | 93        | 34.67e-6      | infeasible | 83.07           | 22440.7        | 86.92       |       |
|                 |                 | Recursive2      | 862.8e-3        | 3.524e-3    | 866.3e-3    | 0.866        | 123       | 28.65e-6      | infeasible | 90.80           | 22150.3        | 115.0       |       |
|                 |                 | Interval        | 25.99e-6        | 2.260       | 2.260       | 2.260        | 320711    | 7.046e-6      | 3.7270     | 100.0           | 100.0          | 100.0       |       |
|                 |                 | Bernstein       | 2.996           | 1.118       | 4.114       | 4.114        | 106597    | 10.49e-6      | 3.7217     | 49.47           | 182.1          | 33.24       |       |
|                 |                 | MinVariance     | 4.398           | 1.360       | 5.759       | 5.759        | 121437    | 11.20e-6      | 3.7221     | 60.20           | 254.8          | 37.86       |       |
|                 | BSplines        | 3.173           | 3.647           | 6.819       | 6.819       | 350145       | 10.41e-6  | 3.7270        | 161.4      | 301.8           | 109.2          |             |       |
|                 | MinNorm         | 4.188           | 1.303           | 5.491       | 5.491       | 114461       | 11.38e-6  | 3.7220        | 57.66      | 243.0           | 35.69          |             |       |
|                 | MinVolume       | 3.574           | 1.456           | 5.030       | 5.030       | 135677       | 10.73e-6  | 3.7225        | 64.43      | 222.6           | 42.31          |             |       |
|                 | ApproxMinVolume | 3.580           | 1.424           | 5.003       | 5.003       | 136049       | 10.46e-6  | 3.7225        | 63.00      | 221.4           | 42.42          |             |       |
|                 | Recursive       | 3.503           | 1.153           | 4.656       | 4.656       | 109979       | 10.48e-6  | 3.7217        | 51.02      | 206.0           | 34.29          |             |       |
|                 | Recursive2      | 3.716           | 1.150           | 4.867       | 4.867       | 109183       | 10.53e-6  | 3.7217        | 50.89      | 215.4           | 34.04          |             |       |
|                 | Interval        | 21.93e-6        | 2.219           | 2.219       | 2.219       | 36:59        | 258043271 | 8.600e-6      | 2.1947     | 100.0           | 100.0          | 100.0       |       |
|                 | Bernstein       | 4.003           | 573.8           | 577.8       | 09:37       | 33494421     | 17.13e-6  | 2.1783        | 25.86      | 26.04           | 12.98          |             |       |
|                 | MinVariance     | 5.150           | 710.2           | 715.4       | 11:55       | 39986051     | 17.76e-6  | 2.1787        | 32.00      | 32.24           | 15.50          |             |       |
|                 | BSplines        | 3.984           | 2725            | 2729        | 45:29       | 167557199    | 16.26e-6  | 2.1849        | 122.8      | 123.0           | 64.93          |             |       |
|                 | MinNorm         | 4.978           | 636.8           | 641.8       | 10:41       | 37060967     | 17.18e-6  | 2.1785        | 28.70      | 28.92           | 14.36          |             |       |
|                 | MinVolume       | 4.483           | 789.2           | 793.6       | 13:13       | 46731837     | 16.89e-6  | 2.1793        | 35.56      | 35.76           | 18.11          |             |       |
|                 | ApproxMinVolume | 4.648           | 819.1           | 823.7       | 13:43       | 46906683     | 17.46e-6  | 2.1793        | 36.91      | 37.12           | 18.18          |             |       |
|                 | Recursive       | 4.019           | 589.0           | 593.0       | 09:53       | 33952573     | 17.35e-6  | 2.1783        | 26.54      | 26.72           | 13.16          |             |       |
|                 | Recursive2      | 4.011           | 598.8           | 602.8       | 10:02       | 35550901     | 16.84e-6  | 2.1783        | 26.98      | 27.16           | 13.78          |             |       |
|                 | Interval        | 28.00e-6        | 446838          | 446838      | 5-04:07:18  | 46148673782  | 9.683e-6  | 1.4639        | 100.0      | 100.0           | 100.0          |             |       |
|                 | Bernstein       | 54.88           | 520018          | 520018      | 6-00:26:58  | 12563231767  | 41.39e-6  | 1.4591        | 116.4      | 116.4           | 27.22          |             |       |
|                 | MinVariance     | 60.26           | 665826          | 665826      | 7-16:57:06  | 15807794175  | 42.12e-6  | 1.4597        | 149.0      | 149.0           | 34.25          |             |       |
|                 | BSplines        | 55.54           | 4128640         | 4128640     | 47-18:50:40 | 100479468274 | 41.09e-6  | 1.4670        | 924.0      | 924.0           | 217.7          |             |       |
|                 | MinNorm         | 59.51           | 593157          | 593157      | 6-20:45:57  | 14262155737  | 41.59e-6  | 1.4594        | 132.7      | 132.7           | 30.90          |             |       |
|                 | MinVolume       | 57.34           | 802157          | 802157      | 9-06:49:17  | 19364907825  | 41.42e-6  | 1.4603        | 179.5      | 179.5           | 41.96          |             |       |
|                 | ApproxMinVolume | 57.53           | 812740          | 812740      | 9-09:45:40  | 19473261169  | 41.74e-6  | 1.4603        | 181.9      | 181.9           | 42.20          |             |       |
|                 | Recursive       | 54.82           | 520266          | 520266      | 6-00:31:06  | 12581302301  | 41.35e-6  | 1.4591        | 116.4      | 116.4           | 27.26          |             |       |
|                 | Recursive2      | 55.18           | 556135          | 556136      | 6-10:28:56  | 13433505549  | 41.40e-6  | 1.4591        | 124.5      | 124.5           | 29.11          |             |       |
|                 | 0.01            | 2               | Interval        | 26.94e-6    | 111.2e-3    | 111.2e-3     | 0.111     | 107           | 1.039e-3   | infeasible      | 100.0          | 100.0       | 100.0 |
|                 |                 |                 | Bernstein       | 2.166       | 4.789e-3    | 2.171        | 2.171     | 69            | 69.40e-6   | infeasible      | 4.308          | 1952.4      | 64.49 |
|                 |                 |                 | MinVariance     | 2.086       | 3.739e-3    | 2.089        | 2.089     | 63            | 59.35e-6   | infeasible      | 3.364          | 1879.2      | 58.88 |
|                 |                 |                 | BSplines        | 2.167       | 8.552e-3    | 2.176        | 2.176     | 113           | 75.68e-6   | infeasible      | 7.693          | 1957.0      | 105.6 |
|                 |                 |                 | MinNorm         | 2.167       | 12.75e-3    | 2.179        | 2.179     | 61            | 209.0e-6   | infeasible      | 11.47          | 1960.1      | 57.01 |
|                 |                 |                 | MinVolume       | 2.167       | 9.690e-3    | 2.177        | 2.177     | 69            | 140.4e-6   | infeasible      | 8.717          | 1957.8      | 64.49 |
|                 |                 |                 | ApproxMinVolume | 2.152       | 11.40e-3    | 2.164        | 2.164     | 69            | 165.2e-6   | infeasible      | 10.26          | 1946.0      | 64.49 |
|                 |                 |                 | Recursive       | 2.152       | 9.382e-3    | 2.161        | 2.161     | 93            | 100.9e-6   | infeasible      | 8.440          | 1944.0      | 86.92 |
|                 |                 |                 | Recursive2      | 2.077       | 12.47e-3    | 2.090        | 2.090     | 123           | 101.4e-6   | infeasible      | 11.22          | 1879.4      | 115.0 |
| Interval        |                 |                 | 28.85e-6        | 307.9e-3    | 307.9e-3    | 0.308        | 40963     | 7.516e-6      | 3.8013     | 100.0           | 100.0          | 100.0       |       |
| Bernstein       |                 |                 | 3.003           | 153.6e-3    | 3.156       | 3.156        | 13987     | 10.98e-6      | 3.7604     | 49.89           | 1025.2         | 34.15       |       |
| MinVariance     |                 |                 | 4.479           | 198.5e-3    | 4.677       | 4.677        | 15869     | 12.51e-6      | 3.7636     | 64.47           | 1519.2         | 38.74       |       |
| BSplines        |                 | 3.213           | 534.8e-3        | 3.748       | 3.748       | 48657        | 10.99e-6  | 3.8029        | 173.7      | 1217.2          | 118.8          |             |       |
| MinNorm         |                 | 4.249           | 189.9e-3        | 4.439       | 4.439       | 14961        | 12.69e-6  | 3.7612        | 61.67      | 1441.8          | 36.52          |             |       |
| MinVolume       |                 | 3.633           | 205.4e-3        | 3.838       | 3.838       | 18009        | 11.41e-6  | 3.7668        | 66.72      | 1246.6          | 43.96          |             |       |
| ApproxMinVolume |                 | 3.600           | 202.5e-3        | 3.802       | 3.802       | 18053        | 11.22e-6  | 3.7668        | 65.77      | 1234.9          | 44.07          |             |       |
| Recursive       |                 | 3.502           | 185.9e-3        | 3.688       | 3.688       | 16495        | 11.27e-6  | 3.7604        | 60.40      | 1198.0          | 40.27          |             |       |
| Recursive2      |                 | 3.720           | 189.2e-3        | 3.909       | 3.909       | 16503        | 11.46e-6  | 3.7604        | 61.46      | 1269.7          | 40.29          |             |       |
| Interval        |                 | 25.03e-6        | 150.5           | 150.5       | 02:30       | 17689035     | 8.507e-6  | 2.3987        | 100.0      | 100.0           | 100.0          |             |       |
| Bernstein       |                 | 4.091           | 36.03           | 40.13       | 40.126      | 2054341      | 17.54e-6  | 2.2545        | 23.95      | 26.66           | 11.61          |             |       |
| MinVariance     |                 | 5.224           | 42.35           | 47.57       | 47.575      | 2415005      | 17.54e-6  | 2.2579        | 28.14      | 31.61           | 13.65          |             |       |
| BSplines        |                 | 4.060           | 176.6           | 180.6       | 03:00       | 10142423     | 17.41e-6  | 2.3074        | 117.3      | 120.0           | 57.34          |             |       |
| MinNorm         |                 | 4.914           | 39.54           | 44.46       | 44.456      | 2238933      | 17.66e-6  | 2.2569        | 26.28      | 29.54           | 12.66          |             |       |
| MinVolume       |                 | 4.603           | 56.52           | 61.12       | 01:01       | 2829311      | 19.98e-6  | 2.2634        | 37.56      | 40.62           | 15.99          |             |       |
| ApproxMinVolume |                 | 4.716           | 50.48           | 55.20       | 55.197      | 2838669      | 17.78e-6  | 2.2634        | 33.54      | 36.68           | 16.05          |             |       |
| Recursive       |                 | 4.110           | 42.38           | 46.49       | 46.492      | 2418191      | 17.53e-6  | 2.2545        | 28.16      | 30.89           | 13.67          |             |       |
| Recursive2      |                 | 4.111           | 51.67           | 55.78       | 55.781      | 2902143      | 17.80e-6  | 2.2545        | 34.33      | 37.07           | 16.41          |             |       |
| Interval        |                 | 20.98e-6        | 7032            | 7032        | 01:57:12    | 708080401    | 9.931e-6  | 1.5549        | 100.0      | 100.0           | 100.0          |             |       |
| Bernstein       |                 | 55.22           | 8640            | 8695        | 02:24:55    | 200204705    | 43.16e-6  | 1.5150        | 122.9      | 123.7           | 28.27          |             |       |
| MinVariance     |                 | 60.16           | 11126           | 11186       | 03:06:26    | 254680529    | 43.69e-6  | 1.5198        | 158.2      | 159.1           | 35.97          |             |       |
| BSplines        |                 | 54.90           | 72776           | 72831       | 20:13:51    | 1720598853   | 42.30e-6  | 1.5759        | 1034.9     | 1035.7          | 243.0          |             |       |
| MinNorm         |                 | 59.29           | 9779            | 9838        | 02:43:58    | 228013011    | 42.89e-6  | 1.5175        | 139.1      | 139.9           | 32.20          |             |       |
| MinVolume       |                 | 57.09           | 13670           | 13727       | 03:48:47    | 315310113    | 43.36e-6  | 1.5237        | 194.4      | 195.2           | 44.53          |             |       |
| ApproxMinVolume |                 | 57.34           | 13701           | 13759       | 03:49:19    | 317430217    | 43.16e-6  | 1.5239        | 194.8      | 195.7           | 44.83          |             |       |
| Recursive       |                 | 55.13           | 8979            | 9035        | 02:30:35    | 207894885    | 43.19e-6  | 1.5150        | 127.7      | 128.5           | 29.36          |             |       |
| Recursive2      |                 | 54.98           | 14663           | 14718       | 04:05:18    | 345337759    | 42.46e-6  | 1.5150        | 208.5      | 209.3           | 48.77          |             |       |

Table 20: Bisection of problem 2D with problem: 9, type: MinFirst

| precision | ndof | solver          | prep_time       | comput_time | total_time | (D-H:M:S.ms) | nb_iter      | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------|------|-----------------|-----------------|-------------|------------|--------------|--------------|---------------|----------|-----------------|----------------|-------------|-------|
| 0.001     | 2    | Interval        | 37.19e-6        | 6.781e-3    | 6.818e-3   | 0.007        | 671          | 10.11e-6      | 80.714   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 716.8e-3        | 4.455e-3    | 721.3e-3   | 0.721        | 583          | 7.642e-6      | 80.593   | 65.70           | 10578.7        | 86.89       |       |
|           |      | MinVariance     | 714.6e-3        | 4.524e-3    | 719.1e-3   | 0.719        | 583          | 7.760e-6      | 80.595   | 66.72           | 10546.9        | 86.89       |       |
|           |      | BSplines        | 710.5e-3        | 4.763e-3    | 715.3e-3   | 0.715        | 777          | 6.130e-6      | 80.623   | 70.24           | 10491.0        | 115.8       |       |
|           |      | MinNorm         | 732.1e-3        | 4.216e-3    | 736.3e-3   | 0.736        | 583          | 7.231e-6      | 80.594   | 62.17           | 10799.7        | 86.89       |       |
|           |      | MinVolume       | 733.5e-3        | 4.538e-3    | 738.1e-3   | 0.738        | 597          | 7.601e-6      | 80.597   | 66.92           | 10825.0        | 88.97       |       |
|           |      | ApproxMinVolume | 726.7e-3        | 4.123e-3    | 730.8e-3   | 0.731        | 597          | 6.907e-6      | 80.597   | 60.81           | 10719.3        | 88.97       |       |
|           |      | Recursive       | 725.3e-3        | 4.248e-3    | 729.6e-3   | 0.730        | 591          | 7.188e-6      | 80.593   | 62.65           | 10700.4        | 88.08       |       |
|           |      | Recursive2      | 720.7e-3        | 4.555e-3    | 725.3e-3   | 0.725        | 589          | 7.733e-6      | 80.593   | 67.17           | 10637.8        | 87.78       |       |
|           | 3    | Interval        | 30.04e-6        | 3.216       | 3.216      | 3.216        | 393723       | 8.169e-6      | 85.311   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 1.420           | 449.0e-3    | 1.869      | 1.869        | 69973        | 6.417e-6      | 85.168   | 13.96           | 58.12          | 17.77       |       |
|           |      | MinVariance     | 1.519           | 479.7e-3    | 1.999      | 1.999        | 74615        | 6.429e-6      | 85.172   | 14.92           | 62.15          | 18.95       |       |
|           |      | BSplines        | 1.420           | 906.9e-3    | 2.327      | 2.327        | 154947       | 5.853e-6      | 85.215   | 28.20           | 72.36          | 39.35       |       |
|           |      | MinNorm         | 1.538           | 486.7e-3    | 2.025      | 2.025        | 72049        | 6.755e-6      | 85.170   | 15.13           | 62.95          | 18.30       |       |
|           |      | MinVolume       | 1.483           | 526.5e-3    | 2.009      | 2.009        | 80979        | 6.502e-6      | 85.175   | 16.37           | 62.47          | 20.57       |       |
|           |      | ApproxMinVolume | 1.474           | 535.9e-3    | 2.010      | 2.010        | 81135        | 6.605e-6      | 85.175   | 16.66           | 62.51          | 20.61       |       |
|           |      | Recursive       | 1.479           | 465.4e-3    | 1.945      | 1.945        | 70077        | 6.642e-6      | 85.168   | 14.47           | 60.46          | 17.80       |       |
|           |      | Recursive2      | 1.485           | 468.4e-3    | 1.954      | 1.954        | 70347        | 6.658e-6      | 85.168   | 14.56           | 60.75          | 17.87       |       |
|           | 4    | Interval        | 28.85e-6        | 1.922       | 1.922      | 32.02        | 198196587    | 9.701e-6      | 100.24   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 11.05           | 294.6       | 305.7      | 05:05        | 12832141     | 22.96e-6      | 100.11   | 15.32           | 15.90          | 6.474       |       |
|           |      | MinVariance     | 14.42           | 358.7       | 373.2      | 06:13        | 14448465     | 24.83e-6      | 100.12   | 18.66           | 19.41          | 7.290       |       |
|           |      | BSplines        | 11.69           | 998.8       | 1010       | 16:50        | 44530227     | 22.43e-6      | 100.19   | 51.94           | 52.55          | 22.47       |       |
|           |      | MinNorm         | 14.32           | 346.5       | 360.9      | 06:00        | 13694269     | 25.31e-6      | 100.12   | 18.02           | 18.77          | 6.909       |       |
|           |      | MinVolume       | 12.60           | 382.4       | 395.0      | 06:34        | 16109081     | 23.74e-6      | 100.12   | 19.89           | 20.54          | 8.128       |       |
|           |      | ApproxMinVolume | 12.29           | 351.2       | 363.5      | 06:03        | 16152151     | 21.74e-6      | 100.13   | 18.27           | 18.91          | 8.150       |       |
|           |      | Recursive       | 14.73           | 320.8       | 335.5      | 05:35        | 12833477     | 25.00e-6      | 100.11   | 16.68           | 17.45          | 6.475       |       |
|           |      | Recursive2      | 15.54           | 308.9       | 324.5      | 05:24        | 12849603     | 24.04e-6      | 100.11   | 16.07           | 16.88          | 6.483       |       |
|           | 5    | Interval        | 24.00e-6        | 4968410     | 4968410    | 57-12:06:50  | 429100212907 | 11.58e-6      | 116.26   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 72.10           | 576410      | 576410     | 6-16:06:50   | 16067783129  | 35.87e-6      | 116.06   | 11.60           | 11.60          | 3.745       |       |
|           |      | MinVariance     | 75.38           | 657375      | 657375     | 7-14:36:15   | 18084989359  | 36.35e-6      | 116.06   | 13.23           | 13.23          | 4.215       |       |
|           |      | BSplines        | 72.51           | 1938060     | 1938060    | 22-10:21:00  | 57904499645  | 33.47e-6      | 116.16   | 39.01           | 39.01          | 13.49       |       |
|           |      | MinNorm         | 75.07           | 613132      | 613132     | 7-02:18:52   | 17193195041  | 35.66e-6      | 116.06   | 12.34           | 12.34          | 4.007       |       |
|           |      | MinVolume       | 73.50           | 718739      | 718739     | 8-07:38:59   | 20235843440  | 35.52e-6      | 116.07   | 14.47           | 14.47          | 4.716       |       |
|           |      | ApproxMinVolume | 73.54           | 710916      | 710916     | 8-05:28:36   | 20291967886  | 35.03e-6      | 116.07   | 14.31           | 14.31          | 4.729       |       |
|           |      | Recursive       | 72.25           | 563737      | 563738     | 6-12:35:38   | 16107783885  | 35.00e-6      | 116.06   | 11.35           | 11.35          | 3.754       |       |
|           |      | Recursive2      | 72.19           | 573654      | 573654     | 6-15:20:54   | 16154599175  | 35.51e-6      | 116.06   | 11.55           | 11.55          | 3.765       |       |
|           | 0.01 | 2               | Interval        | 26.94e-6    | 69.50e-3   | 69.53e-3     | 0.070        | 265           | 262.3e-6 | 82.162          | 100.0          | 100.0       | 100.0 |
|           |      |                 | Bernstein       | 2.098       | 13.60e-3   | 2.112        | 2.112        | 195           | 69.77e-6 | 81.681          | 19.57          | 3036.9      | 73.58 |
|           |      |                 | MinVariance     | 965.6e-3    | 10.43e-3   | 976.0e-3     | 0.976        | 209           | 49.90e-6 | 81.700          | 15.01          | 1403.7      | 78.87 |
|           |      |                 | BSplines        | 965.7e-3    | 12.53e-3   | 978.3e-3     | 0.978        | 297           | 42.18e-6 | 81.923          | 18.02          | 1406.9      | 112.1 |
|           |      |                 | MinNorm         | 962.1e-3    | 6.176e-3   | 968.3e-3     | 0.968        | 195           | 31.67e-6 | 81.692          | 8.886          | 1392.7      | 73.58 |
|           |      |                 | MinVolume       | 978.0e-3    | 14.78e-3   | 992.8e-3     | 0.993        | 209           | 70.74e-6 | 81.718          | 21.27          | 1427.8      | 78.87 |
|           |      |                 | ApproxMinVolume | 981.4e-3    | 4.838e-3   | 986.3e-3     | 0.986        | 209           | 23.15e-6 | 81.719          | 6.961          | 1418.5      | 78.87 |
|           |      |                 | Recursive       | 1.023       | 5.526e-3   | 1.029        | 1.029        | 203           | 27.22e-6 | 81.681          | 7.950          | 1479.6      | 76.60 |
|           |      |                 | Recursive2      | 952.5e-3    | 4.929e-3   | 957.5e-3     | 0.957        | 201           | 24.52e-6 | 81.681          | 7.091          | 1377.1      | 75.85 |
| 3         |      | Interval        | 24.08e-6        | 623.9e-3    | 623.9e-3   | 0.624        | 75517        | 8.262e-6      | 87.099   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 1.426           | 125.1e-3    | 1.551      | 1.551        | 17463        | 7.162e-6      | 85.900   | 20.05           | 248.6          | 23.12       |       |
|           |      | MinVariance     | 1.451           | 124.9e-3    | 1.575      | 1.575        | 18765        | 6.656e-6      | 85.929   | 20.02           | 252.5          | 24.85       |       |
|           |      | BSplines        | 1.389           | 231.3e-3    | 1.620      | 1.620        | 36935        | 6.263e-6      | 86.271   | 37.08           | 259.7          | 48.91       |       |
|           |      | MinNorm         | 1.522           | 126.1e-3    | 1.648      | 1.648        | 18103        | 6.965e-6      | 85.916   | 20.21           | 264.1          | 23.97       |       |
|           |      | MinVolume       | 1.522           | 136.0e-3    | 1.658      | 1.658        | 19955        | 6.814e-6      | 85.957   | 21.79           | 265.7          | 26.42       |       |
|           |      | ApproxMinVolume | 1.515           | 136.0e-3    | 1.651      | 1.651        | 19995        | 6.802e-6      | 85.958   | 21.80           | 264.6          | 26.48       |       |
|           |      | Recursive       | 1.476           | 120.3e-3    | 1.597      | 1.597        | 17499        | 6.874e-6      | 85.900   | 19.28           | 255.9          | 23.17       |       |
|           |      | Recursive2      | 1.503           | 124.1e-3    | 1.627      | 1.627        | 17611        | 7.048e-6      | 85.900   | 19.89           | 260.8          | 23.32       |       |
| 4         |      | Interval        | 29.09e-6        | 188.1       | 188.1      | 03:08        | 19105663     | 9.845e-6      | 102.12   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 11.10           | 48.34       | 59.44      | 59.438       | 2005097      | 24.11e-6      | 101.19   | 25.70           | 31.60          | 10.49       |       |
|           |      | MinVariance     | 14.58           | 56.95       | 71.53      | 01:11        | 2217569      | 25.68e-6      | 101.24   | 30.28           | 38.03          | 11.61       |       |
|           |      | BSplines        | 11.73           | 131.2       | 142.9      | 02:22        | 5620563      | 23.34e-6      | 101.78   | 69.75           | 75.98          | 29.42       |       |
|           |      | MinNorm         | 14.39           | 54.10       | 68.49      | 01:08        | 2116625      | 25.56e-6      | 101.22   | 28.76           | 36.41          | 11.08       |       |
|           |      | MinVolume       | 12.83           | 60.24       | 73.07      | 01:13        | 2429159      | 24.80e-6      | 101.28   | 32.03           | 38.85          | 12.71       |       |
|           |      | ApproxMinVolume | 12.34           | 60.49       | 72.83      | 01:12        | 2434605      | 24.85e-6      | 101.28   | 32.16           | 38.72          | 12.74       |       |
|           |      | Recursive       | 14.83           | 50.82       | 65.66      | 01:05        | 2006849      | 25.32e-6      | 101.19   | 27.02           | 34.90          | 10.50       |       |
|           |      | Recursive2      | 15.74           | 52.36       | 68.09      | 01:08        | 2016491      | 25.97e-6      | 101.19   | 27.84           | 36.20          | 10.55       |       |
| 5         |      | Interval        | 25.03e-6        | 160487      | 160487     | 1-20:34:47   | 13868976571  | 11.57e-6      | 119.00   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 72.00           | 43819       | 43891      | 12:11:31     | 1073020121   | 40.84e-6      | 117.60   | 27.30           | 27.35          | 7.737       |       |
|           |      | MinVariance     | 75.06           | 48345       | 48420      | 13:27:00     | 1178829583   | 41.01e-6      | 117.67   | 30.12           | 30.17          | 8.500       |       |
|           |      | BSplines        | 72.58           | 116949      | 117022     | 1-08:30:22   | 2884178133   | 40.55e-6      | 118.42   | 72.87           | 72.92          | 20.80       |       |
|           |      | MinNorm         | 75.07           | 46753       | 46828      | 13:00:28     | 1129756421   | 41.38e-6      | 117.64   | 29.13           | 29.18          | 8.146       |       |
|           |      | MinVolume       | 73.23           | 51536       | 51609      | 14:20:09     | 1284918461   | 40.11e-6      | 117.73   | 32.11           | 32.16          | 9.265       |       |
|           |      | ApproxMinVolume | 73.40           | 53663       | 53736      | 14:55:36     | 1287711255   | 41.67e-6      | 117.73   | 33.44           | 33.48          | 9.285       |       |
|           |      | Recursive       | 69.94           | 43679       | 43749      | 12:09:09     | 1073021185   | 40.71e-6      | 117.60   | 27.22           | 27.26          | 7.737       |       |
|           |      | Recursive2      | 72.37           | 46092       | 46165      | 12:49:25     | 1088055199   | 42.36e-6      | 117.60   | 28.72           | 28.77          | 7.845       |       |

Table 21: Bisection of problem 2D with problem: 11, type: MinFirst

| precision | ndof | solver          | prep_time       | comput_time | total_time | (D-H-M:S.ms) | nb_iter     | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------|------|-----------------|-----------------|-------------|------------|--------------|-------------|---------------|----------|-----------------|----------------|-------------|-------|
| 0.001     | 2    | Interval        | 25.03e-6        | 4.878e-3    | 4.903e-3   | 0.005        | 299         | 16.31e-6      | 56.017   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 715.6e-3        | 3.639e-3    | 719.3e-3   | 0.719        | 291         | 12.51e-6      | 56.017   | 74.60           | 14669.7        | 97.32       |       |
|           |      | MinVariance     | 716.5e-3        | 3.320e-3    | 719.8e-3   | 0.720        | 291         | 11.41e-6      | 56.020   | 68.06           | 14681.1        | 97.32       |       |
|           |      | BSplines        | 719.7e-3        | 3.521e-3    | 723.3e-3   | 0.723        | 353         | 9.974e-6      | 56.047   | 72.18           | 14751.2        | 118.1       |       |
|           |      | MinNorm         | 730.4e-3        | 3.550e-3    | 734.0e-3   | 0.734        | 291         | 12.20e-6      | 56.019   | 72.78           | 14970.1        | 97.32       |       |
|           |      | MinVolume       | 727.2e-3        | 3.829e-3    | 731.0e-3   | 0.731        | 291         | 13.16e-6      | 56.022   | 78.49           | 14909.0        | 97.32       |       |
|           |      | ApproxMinVolume | 716.8e-3        | 3.509e-3    | 720.3e-3   | 0.720        | 291         | 12.06e-6      | 56.022   | 71.94           | 14691.4        | 97.32       |       |
|           |      | Recursive       | 724.0e-3        | 3.945e-3    | 728.0e-3   | 0.728        | 291         | 13.56e-6      | 56.017   | 80.87           | 14847.6        | 97.32       |       |
|           |      | Recursive2      | 718.8e-3        | 3.302e-3    | 722.1e-3   | 0.722        | 291         | 11.35e-6      | 56.017   | 67.69           | 14728.4        | 97.32       |       |
|           | 3    | Interval        | 28.85e-6        | 356.7e-3    | 356.7e-3   | 0.357        | 43383       | 8.221e-6      | 66.191   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 1.432           | 165.0e-3    | 1.597      | 1.597        | 24033       | 6.867e-6      | 66.184   | 46.27           | 447.7          | 55.40       |       |
|           |      | MinVariance     | 1.513           | 178.4e-3    | 1.692      | 1.692        | 25667       | 6.951e-6      | 66.188   | 50.03           | 474.3          | 59.16       |       |
|           |      | BSplines        | 1.445           | 315.3e-3    | 1.760      | 1.760        | 50105       | 6.293e-6      | 66.235   | 88.41           | 493.5          | 115.5       |       |
|           |      | MinNorm         | 1.517           | 172.5e-3    | 1.689      | 1.689        | 24875       | 6.934e-6      | 66.186   | 48.36           | 473.6          | 57.34       |       |
|           |      | MinVolume       | 1.479           | 185.6e-3    | 1.665      | 1.665        | 27379       | 6.779e-6      | 66.192   | 52.04           | 466.8          | 63.11       |       |
|           |      | ApproxMinVolume | 1.466           | 185.3e-3    | 1.651      | 1.651        | 27383       | 6.769e-6      | 66.192   | 51.97           | 462.8          | 63.12       |       |
|           |      | Recursive       | 1.466           | 164.6e-3    | 1.630      | 1.630        | 24047       | 6.844e-6      | 66.184   | 46.15           | 457.0          | 55.43       |       |
|           |      | Recursive2      | 1.479           | 168.4e-3    | 1.648      | 1.648        | 24181       | 6.965e-6      | 66.184   | 47.22           | 461.9          | 55.74       |       |
|           | 4    | Interval        | 26.94e-6        | 981.7       | 981.7      | 16.21        | 101228629   | 9.698e-6      | 60.598   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 11.08           | 240.0       | 251.1      | 04:11        | 10735221    | 22.36e-6      | 60.430   | 24.45           | 25.58          | 10.60       |       |
|           |      | MinVariance     | 14.49           | 304.3       | 318.8      | 05:18        | 12083445    | 25.18e-6      | 60.436   | 31.00           | 32.47          | 11.94       |       |
|           |      | BSplines        | 11.63           | 785.8       | 797.4      | 13:17        | 35977821    | 21.84e-6      | 60.512   | 80.05           | 81.23          | 35.54       |       |
|           |      | MinNorm         | 14.34           | 280.6       | 294.9      | 04:54        | 11453841    | 24.50e-6      | 60.433   | 28.58           | 30.04          | 11.31       |       |
|           |      | MinVolume       | 12.62           | 294.3       | 306.9      | 05:06        | 13450631    | 21.88e-6      | 60.443   | 29.98           | 31.26          | 13.29       |       |
|           |      | ApproxMinVolume | 12.33           | 327.7       | 340.0      | 05:40        | 13484069    | 24.30e-6      | 60.443   | 33.38           | 34.64          | 13.32       |       |
|           |      | Recursive       | 14.83           | 267.3       | 282.1      | 04:42        | 10737599    | 24.89e-6      | 60.430   | 27.23           | 28.74          | 10.61       |       |
|           |      | Recursive2      | 15.62           | 261.1       | 276.7      | 04:36        | 10753521    | 24.28e-6      | 60.430   | 26.60           | 28.19          | 10.62       |       |
|           | 5    | Interval        | 31.00e-6        | 1080000     | 1080000    | 12-12:00:00  | 93548792545 | 11.54e-6      | 68.184   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 72.30           | 200761      | 200761     | 2-07:46:01   | 5438985345  | 36.91e-6      | 68.008   | 18.59           | 18.59          | 5.814       |       |
|           |      | MinVariance     | 75.46           | 230761      | 230762     | 2-16:06:02   | 6217429189  | 37.12e-6      | 68.018   | 21.37           | 21.37          | 6.646       |       |
|           |      | BSplines        | 71.76           | 754119      | 754119     | 8-17:28:39   | 22437050555 | 33.61e-6      | 68.128   | 69.83           | 69.83          | 23.98       |       |
|           |      | MinNorm         | 74.83           | 216449      | 216449     | 2-12:07:29   | 5852510559  | 36.98e-6      | 68.013   | 20.04           | 20.04          | 6.256       |       |
|           |      | MinVolume       | 73.48           | 254547      | 254547     | 2-22:42:27   | 7028985835  | 36.21e-6      | 68.027   | 23.57           | 23.57          | 7.514       |       |
|           |      | ApproxMinVolume | 73.27           | 255586      | 255586     | 2-22:59:46   | 7040768432  | 36.30e-6      | 68.027   | 23.67           | 23.67          | 7.526       |       |
|           |      | Recursive       | 72.59           | 202257      | 202257     | 2-08:10:57   | 5429930518  | 37.25e-6      | 68.008   | 18.73           | 18.73          | 5.804       |       |
|           |      | Recursive2      | 71.92           | 199032      | 199032     | 2-07:17:12   | 5469998197  | 36.39e-6      | 68.008   | 18.43           | 18.43          | 5.847       |       |
|           | 0.01 | 2               | Interval        | 27.89e-6    | 3.334e-3   | 3.362e-3     | 0.003       | 137           | 24.34e-6 | 57.100          | 100.0          | 100.0       | 100.0 |
|           |      |                 | Bernstein       | 965.8e-3    | 6.902e-3   | 972.7e-3     | 0.973       | 109           | 63.32e-6 | 57.100          | 207.0          | 28931.7     | 79.56 |
|           |      |                 | MinVariance     | 966.4e-3    | 12.90e-3   | 979.4e-3     | 0.979       | 109           | 118.4e-6 | 57.119          | 387.0          | 29130.7     | 79.56 |
|           |      |                 | BSplines        | 966.9e-3    | 15.68e-3   | 982.6e-3     | 0.983       | 149           | 105.2e-6 | 57.342          | 470.2          | 29227.8     | 108.8 |
|           |      |                 | MinNorm         | 965.7e-3    | 13.96e-3   | 979.7e-3     | 0.980       | 109           | 128.0e-6 | 57.110          | 418.6          | 29140.8     | 79.56 |
|           |      |                 | MinVolume       | 965.4e-3    | 5.669e-3   | 971.1e-3     | 0.971       | 109           | 52.01e-6 | 57.137          | 170.0          | 28883.8     | 79.56 |
|           |      |                 | ApproxMinVolume | 963.6e-3    | 5.885e-3   | 969.5e-3     | 0.970       | 109           | 53.99e-6 | 57.138          | 176.5          | 28838.8     | 79.56 |
|           |      |                 | Recursive       | 960.4e-3    | 6.185e-3   | 966.6e-3     | 0.967       | 109           | 56.74e-6 | 57.100          | 185.5          | 28751.3     | 79.56 |
|           |      |                 | Recursive2      | 958.2e-3    | 6.852e-3   | 965.0e-3     | 0.965       | 109           | 62.86e-6 | 57.100          | 205.5          | 28704.6     | 79.56 |
| 3         |      | Interval        | 30.04e-6        | 172.4e-3    | 172.4e-3   | 0.172        | 19889       | 8.667e-6      | 67.355   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 1.436           | 67.12e-3    | 1.503      | 1.503        | 8995        | 7.462e-6      | 67.294   | 38.94           | 871.9          | 45.23       |       |
|           |      | MinVariance     | 1.513           | 73.50e-3    | 1.586      | 1.586        | 9653        | 7.614e-6      | 67.327   | 42.64           | 920.1          | 48.53       |       |
|           |      | BSplines        | 1.440           | 124.7e-3    | 1.565      | 1.565        | 18429       | 6.769e-6      | 67.709   | 72.37           | 907.7          | 92.66       |       |
|           |      | MinNorm         | 1.529           | 69.39e-3    | 1.598      | 1.598        | 9339        | 7.430e-6      | 67.312   | 40.25           | 927.0          | 46.96       |       |
|           |      | MinVolume       | 1.469           | 74.80e-3    | 1.544      | 1.544        | 10193       | 7.339e-6      | 67.358   | 43.40           | 895.5          | 51.25       |       |
|           |      | ApproxMinVolume | 1.459           | 75.19e-3    | 1.535      | 1.535        | 10199       | 7.372e-6      | 67.359   | 43.62           | 890.2          | 51.28       |       |
|           |      | Recursive       | 1.477           | 66.86e-3    | 1.544      | 1.544        | 9011        | 7.420e-6      | 67.294   | 38.79           | 895.8          | 45.31       |       |
|           |      | Recursive2      | 1.471           | 68.14e-3    | 1.539      | 1.539        | 9143        | 7.453e-6      | 67.294   | 39.53           | 892.6          | 45.97       |       |
| 4         |      | Interval        | 32.19e-6        | 92.62       | 92.62      | 01:32        | 9453841     | 9.798e-6      | 63.021   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 11.22           | 32.11       | 43.33      | 43.332       | 1365633     | 23.52e-6      | 61.646   | 34.67           | 46.78          | 14.45       |       |
|           |      | MinVariance     | 14.55           | 38.54       | 53.09      | 53.091       | 1505723     | 25.59e-6      | 61.697   | 41.61           | 57.32          | 15.93       |       |
|           |      | BSplines        | 11.77           | 84.89       | 96.66      | 01:36        | 3652473     | 23.24e-6      | 62.298   | 91.65           | 104.4          | 38.63       |       |
|           |      | MinNorm         | 14.21           | 36.33       | 50.54      | 50.536       | 1440987     | 25.21e-6      | 61.674   | 39.22           | 54.56          | 15.24       |       |
|           |      | MinVolume       | 12.79           | 41.95       | 54.74      | 54.743       | 1643093     | 25.53e-6      | 61.746   | 45.29           | 59.10          | 17.38       |       |
|           |      | ApproxMinVolume | 12.49           | 40.35       | 52.84      | 52.836       | 1646349     | 24.51e-6      | 61.747   | 43.56           | 57.04          | 17.41       |       |
|           |      | Recursive       | 14.78           | 33.85       | 48.63      | 48.633       | 1367341     | 24.76e-6      | 61.646   | 36.54           | 52.51          | 14.46       |       |
|           |      | Recursive2      | 15.76           | 34.88       | 50.64      | 50.642       | 1377377     | 25.33e-6      | 61.646   | 37.66           | 54.68          | 14.57       |       |
| 5         |      | Interval        | 27.89e-6        | 59962       | 59962      | 16:39:22     | 5023190703  | 11.94e-6      | 71.090   | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 73.11           | 18268       | 18341      | 05:05:41     | 453747671   | 40.26e-6      | 69.785   | 30.47           | 30.59          | 9.033       |       |
|           |      | MinVariance     | 75.51           | 21196       | 21272      | 05:54:32     | 510747573   | 41.50e-6      | 69.861   | 35.35           | 35.48          | 10.17       |       |
|           |      | BSplines        | 72.34           | 61019       | 61091      | 16:58:11     | 1568563887  | 38.90e-6      | 70.743   | 101.8           | 101.9          | 31.23       |       |
|           |      | MinNorm         | 75.08           | 20128       | 20203      | 05:36:43     | 484179977   | 41.57e-6      | 69.826   | 33.57           | 33.69          | 9.639       |       |
|           |      | MinVolume       | 73.76           | 23774       | 23848      | 06:37:28     | 569356941   | 41.76e-6      | 69.932   | 39.65           | 39.77          | 11.33       |       |
|           |      | ApproxMinVolume | 73.21           | 22885       | 22958      | 06:22:38     | 570913097   | 40.09e-6      | 69.934   | 38.17           | 38.29          | 11.37       |       |
|           |      | Recursive       | 72.43           | 18612       | 18684      | 05:11:24     | 454231491   | 40.98e-6      | 69.785   | 31.04           | 31.16          | 9.043       |       |
|           |      | Recursive2      | 72.50           | 19138       | 19211      | 05:20:11     | 466446525   | 41.03e-6      | 69.785   | 31.92           | 32.04          | 9.286       |       |

Table 22: Bisection of problem 2D with problem: 12, type: MinFirst

| precision       | ndof            | solver          | prep_time   | comput_time | total_time  | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------------|-----------------|-----------------|-------------|-------------|-------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|-------|
| 0.001           | 2               | Interval        | 33.14e-6    | 2.741e-3    | 2.774e-3    | 0.003        | 59       | 46.46e-6      | infeasible | 100.0           | 100.0          | 100.0       |       |
|                 |                 | Bernstein       | 715.2e-3    | 2.954e-3    | 718.1e-3    | 0.718        | 41       | 72.05e-6      | infeasible | 107.8           | 25888.0        | 69.49       |       |
|                 |                 | MinVariance     | 718.0e-3    | 3.079e-3    | 721.1e-3    | 0.721        | 41       | 75.10e-6      | infeasible | 112.3           | 25994.1        | 69.49       |       |
|                 |                 | BSplines        | 708.7e-3    | 2.879e-3    | 711.6e-3    | 0.712        | 41       | 70.22e-6      | infeasible | 105.0           | 25651.4        | 69.49       |       |
|                 |                 | MinNorm         | 717.4e-3    | 3.403e-3    | 720.8e-3    | 0.721        | 41       | 83.00e-6      | infeasible | 124.2           | 25984.8        | 69.49       |       |
|                 |                 | MinVolume       | 713.5e-3    | 3.582e-3    | 717.1e-3    | 0.717        | 41       | 87.37e-6      | infeasible | 130.7           | 25852.2        | 69.49       |       |
|                 |                 | ApproxMinVolume | 714.9e-3    | 3.151e-3    | 718.0e-3    | 0.718        | 41       | 76.85e-6      | infeasible | 115.0           | 25884.8        | 69.49       |       |
|                 |                 | Recursive       | 702.8e-3    | 3.903e-3    | 706.7e-3    | 0.707        | 41       | 95.19e-6      | infeasible | 142.4           | 25477.2        | 69.49       |       |
|                 |                 | Recursive2      | 707.4e-3    | 3.192e-3    | 710.6e-3    | 0.711        | 41       | 77.85e-6      | infeasible | 116.5           | 25614.9        | 69.49       |       |
|                 |                 | Interval        | 34.09e-6    | 77.77e-3    | 77.80e-3    | 0.078        | 8983     | 8.657e-6      | 45.742     | 100.0           | 100.0          | 100.0       |       |
|                 |                 | Bernstein       | 1.434       | 48.36e-3    | 1.482       | 1.482        | 6787     | 7.125e-6      | 45.738     | 62.19           | 1905.3         | 75.55       |       |
|                 |                 | MinVariance     | 1.537       | 61.76e-3    | 1.599       | 1.599        | 7429     | 8.313e-6      | 45.742     | 79.41           | 2054.8         | 82.70       |       |
|                 | BSplines        | 1.468           | 84.18e-3    | 1.552       | 1.552       | 12745        | 6.605e-6 | 45.794        | 108.2      | 1994.8          | 141.9          |             |       |
|                 | MinNorm         | 1.531           | 51.59e-3    | 1.583       | 1.583       | 7075         | 7.292e-6 | 45.740        | 66.34      | 2034.5          | 78.76          |             |       |
|                 | MinVolume       | 1.464           | 55.01e-3    | 1.519       | 1.519       | 8017         | 6.861e-6 | 45.746        | 70.73      | 1953.0          | 89.25          |             |       |
|                 | ApproxMinVolume | 1.474           | 56.34e-3    | 1.531       | 1.531       | 8021         | 7.024e-6 | 45.746        | 72.45      | 1967.4          | 89.29          |             |       |
|                 | Recursive       | 1.470           | 48.74e-3    | 1.519       | 1.519       | 6787         | 7.182e-6 | 45.738        | 62.68      | 1952.1          | 75.55          |             |       |
|                 | Recursive2      | 1.469           | 50.85e-3    | 1.520       | 1.520       | 6867         | 7.404e-6 | 45.738        | 65.38      | 1953.4          | 76.44          |             |       |
|                 | Interval        | 29.09e-6        | 40.04       | 40.04       | 40.041      | 4047819      | 9.892e-6 | 35.418        | 100.0      | 100.0           | 100.0          |             |       |
|                 | Bernstein       | 11.04           | 20.32       | 31.36       | 31.359      | 884087       | 22.98e-6 | 35.363        | 50.74      | 78.32           | 21.84          |             |       |
|                 | MinVariance     | 14.35           | 25.43       | 39.78       | 39.780      | 991345       | 25.65e-6 | 35.368        | 63.51      | 99.35           | 24.49          |             |       |
|                 | BSplines        | 11.76           | 71.92       | 83.68       | 01:23       | 3122817      | 23.03e-6 | 35.424        | 179.6      | 209.0           | 77.15          |             |       |
|                 | MinNorm         | 14.40           | 23.50       | 37.90       | 37.897      | 940015       | 25.00e-6 | 35.365        | 58.69      | 94.65           | 23.22          |             |       |
|                 | MinVolume       | 12.60           | 27.34       | 39.94       | 39.940      | 1101689      | 24.82e-6 | 35.372        | 68.29      | 99.75           | 27.22          |             |       |
|                 | ApproxMinVolume | 12.32           | 26.83       | 39.15       | 39.154      | 1104841      | 24.28e-6 | 35.372        | 67.01      | 97.79           | 27.29          |             |       |
|                 | Recursive       | 14.75           | 21.96       | 36.71       | 36.707      | 884655       | 24.82e-6 | 35.363        | 54.84      | 91.67           | 21.86          |             |       |
|                 | Recursive2      | 15.44           | 21.88       | 37.33       | 37.326      | 888681       | 24.62e-6 | 35.363        | 54.65      | 93.22           | 21.95          |             |       |
|                 | Interval        | 28.00e-6        | 1246720     | 1246720     | 14:10:18.40 | 110942776513 | 11.24e-6 | 35.082        | 100.0      | 100.0           | 100.0          |             |       |
|                 | Bernstein       | 71.59           | 844226      | 844226      | 9:18:30.26  | 23687231457  | 35.64e-6 | 34.945        | 67.72      | 67.72           | 21.35          |             |       |
|                 | MinVariance     | 75.31           | 948177      | 948177      | 10:23:22.57 | 26508599413  | 35.77e-6 | 34.953        | 76.05      | 76.05           | 23.89          |             |       |
|                 | BSplines        | 71.83           | 2605510     | 2605510     | 30:03:45.10 | 79706071050  | 32.69e-6 | 35.045        | 209.0      | 209.0           | 71.84          |             |       |
|                 | MinNorm         | 74.79           | 902753      | 902754      | 10:10:45.54 | 25193676565  | 35.83e-6 | 34.949        | 72.41      | 72.41           | 22.71          |             |       |
|                 | MinVolume       | 73.43           | 1037130     | 1037130     | 12:00:05.30 | 29414193493  | 35.26e-6 | 34.961        | 83.19      | 83.19           | 26.51          |             |       |
|                 | ApproxMinVolume | 73.37           | 1042450     | 1042450     | 12:01:34.10 | 29491066469  | 35.35e-6 | 34.961        | 83.62      | 83.62           | 26.58          |             |       |
|                 | Recursive       | 72.59           | 853486      | 853486      | 9:21:04.46  | 23739565473  | 35.95e-6 | 34.945        | 68.46      | 68.46           | 21.40          |             |       |
|                 | Recursive2      | 72.35           | 855328      | 855328      | 9:21:35.28  | 23834461967  | 35.89e-6 | 34.945        | 68.61      | 68.61           | 21.48          |             |       |
|                 | 0.01            | 2               | Interval    | 32.19e-6    | 3.798e-3    | 3.830e-3     | 0.004    | 59            | 64.37e-6   | infeasible      | 100.0          | 100.0       | 100.0 |
|                 |                 |                 | Bernstein   | 765.8e-3    | 6.239e-3    | 772.0e-3     | 0.772    | 41            | 152.2e-6   | infeasible      | 164.3          | 20156.7     | 69.49 |
|                 |                 |                 | MinVariance | 769.2e-3    | 7.156e-3    | 776.4e-3     | 0.776    | 41            | 174.5e-6   | infeasible      | 188.4          | 20269.5     | 69.49 |
|                 |                 |                 | BSplines    | 766.2e-3    | 3.621e-3    | 769.9e-3     | 0.770    | 41            | 88.31e-6   | infeasible      | 95.34          | 20100.1     | 69.49 |
| MinNorm         |                 |                 | 765.4e-3    | 9.018e-3    | 774.4e-3    | 0.774        | 41       | 220.0e-6      | infeasible | 237.4           | 20218.5        | 69.49       |       |
| MinVolume       |                 |                 | 764.8e-3    | 4.676e-3    | 769.5e-3    | 0.769        | 41       | 114.0e-6      | infeasible | 123.1           | 20089.4        | 69.49       |       |
| ApproxMinVolume |                 |                 | 765.9e-3    | 4.660e-3    | 770.6e-3    | 0.771        | 41       | 113.7e-6      | infeasible | 122.7           | 20119.1        | 69.49       |       |
| Recursive       |                 |                 | 765.8e-3    | 6.749e-3    | 772.6e-3    | 0.773        | 41       | 164.6e-6      | infeasible | 177.7           | 20170.6        | 69.49       |       |
| Recursive2      |                 |                 | 750.1e-3    | 5.040e-3    | 755.2e-3    | 0.755        | 41       | 122.9e-6      | infeasible | 132.7           | 19715.9        | 69.49       |       |
| Interval        |                 |                 | 29.09e-6    | 35.99e-3    | 36.02e-3    | 0.036        | 4027     | 8.938e-6      | 46.854     | 100.0           | 100.0          | 100.0       |       |
| Bernstein       |                 |                 | 1.452       | 23.00e-3    | 1.475       | 1.475        | 2903     | 7.924e-6      | 46.813     | 63.91           | 4094.0         | 72.09       |       |
| MinVariance     |                 |                 | 1.536       | 24.88e-3    | 1.561       | 1.561        | 3149     | 7.901e-6      | 46.849     | 69.13           | 4334.4         | 78.20       |       |
| BSplines        |                 | 1.438           | 34.33e-3    | 1.472       | 1.472       | 4851         | 7.076e-6 | 47.265        | 95.38      | 4086.5          | 120.5          |             |       |
| MinNorm         |                 | 1.526           | 23.96e-3    | 1.550       | 1.550       | 3033         | 7.898e-6 | 46.833        | 66.56      | 4303.4          | 75.32          |             |       |
| MinVolume       |                 | 1.489           | 25.37e-3    | 1.514       | 1.514       | 3307         | 7.673e-6 | 46.883        | 70.50      | 4203.5          | 82.12          |             |       |
| ApproxMinVolume |                 | 1.473           | 26.79e-3    | 1.500       | 1.500       | 3307         | 8.102e-6 | 46.884        | 74.44      | 4164.8          | 82.12          |             |       |
| Recursive       |                 | 1.479           | 23.12e-3    | 1.503       | 1.503       | 2903         | 7.964e-6 | 46.813        | 64.23      | 4171.2          | 72.09          |             |       |
| Recursive2      |                 | 1.482           | 23.73e-3    | 1.506       | 1.506       | 2971         | 7.986e-6 | 46.813        | 65.92      | 4179.6          | 73.78          |             |       |
| Interval        |                 | 35.05e-6        | 12.89       | 12.89       | 12.890      | 1281177      | 10.06e-6 | 36.786        | 100.0      | 100.0           | 100.0          |             |       |
| Bernstein       |                 | 11.20           | 6.961       | 18.16       | 18.163      | 310257       | 22.44e-6 | 36.315        | 54.00      | 140.9           | 24.22          |             |       |
| MinVariance     |                 | 14.69           | 9.455       | 24.14       | 24.142      | 343109       | 27.56e-6 | 36.354        | 73.35      | 187.3           | 26.78          |             |       |
| BSplines        |                 | 11.78           | 21.69       | 33.47       | 33.470      | 896567       | 24.20e-6 | 36.812        | 168.3      | 259.7           | 69.98          |             |       |
| MinNorm         |                 | 14.49           | 8.509       | 22.99       | 22.995      | 328021       | 25.94e-6 | 36.337        | 66.01      | 178.4           | 25.60          |             |       |
| MinVolume       |                 | 12.71           | 9.624       | 22.33       | 22.331      | 376629       | 25.55e-6 | 36.392        | 74.66      | 173.2           | 29.40          |             |       |
| ApproxMinVolume |                 | 12.30           | 9.460       | 21.76       | 21.764      | 377447       | 25.06e-6 | 36.392        | 73.39      | 168.8           | 29.46          |             |       |
| Recursive       |                 | 14.89           | 7.417       | 22.31       | 22.307      | 310731       | 23.87e-6 | 36.315        | 57.54      | 173.1           | 24.25          |             |       |
| Recursive2      |                 | 15.57           | 8.182       | 23.76       | 23.756      | 315263       | 25.95e-6 | 36.315        | 63.48      | 184.3           | 24.61          |             |       |
| Interval        |                 | 30.04e-6        | 17303       | 17303       | 04:48:23    | 1503905297   | 11.51e-6 | 37.512        | 100.0      | 100.0           | 100.0          |             |       |
| Bernstein       |                 | 72.08           | 19581       | 19653       | 05:27:33    | 494270911    | 39.62e-6 | 36.421        | 113.2      | 113.6           | 32.87          |             |       |
| MinVariance     |                 | 75.25           | 21644       | 21719       | 06:01:59    | 538540131    | 40.19e-6 | 36.485        | 125.1      | 125.5           | 35.81          |             |       |
| BSplines        |                 | 72.41           | 44562       | 44634       | 12:23:54    | 1203225521   | 37.04e-6 | 37.240        | 257.5      | 258.0           | 80.01          |             |       |
| MinNorm         |                 | 74.93           | 20680       | 20755       | 05:45:55    | 518072165    | 39.92e-6 | 36.456        | 119.5      | 119.9           | 34.45          |             |       |
| MinVolume       |                 | 73.29           | 23076       | 23149       | 06:25:49    | 582455005    | 39.62e-6 | 36.545        | 133.4      | 133.8           | 38.73          |             |       |
| ApproxMinVolume |                 | 73.53           | 23429       | 23502       | 06:31:42    | 583598409    | 40.15e-6 | 36.547        | 135.4      | 135.8           | 38.81          |             |       |
| Recursive       |                 | 72.78           | 20300       | 20372       | 05:39:32    | 495554445    | 40.96e-6 | 36.421        | 117.3      | 117.7           | 32.95          |             |       |
| Recursive2      |                 | 72.18           | 20136       | 20208       | 05:36:48    | 508249521    | 39.62e-6 | 36.421        | 116.4      | 116.8           | 33.80          |             |       |

Table 23: Bisection of problem 2D with problem: 13, type: MinFirst



| precision       | ndof            | solver          | prep_time | comput_time | total_time  | (D:H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------------|-----------|-------------|-------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | 2               | Interval        | 29.09e-6  | 4.641e-3    | 4.670e-3    | 0.005        | 309      | 15.02e-6      | 56.197   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 806.1e-3  | 3.935e-3    | 810.0e-3    | 0.810        | 285      | 13.81e-6      | 56.197   | 84.79           | 17345.3        | 92.23       |
|                 |                 | MinVariance     | 844.3e-3  | 3.985e-3    | 848.3e-3    | 0.848        | 305      | 13.07e-6      | 56.200   | 85.86           | 18164.2        | 98.71       |
|                 |                 | BSplines        | 843.0e-3  | 5.383e-3    | 848.4e-3    | 0.848        | 581      | 9.265e-6      | 56.287   | 116.0           | 18166.0        | 188.0       |
|                 |                 | MinNorm         | 822.5e-3  | 4.220e-3    | 826.7e-3    | 0.827        | 297      | 14.21e-6      | 56.199   | 90.93           | 17702.1        | 96.12       |
|                 |                 | MinVolume       | 852.4e-3  | 4.451e-3    | 856.9e-3    | 0.857        | 319      | 13.95e-6      | 56.202   | 95.91           | 18348.1        | 103.2       |
|                 |                 | ApproxMinVolume | 832.0e-3  | 4.921e-3    | 837.0e-3    | 0.837        | 319      | 15.43e-6      | 56.202   | 106.0           | 17922.0        | 103.2       |
|                 |                 | Recursive       | 868.8e-3  | 4.085e-3    | 872.9e-3    | 0.873        | 347      | 11.77e-6      | 56.197   | 88.02           | 18691.2        | 112.3       |
|                 |                 | Recursive2      | 883.3e-3  | 4.488e-3    | 887.8e-3    | 0.888        | 301      | 14.91e-6      | 56.197   | 96.70           | 19009.5        | 97.41       |
|                 |                 | Interval        | 33.14e-6  | 488.4e-3    | 488.4e-3    | 0.488        | 57467    | 8.498e-6      | 66.964   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 3.381     | 324.6e-3    | 3.706       | 3.706        | 27483    | 11.81e-6      | 66.956   | 66.46           | 758.7          | 47.82       |
|                 |                 | MinVariance     | 4.675     | 431.3e-3    | 5.106       | 5.106        | 32117    | 13.43e-6      | 66.960   | 88.30           | 1045.5         | 55.89       |
|                 | BSplines        | 3.546           | 1.496     | 5.042       | 5.042       | 131623       | 11.37e-6 | 67.056        | 306.4    | 1032.4          | 229.0          |             |
|                 | MinNorm         | 4.474           | 385.8e-3  | 4.859       | 4.859       | 29939        | 12.89e-6 | 66.958        | 78.99    | 994.9           | 52.10          |             |
|                 | MinVolume       | 3.931           | 497.1e-3  | 4.428       | 4.428       | 43345        | 11.47e-6 | 66.964        | 101.8    | 906.7           | 75.43          |             |
|                 | ApproxMinVolume | 3.972           | 507.7e-3  | 4.480       | 4.480       | 43481        | 11.68e-6 | 66.964        | 104.0    | 917.2           | 75.66          |             |
|                 | Recursive       | 3.871           | 388.9e-3  | 4.260       | 4.260       | 30669        | 12.68e-6 | 66.956        | 79.63    | 872.1           | 53.37          |             |
|                 | Recursive2      | 4.411           | 363.1e-3  | 4.774       | 4.774       | 28249        | 12.86e-6 | 66.956        | 74.36    | 977.5           | 49.16          |             |
|                 | Interval        | 22.89e-6        | 1380      | 1380        | 23.00       | 138635525    | 9.960e-6 | 61.348        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 11.50           | 407.6     | 419.1       | 06:59       | 14467777     | 28.17e-6 | 61.169        | 29.52    | 30.35           | 10.44          |             |
|                 | MinVariance     | 15.26           | 580.8     | 596.0       | 09:56       | 18199891     | 31.91e-6 | 61.181        | 42.06    | 43.16           | 13.13          |             |
|                 | BSplines        | 12.00           | 2987      | 2999        | 49:59       | 107209103    | 27.87e-6 | 61.321        | 216.4    | 217.2           | 77.33          |             |
|                 | MinNorm         | 14.75           | 509.1     | 523.9       | 08:43       | 16555737     | 30.75e-6 | 61.175        | 36.87    | 37.94           | 11.94          |             |
|                 | MinVolume       | 13.23           | 673.6     | 686.8       | 11:26       | 22183567     | 30.36e-6 | 61.193        | 48.78    | 49.74           | 16.00          |             |
|                 | ApproxMinVolume | 12.96           | 666.9     | 679.8       | 11:19       | 22275089     | 29.94e-6 | 61.193        | 48.29    | 49.23           | 16.07          |             |
|                 | Recursive       | 15.72           | 466.4     | 482.1       | 08:02       | 15596101     | 29.90e-6 | 61.169        | 33.78    | 34.91           | 11.25          |             |
|                 | Recursive2      | 15.69           | 478.5     | 494.2       | 08:14       | 15645561     | 30.59e-6 | 61.169        | 34.66    | 35.79           | 11.29          |             |
|                 | Interval        | 29.00e-6        | 1735250   | 1735250     | 20:02:00:50 | 151278296982 | 11.47e-6 | 69.047        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 73.84           | 389912    | 389913      | 4:12:18:33  | 7632334923   | 51.09e-6 | 68.855        | 22.47    | 22.47           | 5.045          |             |
|                 | MinVariance     | 77.97           | 517962    | 517962      | 5:23:52:42  | 10073952213  | 51.42e-6 | 68.873        | 29.85    | 29.85           | 6.659          |             |
|                 | BSplines        | 73.72           | 4820730   | 4820730     | 55:19:05:30 | 94535010765  | 50.99e-6 | 69.079        | 277.8    | 277.8           | 62.49          |             |
|                 | MinNorm         | 77.49           | 463219    | 463219      | 5:08:40:19  | 8851231154   | 52.33e-6 | 68.865        | 26.69    | 26.69           | 5.851          |             |
|                 | MinVolume       | 75.61           | 656363    | 656363      | 7:14:19:23  | 12904267211  | 50.86e-6 | 68.890        | 37.83    | 37.83           | 8.530          |             |
|                 | ApproxMinVolume | 75.61           | 668362    | 668362      | 7:17:39:22  | 12984726841  | 51.47e-6 | 68.890        | 38.52    | 38.52           | 8.583          |             |
|                 | Recursive       | 73.69           | 450767    | 450767      | 5:05:12:47  | 8626207494   | 52.26e-6 | 68.855        | 25.98    | 25.98           | 5.702          |             |
|                 | Recursive2      | 74.10           | 415463    | 415463      | 4:19:24:23  | 8053202773   | 51.59e-6 | 68.855        | 23.94    | 23.94           | 5.323          |             |
|                 | Interval        | 30.99e-6        | 3.422e-3  | 3.453e-3    | 0.003       | 143          | 23.93e-6 | 57.100        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 1.076           | 4.596e-3  | 1.081       | 1.081       | 105          | 43.77e-6 | 57.100        | 134.3    | 13297.0         | 73.43          |             |
|                 | MinVariance     | 1.076           | 5.231e-3  | 1.081       | 1.081       | 113          | 46.29e-6 | 57.119        | 152.9    | 31315.2         | 79.02          |             |
|                 | BSplines        | 1.076           | 4.616e-3  | 1.081       | 1.081       | 279          | 16.54e-6 | 57.825        | 134.9    | 31296.7         | 195.1          |             |
|                 | MinNorm         | 2.260           | 10.44e-3  | 2.271       | 2.271       | 109          | 95.79e-6 | 57.110        | 305.1    | 65758.7         | 76.22          |             |
|                 | MinVolume       | 2.260           | 9.986e-3  | 2.270       | 2.270       | 127          | 78.63e-6 | 57.137        | 291.8    | 65742.7         | 88.81          |             |
|                 | ApproxMinVolume | 2.262           | 12.37e-3  | 2.274       | 2.274       | 127          | 97.39e-6 | 57.138        | 361.4    | 65854.5         | 88.81          |             |
|                 | Recursive       | 2.261           | 11.65e-3  | 2.273       | 2.273       | 163          | 71.45e-6 | 57.100        | 340.4    | 65823.8         | 114.0          |             |
|                 | Recursive2      | 2.260           | 8.224e-3  | 2.268       | 2.268       | 121          | 67.97e-6 | 57.100        | 240.3    | 65685.7         | 84.62          |             |
|                 | Interval        | 23.84e-6        | 184.2e-3  | 184.2e-3    | 0.184       | 21921        | 8.402e-6 | 68.518        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 3.390           | 125.6e-3  | 3.516       | 3.516       | 9905         | 12.68e-6 | 68.447        | 68.21    | 1908.8          | 45.18          |             |
|                 | MinVariance     | 4.704           | 153.9e-3  | 4.858       | 4.858       | 11315        | 13.60e-6 | 68.480        | 83.57    | 2637.3          | 51.62          |             |
| BSplines        | 3.580           | 539.7e-3        | 4.120     | 4.120       | 43593       | 12.38e-6     | 69.244   | 293.0         | 2236.5   | 198.9           |                |             |
| MinNorm         | 4.508           | 138.4e-3        | 4.646     | 4.646       | 10539       | 13.13e-6     | 68.465   | 75.16         | 2522.5   | 48.08           |                |             |
| MinVolume       | 3.926           | 163.7e-3        | 4.089     | 4.089       | 13067       | 12.53e-6     | 68.511   | 88.89         | 2220.1   | 59.61           |                |             |
| ApproxMinVolume | 3.982           | 173.0e-3        | 4.155     | 4.155       | 13105       | 13.20e-6     | 68.512   | 93.94         | 2255.6   | 59.78           |                |             |
| Recursive       | 3.876           | 174.0e-3        | 4.050     | 4.050       | 13539       | 12.86e-6     | 68.447   | 94.50         | 2198.8   | 61.76           |                |             |
| Recursive2      | 4.352           | 138.7e-3        | 4.491     | 4.491       | 10825       | 12.81e-6     | 68.447   | 75.29         | 2438.2   | 49.38           |                |             |
| Interval        | 30.04e-6        | 100.7           | 100.7     | 01:40       | 9683383     | 10.40e-6     | 63.828   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 11.65           | 42.43           | 54.08     | 54.077      | 1446751     | 29.33e-6     | 62.291   | 42.12         | 53.69    | 14.94           |                |             |
| MinVariance     | 15.39           | 57.88           | 73.27     | 01:13       | 1767887     | 32.74e-6     | 62.387   | 57.47         | 72.74    | 18.26           |                |             |
| BSplines        | 12.12           | 282.2           | 294.3     | 04:54       | 9458327     | 29.83e-6     | 63.535   | 280.1         | 292.2    | 97.68           |                |             |
| MinNorm         | 15.10           | 50.92           | 66.03     | 01:06       | 1597585     | 31.88e-6     | 62.323   | 50.56         | 65.55    | 16.50           |                |             |
| MinVolume       | 13.19           | 65.02           | 78.21     | 01:18       | 2149515     | 30.25e-6     | 62.454   | 64.55         | 77.65    | 22.20           |                |             |
| ApproxMinVolume | 13.09           | 65.02           | 78.11     | 01:18       | 2158605     | 30.12e-6     | 62.457   | 64.55         | 77.54    | 22.29           |                |             |
| Recursive       | 15.90           | 61.37           | 77.26     | 01:17       | 1987681     | 30.87e-6     | 62.291   | 60.93         | 76.71    | 20.53           |                |             |
| Recursive2      | 15.85           | 66.40           | 82.25     | 01:22       | 2143553     | 30.98e-6     | 62.291   | 65.93         | 81.66    | 22.14           |                |             |
| Interval        | 35.05e-6        | 53601           | 53601     | 14:53:21    | 4690020111  | 11.43e-6     | 72.049   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 99.05           | 22184           | 22283     | 06:11:23    | 416835919   | 53.22e-6     | 70.530   | 41.39         | 41.57    | 8.888           |                |             |
| MinVariance     | 103.2           | 29305           | 29408     | 08:10:08    | 542403241   | 54.03e-6     | 70.642   | 54.67         | 54.87    | 11.57           |                |             |
| BSplines        | 98.89           | 267826          | 267826    | 3:02:23:46  | 5106155908  | 52.45e-6     | 72.337   | 499.7         | 499.7    | 108.9           |                |             |
| MinNorm         | 102.6           | 25621           | 25724     | 07:08:44    | 476926901   | 53.72e-6     | 70.568   | 47.80         | 47.99    | 10.17           |                |             |
| MinVolume       | 101.5           | 40851           | 40953     | 11:22:33    | 706534125   | 57.82e-6     | 70.758   | 76.21         | 76.40    | 15.06           |                |             |
| ApproxMinVolume | 100.9           | 37214           | 37315     | 10:21:55    | 710494281   | 52.38e-6     | 70.760   | 69.43         | 69.62    | 15.15           |                |             |
| Recursive       | 98.93           | 39381           | 39480     | 10:58:00    | 752622697   | 52.33e-6     | 70.530   | 73.47         | 73.66    | 16.05           |                |             |
| Recursive2      | 99.34           | 32623           | 32722     | 09:05:22    | 619323045   | 52.68e-6     | 70.530   | 60.86         | 61.05    | 13.21           |                |             |

Table 25: Bisection of problem 2D with problem: 15, type: MinFirst

| precision       | ndof            | solver          | prep_time   | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------------|-----------------|-----------------|-------------|-------------|------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|-------|
| 0.001           | 2               | Interval        | 30.99e-6    | 3.115e-3    | 3.146e-3   | 0.003        | 89       | 35.00e-6      | infeasible | 100.0           | 100.0          | 100.0       |       |
|                 |                 | Bernstein       | 855.7e-3    | 3.047e-3    | 858.8e-3   | 0.859        | 65       | 46.88e-6      | infeasible | 97.82           | 27297.8        | 73.03       |       |
|                 |                 | MinVariance     | 901.0e-3    | 3.460e-3    | 904.4e-3   | 0.904        | 61       | 56.72e-6      | infeasible | 111.1           | 28749.6        | 68.54       |       |
|                 |                 | BSplines        | 840.7e-3    | 3.297e-3    | 844.1e-3   | 0.844        | 111      | 29.70e-6      | infeasible | 105.8           | 26830.0        | 124.7       |       |
|                 |                 | MinNorm         | 891.9e-3    | 3.185e-3    | 895.1e-3   | 0.895        | 59       | 53.98e-6      | infeasible | 102.3           | 28451.7        | 66.29       |       |
|                 |                 | MinVolume       | 882.0e-3    | 3.016e-3    | 885.0e-3   | 0.885        | 69       | 43.71e-6      | infeasible | 96.82           | 28131.4        | 77.53       |       |
|                 |                 | ApproxMinVolume | 891.4e-3    | 3.407e-3    | 894.8e-3   | 0.895        | 69       | 49.38e-6      | infeasible | 109.4           | 28442.5        | 77.53       |       |
|                 |                 | Recursive       | 867.5e-3    | 3.394e-3    | 871.0e-3   | 0.871        | 91       | 37.30e-6      | infeasible | 109.0           | 27685.0        | 102.2       |       |
|                 |                 | Recursive2      | 846.2e-3    | 3.417e-3    | 849.6e-3   | 0.850        | 119      | 28.71e-6      | infeasible | 109.7           | 27006.6        | 133.7       |       |
|                 |                 | Interval        | 28.85e-6    | 426.2e-3    | 426.3e-3   | 0.426        | 50577    | 8.428e-6      | 47.608     | 100.0           | 100.0          | 100.0       |       |
|                 |                 | Bernstein       | 3.404       | 397.4e-3    | 3.801      | 3.801        | 31069    | 12.79e-6      | 47.567     | 93.23           | 891.8          | 61.43       |       |
|                 |                 | MinVariance     | 4.727       | 503.3e-3    | 5.230      | 5.230        | 35549    | 14.16e-6      | 47.571     | 118.1           | 1227.0         | 70.29       |       |
|                 |                 | BSplines        | 3.600       | 1.390       | 4.990      | 4.990        | 109921   | 12.65e-6      | 47.694     | 326.2           | 1170.6         | 217.3       |       |
|                 |                 | MinNorm         | 4.511       | 447.3e-3    | 4.958      | 4.958        | 33345    | 13.41e-6      | 47.569     | 104.9           | 1163.2         | 65.93       |       |
|                 | MinVolume       | 3.919           | 522.1e-3    | 4.441       | 4.441      | 39941        | 13.07e-6 | 47.575        | 122.5      | 1041.8          | 78.97          |             |       |
|                 | ApproxMinVolume | 3.914           | 531.4e-3    | 4.445       | 4.445      | 40003        | 13.28e-6 | 47.576        | 124.7      | 1042.8          | 79.09          |             |       |
|                 | Recursive       | 3.872           | 411.9e-3    | 4.284       | 4.284      | 32383        | 12.72e-6 | 47.567        | 96.63      | 1005.0          | 64.03          |             |       |
|                 | Recursive2      | 4.388           | 423.5e-3    | 4.812       | 4.812      | 31859        | 13.29e-6 | 47.567        | 99.37      | 1128.8          | 62.99          |             |       |
|                 | Interval        | 30.99e-6        | 115.5       | 115.5       | 0.155      | 11486469     | 10.06e-6 | 35.729        | 100.0      | 100.0           | 100.0          |             |       |
|                 | Bernstein       | 11.52           | 72.19       | 83.70       | 0.123      | 2535375      | 28.47e-6 | 35.671        | 62.48      | 72.45           | 22.07          |             |       |
|                 | MinVariance     | 15.28           | 105.4       | 120.7       | 0.200      | 3274205      | 32.19e-6 | 35.679        | 91.23      | 104.5           | 28.50          |             |       |
|                 | BSplines        | 11.96           | 659.5       | 671.4       | 1.111      | 22128483     | 29.80e-6 | 35.777        | 570.8      | 581.2           | 192.6          |             |       |
|                 | MinNorm         | 14.77           | 92.31       | 107.1       | 0.147      | 2909521      | 31.73e-6 | 35.677        | 79.91      | 92.69           | 25.33          |             |       |
|                 | MinVolume       | 13.16           | 120.9       | 134.0       | 0.214      | 4025573      | 30.03e-6 | 35.685        | 104.6      | 116.0           | 35.05          |             |       |
|                 | ApproxMinVolume | 12.92           | 120.6       | 133.6       | 0.213      | 4043095      | 29.84e-6 | 35.685        | 104.4      | 115.6           | 35.20          |             |       |
|                 | Recursive       | 15.92           | 82.29       | 98.21       | 0.138      | 2666987      | 30.85e-6 | 35.671        | 71.23      | 85.01           | 23.22          |             |       |
|                 | Recursive2      | 15.78           | 87.01       | 102.8       | 0.142      | 2817047      | 30.89e-6 | 35.671        | 75.31      | 88.97           | 24.52          |             |       |
|                 | Interval        | 28.00e-6        | 687650      | 687650      | 7-23:00:50 | 59005613511  | 11.65e-6 | 35.750        | 100.0      | 100.0           | 100.0          |             |       |
|                 | Bernstein       | 73.58           | 456398      | 456398      | 5-06:46:38 | 9110726498   | 50.09e-6 | 35.610        | 66.37      | 66.37           | 15.44          |             |       |
|                 | MinVariance     | 78.14           | 624199      | 624199      | 7-05:23:19 | 12365380847  | 50.48e-6 | 35.626        | 90.77      | 90.77           | 20.96          |             |       |
|                 | BSplines        | 73.70           | in progress | > 60 days   |            |              |          |               |            |                 |                |             |       |
|                 | MinNorm         | 77.24           | 549998      | 549998      | 6-08:46:38 | 10896867091  | 50.47e-6 | 35.618        | 79.98      | 79.98           | 18.47          |             |       |
|                 | MinVolume       | 75.64           | 793283      | 793283      | 9-04:21:23 | 16186187558  | 49.01e-6 | 35.641        | 115.4      | 115.4           | 27.43          |             |       |
|                 | ApproxMinVolume | 75.37           | 825185      | 825185      | 9-13:13:05 | 16301546790  | 50.62e-6 | 35.642        | 120.0      | 120.0           | 27.63          |             |       |
|                 | Recursive       | 74.17           | 488684      | 488684      | 5-15:44:44 | 9897003665   | 49.38e-6 | 35.610        | 71.07      | 71.07           | 16.77          |             |       |
|                 | Recursive2      | 73.95           | 484539      | 484539      | 5-14:35:39 | 9592032214   | 50.51e-6 | 35.610        | 70.46      | 70.46           | 16.26          |             |       |
|                 | 0.01            | 2               | Interval    | 26.94e-6    | 77.60e-3   | 77.62e-3     | 0.078    | 89            | 871.9e-6   | infeasible      | 100.0          | 100.0       | 100.0 |
|                 |                 |                 | Bernstein   | 2.260       | 9.079e-3   | 2.269        | 2.269    | 65            | 139.7e-6   | infeasible      | 11.70          | 2923.4      | 73.03 |
|                 |                 |                 | MinVariance | 2.260       | 8.334e-3   | 2.268        | 2.268    | 61            | 136.6e-6   | infeasible      | 10.74          | 2922.2      | 68.54 |
|                 |                 |                 | BSplines    | 2.260       | 6.923e-3   | 2.267        | 2.267    | 111           | 62.37e-6   | infeasible      | 8.922          | 2920.1      | 124.7 |
| MinNorm         |                 |                 | 2.260       | 5.535e-3    | 2.265      | 2.265        | 59       | 93.82e-6      | infeasible | 7.133           | 2918.1         | 66.29       |       |
| MinVolume       |                 |                 | 2.260       | 4.952e-3    | 2.265      | 2.265        | 69       | 71.77e-6      | infeasible | 6.382           | 2917.3         | 77.53       |       |
| ApproxMinVolume |                 |                 | 2.260       | 7.288e-3    | 2.267      | 2.267        | 69       | 105.6e-6      | infeasible | 9.392           | 2920.7         | 77.53       |       |
| Recursive       |                 |                 | 2.260       | 3.604e-3    | 2.263      | 2.263        | 91       | 39.60e-6      | infeasible | 4.644           | 2915.9         | 102.2       |       |
| Recursive2      |                 |                 | 2.260       | 4.522e-3    | 2.264      | 2.264        | 119      | 38.00e-6      | infeasible | 5.828           | 2917.0         | 133.7       |       |
| Interval        |                 |                 | 29.80e-6    | 61.14e-3    | 61.17e-3   | 0.061        | 6937     | 8.813e-6      | 49.139     | 100.0           | 100.0          | 100.0       |       |
| Bernstein       |                 |                 | 3.372       | 52.64e-3    | 3.425      | 3.425        | 4049     | 13.00e-6      | 48.529     | 86.10           | 5599.5         | 58.37       |       |
| MinVariance     |                 |                 | 4.661       | 65.62e-3    | 4.727      | 4.727        | 4805     | 13.66e-6      | 48.565     | 107.3           | 7727.5         | 69.27       |       |
| BSplines        |                 |                 | 3.583       | 190.1e-3    | 3.774      | 3.774        | 15215    | 12.49e-6      | 49.543     | 310.9           | 6169.2         | 219.3       |       |
| MinNorm         |                 |                 | 4.577       | 63.43e-3    | 4.640      | 4.640        | 4585     | 13.83e-6      | 48.549     | 103.7           | 7585.8         | 66.09       |       |
| MinVolume       |                 | 3.968           | 70.94e-3    | 4.039       | 4.039      | 5373         | 13.20e-6 | 48.882        | 116.0      | 6603.0          | 77.45          |             |       |
| ApproxMinVolume |                 | 3.947           | 71.72e-3    | 4.018       | 4.018      | 5387         | 13.31e-6 | 48.882        | 117.3      | 6569.6          | 77.66          |             |       |
| Recursive       |                 | 3.890           | 65.56e-3    | 3.956       | 3.956      | 5089         | 12.88e-6 | 48.529        | 107.2      | 6467.6          | 73.36          |             |       |
| Recursive2      |                 | 4.384           | 67.08e-3    | 4.451       | 4.451      | 4763         | 14.08e-6 | 48.529        | 109.7      | 7276.8          | 68.66          |             |       |
| Interval        |                 | 42.92e-6        | 26.24       | 26.24       | 26.241     | 2692783      | 9.745e-6 | 37.082        | 100.0      | 100.0           | 100.0          |             |       |
| Bernstein       |                 | 11.59           | 13.66       | 25.25       | 25.248     | 508285       | 26.87e-6 | 36.580        | 52.04      | 96.21           | 18.88          |             |       |
| MinVariance     |                 | 15.17           | 22.47       | 37.64       | 37.643     | 697329       | 32.22e-6 | 36.618        | 85.62      | 143.4           | 25.90          |             |       |
| BSplines        |                 | 12.15           | 139.3       | 151.4       | 0.231      | 4614157      | 30.18e-6 | 37.505        | 530.8      | 577.0           | 171.4          |             |       |
| MinNorm         |                 | 15.05           | 19.72       | 34.77       | 34.770     | 616243       | 32.00e-6 | 36.601        | 75.16      | 132.5           | 22.88          |             |       |
| MinVolume       |                 | 13.19           | 24.00       | 37.19       | 37.189     | 841707       | 28.52e-6 | 36.702        | 91.47      | 141.7           | 31.26          |             |       |
| ApproxMinVolume |                 | 13.13           | 25.89       | 39.02       | 39.025     | 845367       | 30.63e-6 | 36.703        | 98.67      | 148.7           | 31.39          |             |       |
| Recursive       |                 | 15.81           | 19.06       | 34.87       | 34.869     | 622349       | 30.63e-6 | 36.580        | 72.64      | 132.9           | 23.11          |             |       |
| Recursive2      |                 | 15.76           | 23.48       | 39.24       | 39.242     | 779781       | 30.12e-6 | 36.580        | 89.49      | 149.5           | 28.96          |             |       |
| Interval        |                 | 38.15e-6        | 19061       | 19061       | 05:17:41   | 1600276821   | 11.91e-6 | 38.245        | 100.0      | 100.0           | 100.0          |             |       |
| Bernstein       |                 | 99.13           | 21513       | 21612       | 06:00:12   | 403299393    | 53.34e-6 | 37.086        | 112.9      | 113.4           | 25.20          |             |       |
| MinVariance     |                 | 103.3           | 27265       | 27368       | 07:36:08   | 499019297    | 54.64e-6 | 37.202        | 143.0      | 143.6           | 31.18          |             |       |
| BSplines        |                 | 99.07           | 156957      | 156958      | 1-19:35:58 | 2979278014   | 52.68e-6 | 38.704        | 823.4      | 823.4           | 186.2          |             |       |
| MinNorm         |                 | 103.1           | 24685       | 24788       | 06:53:08   | 454194825    | 54.35e-6 | 37.151        | 129.5      | 130.0           | 28.38          |             |       |
| MinVolume       |                 | 101.6           | 32489       | 32591       | 09:03:11   | 618067121    | 52.57e-6 | 37.312        | 170.4      | 171.0           | 38.62          |             |       |
| ApproxMinVolume |                 | 101.6           | 33062       | 33163       | 09:12:43   | 621500519    | 53.20e-6 | 37.313        | 173.5      | 174.0           | 38.84          |             |       |
| Recursive       |                 | 98.29           | 29944       | 30043       | 08:20:43   | 561904601    | 53.29e-6 | 37.086        | 157.1      | 157.6           | 35.11          |             |       |
| Recursive2      |                 | 99.04           | 26455       | 26555       | 07:22:35   | 492025219    | 53.77e-6 | 37.086        | 138.8      | 139.3           | 30.75          |             |       |

Table 26: Bisection of problem 2D with problem: 16, type: MinFirst





| precision       | ndof            | solver          | prep_time | comput_time | total_time  | (D:H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------------|-----------|-------------|-------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | 2               | Interval        | 33.86e-6  | 3.524e-3    | 3.558e-3    | 0.004        | 159      | 22.16e-6      | 53.397   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 845.4e-3  | 3.615e-3    | 849.1e-3    | 0.849        | 159      | 22.74e-6      | 53.397   | 102.6           | 23864.2        | 100.0       |
|                 |                 | MinVariance     | 938.1e-3  | 3.988e-3    | 942.1e-3    | 0.942        | 183      | 21.79e-6      | 53.400   | 113.2           | 26479.7        | 115.1       |
|                 |                 | BSplines        | 864.9e-3  | 4.343e-3    | 869.3e-3    | 0.869        | 391      | 11.11e-6      | 53.486   | 123.2           | 24431.6        | 245.9       |
|                 |                 | MinNorm         | 883.9e-3  | 3.612e-3    | 887.6e-3    | 0.888        | 179      | 20.18e-6      | 53.398   | 102.5           | 24946.3        | 112.6       |
|                 |                 | MinVolume       | 895.2e-3  | 3.794e-3    | 899.1e-3    | 0.899        | 207      | 18.33e-6      | 53.402   | 107.7           | 25269.0        | 130.2       |
|                 |                 | ApproxMinVolume | 797.8e-3  | 3.821e-3    | 801.6e-3    | 0.802        | 207      | 18.46e-6      | 53.402   | 108.4           | 22530.3        | 130.2       |
|                 |                 | Recursive       | 762.9e-3  | 3.896e-3    | 766.8e-3    | 0.767        | 175      | 22.26e-6      | 53.397   | 110.6           | 21553.0        | 110.1       |
|                 |                 | Recursive2      | 855.9e-3  | 3.462e-3    | 859.4e-3    | 0.859        | 179      | 19.34e-6      | 53.397   | 98.23           | 24154.3        | 112.6       |
|                 |                 | Interval        | 30.04e-6  | 697.9e-3    | 698.0e-3    | 0.698        | 83677    | 8.341e-6      | 64.042   | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 3.415     | 562.0e-3    | 3.977       | 3.977        | 44791    | 12.55e-6      | 64.037   | 80.52           | 569.7          | 53.53       |
|                 |                 | MinVariance     | 4.665     | 703.4e-3    | 5.368       | 5.368        | 53905    | 13.05e-6      | 64.041   | 100.8           | 769.1          | 64.42       |
|                 | BSplines        | 3.564           | 2.401     | 5.965       | 5.965       | 197089       | 12.18e-6 | 64.137        | 344.1    | 854.6           | 235.5          |             |
|                 | MinNorm         | 4.522           | 626.5e-3  | 5.149       | 5.149       | 49279        | 12.71e-6 | 64.039        | 89.76    | 737.7           | 58.89          |             |
|                 | MinVolume       | 3.978           | 777.3e-3  | 4.756       | 4.756       | 61811        | 12.58e-6 | 64.045        | 111.4    | 681.4           | 73.87          |             |
|                 | ApproxMinVolume | 3.970           | 772.9e-3  | 4.743       | 4.743       | 62071        | 12.45e-6 | 64.045        | 110.7    | 679.6           | 74.18          |             |
|                 | Recursive       | 3.871           | 589.0e-3  | 4.460       | 4.460       | 47183        | 12.48e-6 | 64.037        | 84.40    | 639.0           | 56.39          |             |
|                 | Recursive2      | 4.379           | 588.2e-3  | 4.967       | 4.967       | 45353        | 12.97e-6 | 64.037        | 84.28    | 711.7           | 54.20          |             |
|                 | Interval        | 25.99e-6        | 948.9     | 948.9       | 15.48       | 104660775    | 9.066e-6 | 55.997        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 11.49           | 411.4     | 422.9       | 07:02       | 14375207     | 28.62e-6 | 55.850        | 43.36    | 44.57           | 13.74          |             |
|                 | MinVariance     | 15.20           | 573.9     | 589.1       | 09:49       | 17846991     | 32.15e-6 | 55.862        | 60.48    | 62.08           | 17.05          |             |
|                 | BSplines        | 11.94           | 2949      | 2961        | 49:21       | 98077069     | 30.07e-6 | 56.000        | 310.8    | 312.1           | 93.71          |             |
|                 | MinNorm         | 14.89           | 475.0     | 489.9       | 08:09       | 16242609     | 29.25e-6 | 55.857        | 50.06    | 51.63           | 15.52          |             |
|                 | MinVolume       | 13.23           | 669.7     | 682.9       | 11:22       | 21472161     | 31.19e-6 | 55.873        | 70.57    | 71.97           | 20.52          |             |
|                 | ApproxMinVolume | 13.06           | 656.1     | 669.2       | 11:09       | 21564243     | 30.43e-6 | 55.874        | 69.15    | 70.52           | 20.60          |             |
|                 | Recursive       | 15.95           | 474.9     | 490.9       | 08:10       | 15010869     | 31.64e-6 | 55.850        | 50.05    | 51.73           | 14.34          |             |
|                 | Recursive2      | 15.60           | 462.5     | 478.1       | 07:58       | 15089753     | 30.65e-6 | 55.850        | 48.75    | 50.39           | 14.42          |             |
|                 | Interval        | 28.00e-6        | 1350520   | 1350520     | 15-15:08:40 | 121375517999 | 11.13e-6 | 63.149        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 73.56           | 323217    | 323217      | 3-17:46:57  | 6280505880   | 51.46e-6 | 62.923        | 23.93    | 23.93           | 5.174          |             |
|                 | MinVariance     | 77.90           | 428194    | 428194      | 4-22:56:34  | 8237514581   | 51.98e-6 | 62.941        | 31.71    | 31.71           | 6.787          |             |
|                 | BSplines        | 73.71           | 3722210   | 3722210     | 43-01:56:50 | 73456512169  | 50.67e-6 | 63.145        | 275.6    | 275.6           | 60.52          |             |
|                 | MinNorm         | 77.17           | 374380    | 374380      | 4-07:59:40  | 7265029483   | 51.53e-6 | 62.933        | 27.72    | 27.72           | 5.986          |             |
|                 | MinVolume       | 76.33           | 532077    | 532077      | 6-03:47:57  | 10483140457  | 50.76e-6 | 62.957        | 39.40    | 39.40           | 8.637          |             |
|                 | ApproxMinVolume | 75.81           | 531701    | 531701      | 6-03:41:41  | 10539658705  | 50.45e-6 | 62.958        | 39.37    | 39.37           | 8.684          |             |
|                 | Recursive       | 74.04           | 348973    | 348973      | 4-00:56:13  | 6806633015   | 51.27e-6 | 62.923        | 25.84    | 25.84           | 5.608          |             |
|                 | Recursive2      | 74.20           | 333486    | 333486      | 3-20:38:06  | 6482265500   | 51.45e-6 | 62.923        | 24.69    | 24.69           | 5.341          |             |
|                 | Interval        | 32.90e-6        | 118.3e-3  | 118.4e-3    | 0.118       | 319          | 371.0e-6 | 54.227        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 2.259           | 7.758e-3  | 2.267       | 2.267       | 291          | 26.66e-6 | 54.227        | 6.555    | 1915.2          | 91.22          |             |
|                 | MinVariance     | 2.259           | 11.63e-3  | 2.271       | 2.271       | 295          | 39.42e-6 | 54.246        | 9.827    | 1918.4          | 92.48          |             |
|                 | BSplines        | 2.260           | 12.10e-3  | 2.272       | 2.272       | 551          | 21.96e-6 | 54.941        | 10.23    | 1919.0          | 172.7          |             |
|                 | MinNorm         | 2.259           | 9.963e-3  | 2.269       | 2.269       | 293          | 34.00e-6 | 54.237        | 8.419    | 1917.0          | 91.85          |             |
|                 | MinVolume       | 2.259           | 9.081e-3  | 2.268       | 2.268       | 315          | 28.83e-6 | 54.264        | 7.673    | 1916.1          | 98.75          |             |
|                 | ApproxMinVolume | 2.260           | 10.79e-3  | 2.270       | 2.270       | 317          | 34.02e-6 | 54.264        | 9.113    | 1918.0          | 99.37          |             |
|                 | Recursive       | 1.511           | 6.837e-3  | 1.518       | 1.518       | 343          | 19.93e-6 | 54.227        | 5.777    | 1282.5          | 107.5          |             |
|                 | Recursive2      | 1.528           | 11.09e-3  | 1.539       | 1.539       | 305          | 36.36e-6 | 54.227        | 9.372    | 1300.0          | 95.61          |             |
|                 | Interval        | 28.13e-6        | 300.6e-3  | 300.6e-3    | 0.301       | 37029        | 8.117e-6 | 65.393        | 100.0    | 100.0           | 100.0          |             |
|                 | Bernstein       | 3.311           | 198.3e-3  | 3.509       | 3.509       | 15969        | 12.42e-6 | 65.347        | 65.99    | 1167.3          | 43.13          |             |
|                 | MinVariance     | 4.638           | 281.2e-3  | 4.919       | 4.919       | 20523        | 13.70e-6 | 65.380        | 93.57    | 1636.4          | 55.42          |             |
| BSplines        | 3.540           | 1.013           | 4.553     | 4.553       | 77799       | 13.02e-6     | 66.153   | 337.1         | 1514.8   | 210.1           |                |             |
| MinNorm         | 4.511           | 224.6e-3        | 4.735     | 4.735       | 17195       | 13.06e-6     | 65.365   | 74.73         | 1575.3   | 46.44           |                |             |
| MinVolume       | 3.933           | 306.6e-3        | 4.240     | 4.240       | 23489       | 13.05e-6     | 65.411   | 102.0         | 1410.5   | 63.43           |                |             |
| ApproxMinVolume | 3.942           | 301.3e-3        | 4.243     | 4.243       | 23553       | 12.79e-6     | 65.412   | 100.2         | 1411.5   | 63.61           |                |             |
| Recursive       | 3.756           | 239.8e-3        | 3.996     | 3.996       | 18879       | 12.70e-6     | 65.347   | 79.77         | 1329.4   | 50.98           |                |             |
| Recursive2      | 4.326           | 209.6e-3        | 4.536     | 4.536       | 16733       | 12.53e-6     | 65.347   | 69.74         | 1508.9   | 45.19           |                |             |
| Interval        | 30.99e-6        | 71.99           | 71.99     | 01:11       | 7423905     | 9.697e-6     | 58.169   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 11.59           | 37.99           | 49.58     | 49.583      | 1309339     | 29.01e-6     | 56.938   | 52.77         | 68.88    | 17.64           |                |             |
| MinVariance     | 15.47           | 53.44           | 68.91     | 01:08       | 1607239     | 33.25e-6     | 57.038   | 74.23         | 95.73    | 21.65           |                |             |
| BSplines        | 12.01           | 268.4           | 280.4     | 04:40       | 9116619     | 29.44e-6     | 58.127   | 372.8         | 389.5    | 122.8           |                |             |
| MinNorm         | 14.94           | 47.42           | 62.37     | 01:02       | 1453903     | 32.62e-6     | 56.978   | 65.88         | 86.63    | 19.58           |                |             |
| MinVolume       | 13.30           | 62.51           | 75.81     | 01:15       | 1984227     | 31.50e-6     | 57.123   | 86.83         | 105.3    | 26.73           |                |             |
| ApproxMinVolume | 13.04           | 58.06           | 71.10     | 01:11       | 1994227     | 29.11e-6     | 57.124   | 80.65         | 98.77    | 26.86           |                |             |
| Recursive       | 15.90           | 49.92           | 65.82     | 01:05       | 1600277     | 31.20e-6     | 56.938   | 69.35         | 91.43    | 21.56           |                |             |
| Recursive2      | 15.69           | 54.47           | 70.16     | 01:10       | 1777477     | 30.65e-6     | 56.938   | 75.67         | 97.47    | 23.94           |                |             |
| Interval        | 21.93e-6        | 48486           | 48486     | 13:28:06    | 4326690579  | 11.21e-6     | 66.175   | 100.0         | 100.0    | 100.0           |                |             |
| Bernstein       | 96.22           | 19798           | 19894     | 05:31:34    | 379042839   | 52.23e-6     | 64.526   | 40.83         | 41.03    | 8.761           |                |             |
| MinVariance     | 101.0           | 27574           | 27675     | 07:41:15    | 511592157   | 53.90e-6     | 64.680   | 56.87         | 57.08    | 11.82           |                |             |
| BSplines        | 96.91           | 251431          | 251431    | 2-21:50:31  | 4816801329  | 52.20e-6     | 66.278   | 518.6         | 518.6    | 111.3           |                |             |
| MinNorm         | 100.5           | 23435           | 23536     | 06:32:16    | 447578831   | 52.36e-6     | 64.623   | 48.33         | 48.54    | 10.34           |                |             |
| MinVolume       | 100.9           | 34848           | 34949     | 09:42:29    | 661782405   | 52.66e-6     | 64.786   | 71.87         | 72.08    | 15.30           |                |             |
| ApproxMinVolume | 100.1           | 35079           | 35179     | 09:46:19    | 666118385   | 52.66e-6     | 64.791   | 72.35         | 72.56    | 15.40           |                |             |
| Recursive       | 98.30           | 30142           | 30240     | 08:24:00    | 573913681   | 52.52e-6     | 64.526   | 62.17         | 62.37    | 13.26           |                |             |
| Recursive2      | 98.30           | 26723           | 26821     | 07:27:01    | 509400055   | 52.46e-6     | 64.526   | 55.12         | 55.32    | 11.77           |                |             |

Table 28: Bisection of problem 2D with problem: 18, type: MinFirst















| precision | ndof | solver          | prep_time       | comput_time | total_time | (D-H:M:S.ms) | nb_iter     | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------|------|-----------------|-----------------|-------------|------------|--------------|-------------|---------------|------------|-----------------|----------------|-------------|-------|
| 0.001     | 2    | Interval        | 25.99e-6        | 3.508e-3    | 3.534e-3   | 0.004        | 89          | 39.42e-6      | infeasible | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 855.0e-3        | 3.075e-3    | 858.1e-3   | 0.858        | 65          | 47.31e-6      | infeasible | 87.66           | 24279.9        | 73.03       |       |
|           |      | MinVariance     | 948.8e-3        | 3.210e-3    | 952.1e-3   | 0.952        | 61          | 52.62e-6      | infeasible | 91.50           | 26939.2        | 68.54       |       |
|           |      | BSplines        | 870.6e-3        | 3.127e-3    | 873.7e-3   | 0.874        | 111         | 28.17e-6      | infeasible | 89.14           | 24722.0        | 124.7       |       |
|           |      | MinNorm         | 919.0e-3        | 3.190e-3    | 922.2e-3   | 0.922        | 59          | 54.07e-6      | infeasible | 90.93           | 26094.1        | 66.29       |       |
|           |      | MinVolume       | 903.5e-3        | 2.918e-3    | 906.5e-3   | 0.906        | 69          | 42.29e-6      | infeasible | 83.18           | 25649.0        | 77.53       |       |
|           |      | ApproxMinVolume | 894.0e-3        | 3.415e-3    | 897.4e-3   | 0.897        | 69          | 49.49e-6      | infeasible | 97.35           | 25392.4        | 77.53       |       |
|           |      | Recursive       | 867.5e-3        | 3.538e-3    | 871.1e-3   | 0.871        | 91          | 38.88e-6      | infeasible | 100.9           | 24648.1        | 102.2       |       |
|           |      | Recursive2      | 873.2e-3        | 3.271e-3    | 876.4e-3   | 0.876        | 119         | 27.49e-6      | infeasible | 93.24           | 24799.9        | 133.7       |       |
|           | 3    | Interval        | 29.09e-6        | 2.133       | 2.133      | 2.133        | 296425      | 7.197e-6      | 4.0146     | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 2.989           | 981.1e-3    | 3.970      | 3.970        | 96347       | 10.18e-6      | 4.0092     | 45.99           | 186.1          | 32.50       |       |
|           |      | MinVariance     | 4.388           | 1.249       | 5.637      | 5.637        | 110827      | 11.27e-6      | 4.0097     | 58.52           | 264.2          | 37.39       |       |
|           |      | BSplines        | 3.142           | 3.625       | 6.766      | 6.766        | 353175      | 10.26e-6      | 4.0148     | 169.9           | 317.2          | 119.1       |       |
|           |      | MinNorm         | 4.237           | 1.147       | 5.384      | 5.384        | 103835      | 11.05e-6      | 4.0093     | 53.76           | 252.4          | 35.03       |       |
|           |      | MinVolume       | 3.586           | 1.317       | 4.902      | 4.902        | 125293      | 10.51e-6      | 4.0102     | 61.72           | 229.8          | 42.27       |       |
|           |      | ApproxMinVolume | 3.561           | 1.374       | 4.935      | 4.935        | 125641      | 10.93e-6      | 4.0102     | 64.39           | 231.3          | 42.39       |       |
|           |      | Recursive       | 3.506           | 1.076       | 4.581      | 4.581        | 99723       | 10.79e-6      | 4.0092     | 50.42           | 214.7          | 33.64       |       |
|           |      | Recursive2      | 3.711           | 1.100       | 4.811      | 4.811        | 100257      | 10.97e-6      | 4.0092     | 51.55           | 225.5          | 33.82       |       |
|           | 4    | Interval        | 30.99e-6        | 1098        | 1098       | 18.18        | 137707139   | 7.978e-6      | 2.3843     | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 3.996           | 136.3       | 140.3      | 02:20        | 7384597     | 18.46e-6      | 2.3649     | 12.41           | 12.77          | 5.363       |       |
|           |      | MinVariance     | 5.140           | 162.8       | 167.9      | 02:47        | 8554199     | 19.03e-6      | 2.3654     | 14.82           | 15.29          | 6.212       |       |
|           |      | BSplines        | 3.965           | 570.1       | 574.0      | 09:34        | 31906287    | 17.87e-6      | 2.3716     | 51.89           | 52.25          | 23.17       |       |
|           |      | MinNorm         | 5.019           | 153.3       | 158.3      | 02:38        | 8023337     | 19.10e-6      | 2.3653     | 13.95           | 14.41          | 5.826       |       |
|           |      | MinVolume       | 4.565           | 186.6       | 191.2      | 03:11        | 9785119     | 19.07e-6      | 2.3659     | 16.99           | 17.40          | 7.106       |       |
|           |      | ApproxMinVolume | 4.591           | 183.7       | 188.3      | 03:08        | 9810943     | 18.73e-6      | 2.3659     | 16.72           | 17.14          | 7.124       |       |
|           |      | Recursive       | 4.050           | 143.4       | 147.4      | 02:27        | 7716861     | 18.58e-6      | 2.3649     | 13.05           | 13.42          | 5.604       |       |
|           |      | Recursive2      | 4.053           | 145.5       | 149.6      | 02:29        | 7929785     | 18.35e-6      | 2.3649     | 13.25           | 13.62          | 5.758       |       |
|           | 5    | Interval        | 27.00e-6        | 405792      | 405792     | 4-16:43:12   | 41698312627 | 9.732e-6      | 1.6158     | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 54.92           | 442040      | 442040     | 5-02:47:20   | 10525260016 | 42.00e-6      | 1.6106     | 108.9           | 108.9          | 25.24       |       |
|           |      | MinVariance     | 59.84           | 560127      | 560127     | 6-11:35:27   | 13261984483 | 42.24e-6      | 1.6112     | 138.0           | 138.0          | 31.80       |       |
|           |      | BSplines        | 55.22           | 3480110     | 3480110    | 40-06:41:50  | 84542081447 | 41.16e-6      | 1.6186     | 857.6           | 857.6          | 202.7       |       |
|           |      | MinNorm         | 59.50           | 504001      | 504001     | 5-20:00:01   | 11965685782 | 42.12e-6      | 1.6109     | 124.2           | 124.2          | 28.70       |       |
|           |      | MinVolume       | 57.31           | 674935      | 674935     | 7-19:28:55   | 16261873597 | 41.50e-6      | 1.6118     | 166.3           | 166.3          | 39.00       |       |
|           |      | ApproxMinVolume | 57.20           | 680081      | 680081     | 7-20:54:41   | 16348455509 | 41.60e-6      | 1.6118     | 167.6           | 167.6          | 39.21       |       |
|           |      | Recursive       | 54.74           | 440588      | 440588     | 5-02:23:08   | 10541421976 | 41.80e-6      | 1.6106     | 108.6           | 108.6          | 25.28       |       |
|           |      | Recursive2      | 54.99           | 484821      | 484821     | 5-14:40:21   | 11621586282 | 41.72e-6      | 1.6106     | 119.5           | 119.5          | 27.87       |       |
|           | 0.01 | 2               | Interval        | 30.99e-6    | 3.143e-3   | 3.174e-3     | 0.003       | 89            | 35.31e-6   | infeasible      | 100.0          | 100.0       | 100.0 |
|           |      |                 | Bernstein       | 852.0e-3    | 3.445e-3   | 855.4e-3     | 0.855       | 65            | 53.00e-6   | infeasible      | 109.6          | 26953.0     | 73.03 |
|           |      |                 | MinVariance     | 933.4e-3    | 3.084e-3   | 936.5e-3     | 0.936       | 61            | 50.56e-6   | infeasible      | 98.13          | 29505.6     | 68.54 |
|           |      |                 | BSplines        | 855.2e-3    | 3.523e-3   | 858.8e-3     | 0.859       | 111           | 31.74e-6   | infeasible      | 112.1          | 27057.7     | 124.7 |
|           |      |                 | MinNorm         | 927.2e-3    | 3.378e-3   | 930.6e-3     | 0.931       | 59            | 57.25e-6   | infeasible      | 107.5          | 29321.1     | 66.29 |
|           |      |                 | MinVolume       | 942.3e-3    | 3.229e-3   | 945.5e-3     | 0.946       | 69            | 46.80e-6   | infeasible      | 102.7          | 29791.9     | 77.53 |
|           |      |                 | ApproxMinVolume | 908.4e-3    | 3.232e-3   | 911.6e-3     | 0.912       | 69            | 46.84e-6   | infeasible      | 102.8          | 28722.8     | 77.53 |
|           |      |                 | Recursive       | 869.6e-3    | 3.298e-3   | 872.9e-3     | 0.873       | 91            | 36.24e-6   | infeasible      | 104.9          | 27502.9     | 102.2 |
|           |      |                 | Recursive2      | 873.4e-3    | 3.426e-3   | 876.8e-3     | 0.877       | 119           | 28.79e-6   | infeasible      | 109.0          | 27626.5     | 133.7 |
| 3         |      | Interval        | 28.85e-6        | 262.8e-3    | 262.8e-3   | 0.263        | 35691       | 7.363e-6      | 4.0953     | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 2.991           | 142.6e-3    | 3.133      | 3.133        | 12859       | 11.09e-6      | 4.0529     | 54.28           | 1192.1         | 36.03       |       |
|           |      | MinVariance     | 4.424           | 171.5e-3    | 4.595      | 4.595        | 14453       | 11.87e-6      | 4.0544     | 65.26           | 1748.3         | 40.49       |       |
|           |      | BSplines        | 3.193           | 481.6e-3    | 3.674      | 3.674        | 45133       | 10.67e-6      | 4.0987     | 183.2           | 1398.0         | 126.5       |       |
|           |      | MinNorm         | 4.222           | 158.6e-3    | 4.381      | 4.381        | 13565       | 11.69e-6      | 4.0537     | 60.36           | 1666.7         | 38.01       |       |
|           |      | MinVolume       | 3.578           | 185.1e-3    | 3.764      | 3.764        | 16767       | 11.04e-6      | 4.0599     | 70.43           | 1431.9         | 46.98       |       |
|           |      | ApproxMinVolume | 3.616           | 192.1e-3    | 3.808      | 3.808        | 16817       | 11.42e-6      | 4.0599     | 73.09           | 1448.9         | 47.12       |       |
|           |      | Recursive       | 3.513           | 176.1e-3    | 3.689      | 3.689        | 15323       | 11.49e-6      | 4.0529     | 66.99           | 1403.7         | 42.93       |       |
|           |      | Recursive2      | 3.701           | 183.4e-3    | 3.884      | 3.884        | 16075       | 11.41e-6      | 4.0529     | 69.80           | 1478.0         | 45.04       |       |
| 4         |      | Interval        | 25.99e-6        | 123.8       | 123.8      | 02:03        | 14560057    | 8.500e-6      | 2.6074     | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 4.029           | 19.42       | 23.45      | 23.452       | 1038455     | 18.70e-6      | 2.4438     | 15.69           | 18.95          | 7.132       |       |
|           |      | MinVariance     | 5.157           | 23.09       | 28.24      | 28.243       | 1220073     | 18.92e-6      | 2.4487     | 18.65           | 22.82          | 8.380       |       |
|           |      | BSplines        | 3.972           | 80.85       | 84.82      | 01:24        | 4533581     | 17.83e-6      | 2.4946     | 65.32           | 68.53          | 31.14       |       |
|           |      | MinNorm         | 5.042           | 21.43       | 26.47      | 26.467       | 1118335     | 19.16e-6      | 2.4463     | 17.31           | 21.38          | 7.681       |       |
|           |      | MinVolume       | 4.478           | 26.15       | 30.63      | 30.625       | 1390943     | 18.80e-6      | 2.4509     | 21.13           | 24.74          | 9.553       |       |
|           |      | ApproxMinVolume | 4.600           | 25.80       | 30.40      | 30.404       | 1395155     | 18.50e-6      | 2.4510     | 20.85           | 24.57          | 9.582       |       |
|           |      | Recursive       | 4.022           | 23.82       | 27.84      | 27.838       | 1317881     | 18.07e-6      | 2.4438     | 19.24           | 22.49          | 9.051       |       |
|           |      | Recursive2      | 4.055           | 27.91       | 31.97      | 31.966       | 1486525     | 18.78e-6      | 2.4438     | 22.55           | 25.83          | 10.21       |       |
| 5         |      | Interval        | 32.90e-6        | 5334        | 5334       | 01:28:54     | 552015029   | 9.664e-6      | 1.7103     | 100.0           | 100.0          | 100.0       |       |
|           |      | Bernstein       | 72.48           | 6473        | 6545       | 01:49:05     | 149768043   | 43.22e-6      | 1.6674     | 121.3           | 122.7          | 27.13       |       |
|           |      | MinVariance     | 77.92           | 8162        | 8240       | 02:17:20     | 187719485   | 43.48e-6      | 1.6722     | 153.0           | 154.5          | 34.01       |       |
|           |      | BSplines        | 73.05           | 49584       | 49657      | 13:47:37     | 1165915905  | 42.53e-6      | 1.7288     | 929.5           | 930.9          | 211.2       |       |
|           |      | MinNorm         | 77.51           | 7223        | 7301       | 02:01:41     | 169241603   | 42.68e-6      | 1.6702     | 135.4           | 136.9          | 30.66       |       |
|           |      | MinVolume       | 75.30           | 9778        | 9854       | 02:44:14     | 230353345   | 42.45e-6      | 1.6761     | 183.3           | 184.7          | 41.73       |       |
|           |      | ApproxMinVolume | 75.22           | 9792        | 9867       | 02:44:27     | 231925331   | 42.22e-6      | 1.6762     | 183.6           | 185.0          | 42.01       |       |
|           |      | Recursive       | 72.15           | 6773        | 6845       | 01:54:05     | 150770339   | 42.39e-6      | 1.6674     | 127.0           | 128.3          | 28.94       |       |
|           |      | Recursive2      | 72.55           | 12787       | 12860      | 03:34:20     | 292924389   | 43.66e-6      | 1.6674     | 239.7           | 241.1          | 53.06       |       |

Table 35: Bisection of problem 2D with problem: 6, type: MaxFirst



















| precision       | ndof            | solver          | prep_time   | comput_time | total_time  | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------------|-------------|-------------|-------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|
| 0.001           | 2               | Interval        | 25.03e-6    | 3.119e-3    | 3.144e-3    | 0.003        | 89       | 35.04e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 855.6e-3    | 3.958e-3    | 859.6e-3    | 0.860        | 65       | 60.89e-6      | infeasible | 126.9           | 27341.1        | 73.03       |
|                 |                 | MinVariance     | 918.3e-3    | 3.876e-3    | 922.1e-3    | 0.922        | 61       | 63.54e-6      | infeasible | 124.3           | 29330.0        | 68.54       |
|                 |                 | BSplines        | 874.8e-3    | 3.362e-3    | 878.2e-3    | 0.878        | 111      | 30.29e-6      | infeasible | 107.8           | 27931.3        | 124.7       |
|                 |                 | MinNorm         | 910.2e-3    | 3.517e-3    | 913.7e-3    | 0.914        | 59       | 59.61e-6      | infeasible | 112.8           | 29061.8        | 66.29       |
|                 |                 | MinVolume       | 882.7e-3    | 3.689e-3    | 886.4e-3    | 0.886        | 69       | 53.46e-6      | infeasible | 118.3           | 28194.1        | 77.53       |
|                 |                 | ApproxMinVolume | 879.9e-3    | 3.200e-3    | 883.1e-3    | 0.883        | 69       | 46.38e-6      | infeasible | 102.6           | 28088.0        | 77.53       |
|                 |                 | Recursive       | 800.3e-3    | 3.423e-3    | 803.7e-3    | 0.804        | 91       | 37.62e-6      | infeasible | 109.7           | 25563.4        | 102.2       |
|                 |                 | Recursive2      | 778.2e-3    | 4.367e-3    | 782.5e-3    | 0.783        | 119      | 36.70e-6      | infeasible | 140.0           | 24889.8        | 133.7       |
|                 |                 | Interval        | 24.80e-6    | 463.5e-3    | 463.6e-3    | 0.464        | 53441    | 8.67e-6       | 47.608     | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 3.390       | 288.9e-3    | 3.679       | 3.679        | 41013    | 7.045e-6      | 47.567     | 62.33           | 793.7          | 76.74       |
|                 |                 | MinVariance     | 4.694       | 334.9e-3    | 5.029       | 5.029        | 43683    | 7.667e-6      | 47.571     | 72.26           | 1084.8         | 81.74       |
|                 |                 | BSplines        | 3.548       | 1.057       | 4.605       | 4.605        | 132407   | 7.983e-6      | 47.694     | 228.0           | 993.5          | 247.8       |
|                 |                 | MinNorm         | 4.513       | 310.3e-3    | 4.824       | 4.824        | 42199    | 7.353e-6      | 47.569     | 66.94           | 1040.6         | 78.96       |
|                 | MinVolume       | 3.923           | 362.9e-3    | 4.286       | 4.286       | 49189        | 7.378e-6 | 47.575        | 78.29      | 924.5           | 92.04          |             |
|                 | ApproxMinVolume | 3.884           | 365.7e-3    | 4.249       | 4.249       | 49229        | 7.429e-6 | 47.576        | 78.90      | 916.7           | 92.12          |             |
|                 | Recursive       | 3.880           | 292.2e-3    | 4.172       | 4.172       | 41695        | 7.009e-6 | 47.567        | 63.04      | 900.0           | 78.02          |             |
|                 | Recursive2      | 4.332           | 319.3e-3    | 4.651       | 4.651       | 42411        | 7.528e-6 | 47.567        | 68.88      | 1003.4          | 79.36          |             |
|                 | Interval        | 33.86e-6        | 95.09       | 95.09       | 01:35       | 9088859      | 10.46e-6 | 35.729        | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 11.93           | 33.32       | 45.25       | 45.247      | 1262695      | 26.39e-6 | 35.671        | 35.04      | 47.58           | 13.89          |             |
|                 | MinVariance     | 16.46           | 47.02       | 63.48       | 01:03       | 1572127      | 29.91e-6 | 35.679        | 49.45      | 66.76           | 17.30          |             |
|                 | BSplines        | 12.47           | 292.3       | 304.8       | 05:04       | 10251133     | 28.51e-6 | 35.777        | 307.4      | 320.5           | 112.8          |             |
|                 | MinNorm         | 15.73           | 42.35       | 58.09       | 58.089      | 1449681      | 29.22e-6 | 35.677        | 44.54      | 61.09           | 15.95          |             |
|                 | MinVolume       | 13.24           | 53.45       | 66.69       | 01:06       | 1881215      | 28.41e-6 | 35.685        | 56.21      | 70.14           | 20.70          |             |
|                 | ApproxMinVolume | 12.91           | 48.93       | 61.84       | 01:01       | 1889165      | 25.90e-6 | 35.685        | 51.45      | 65.03           | 20.79          |             |
|                 | Recursive       | 15.73           | 40.61       | 56.33       | 56.335      | 1408319      | 28.84e-6 | 35.671        | 42.71      | 59.24           | 15.50          |             |
|                 | Recursive2      | 15.62           | 37.55       | 53.16       | 53.164      | 1327707      | 28.28e-6 | 35.671        | 39.48      | 55.91           | 14.61          |             |
|                 | Interval        | 29.00e-6        | 866015      | 866015      | 10:00:33.35 | 75199201798  | 11.52e-6 | 35.750        | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 73.44           | 716244      | 716244      | 8:06:57.24  | 14233161783  | 50.32e-6 | 35.610        | 82.71      | 82.71           | 18.93          |             |
|                 | MinVariance     | 77.30           | 976996      | 976996      | 11:07:23.16 | 18975920139  | 51.49e-6 | 35.626        | 112.8      | 112.8           | 25.23          |             |
|                 | BSplines        | 73.86           | in progress | > 60 days   |             |              |          |               |            |                 |                |             |
|                 | MinNorm         | 77.87           | 854982      | 854982      | 9:21:29.42  | 16650102303  | 51.35e-6 | 35.618        | 98.73      | 98.73           | 22.14          |             |
|                 | MinVolume       | 75.83           | 1237620     | 1237620     | 14:07:47.00 | 24436785553  | 50.65e-6 | 35.641        | 142.9      | 142.9           | 32.50          |             |
|                 | ApproxMinVolume | 75.14           | 1251030     | 1251030     | 14:11:30.30 | 24594361719  | 50.87e-6 | 35.642        | 144.5      | 144.5           | 32.71          |             |
|                 | Recursive       | 73.78           | 772313      | 772313      | 8:22:31.53  | 15310245251  | 50.44e-6 | 35.610        | 89.18      | 89.18           | 20.36          |             |
|                 | Recursive2      | 74.26           | 749059      | 749059      | 8:16:04.19  | 14828816662  | 50.51e-6 | 35.610        | 86.49      | 86.49           | 19.72          |             |
|                 | Interval        | 27.18e-6        | 2.887e-3    | 2.914e-3    | 0.003       | 89           | 32.44e-6 | infeasible    | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 832.8e-3        | 3.245e-3    | 836.0e-3    | 0.836       | 65           | 49.92e-6 | infeasible    | 112.4      | 28687.6         | 73.03          |             |
|                 | MinVariance     | 896.5e-3        | 3.170e-3    | 899.7e-3    | 0.900       | 61           | 51.97e-6 | infeasible    | 109.8      | 30874.2         | 68.54          |             |
|                 | BSplines        | 844.1e-3        | 3.699e-3    | 847.9e-3    | 0.848       | 111          | 33.32e-6 | infeasible    | 128.1      | 29094.0         | 124.7          |             |
|                 | MinNorm         | 896.7e-3        | 4.362e-3    | 901.1e-3    | 0.901       | 59           | 73.93e-6 | infeasible    | 151.1      | 30920.2         | 66.29          |             |
|                 | MinVolume       | 886.1e-3        | 5.657e-3    | 891.8e-3    | 0.892       | 69           | 81.99e-6 | infeasible    | 196.0      | 30602.2         | 77.53          |             |
|                 | ApproxMinVolume | 882.8e-3        | 4.285e-3    | 887.1e-3    | 0.887       | 69           | 62.10e-6 | infeasible    | 148.4      | 30441.5         | 77.53          |             |
|                 | Recursive       | 860.2e-3        | 4.379e-3    | 864.6e-3    | 0.865       | 91           | 48.12e-6 | infeasible    | 151.7      | 29668.2         | 102.2          |             |
|                 | Recursive2      | 866.0e-3        | 3.594e-3    | 869.6e-3    | 0.870       | 119          | 30.20e-6 | infeasible    | 124.5      | 29841.9         | 133.7          |             |
|                 | Interval        | 30.04e-6        | 82.68e-3    | 82.71e-3    | 0.083       | 9493         | 8.709e-6 | 49.139        | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 3.281           | 56.38e-3    | 3.338       | 3.338       | 5891         | 9.570e-6 | 48.529        | 68.19      | 4035.5          | 62.06          |             |
|                 | MinVariance     | 4.611           | 63.06e-3    | 4.674       | 4.674       | 6441         | 9.790e-6 | 48.565        | 76.27      | 5651.0          | 67.85          |             |
|                 | BSplines        | 3.513           | 201.9e-3    | 3.715       | 3.715       | 20773        | 9.717e-6 | 49.543        | 244.2      | 4492.0          | 218.8          |             |
| MinNorm         | 4.383           | 58.80e-3        | 4.442       | 4.442       | 6151        | 9.560e-6     | 48.549   | 71.13         | 5371.0     | 64.80           |                |             |
| MinVolume       | 3.872           | 79.50e-3        | 3.951       | 3.951       | 7877        | 10.09e-6     | 48.882   | 96.15         | 4777.3     | 82.98           |                |             |
| ApproxMinVolume | 3.767           | 75.22e-3        | 3.842       | 3.842       | 7887        | 9.537e-6     | 48.882   | 90.98         | 4645.6     | 83.08           |                |             |
| Recursive       | 3.868           | 66.66e-3        | 3.935       | 3.935       | 6561        | 10.16e-6     | 48.529   | 80.62         | 4757.9     | 69.11           |                |             |
| Recursive2      | 4.343           | 73.77e-3        | 4.416       | 4.416       | 7253        | 10.17e-6     | 48.529   | 89.23         | 5339.8     | 76.40           |                |             |
| Interval        | 31.95e-6        | 78.05           | 78.05       | 01:18       | 7838819     | 9.957e-6     | 37.082   | 100.0         | 100.0      | 100.0           |                |             |
| Bernstein       | 11.69           | 14.23           | 25.92       | 25.920      | 487613      | 29.18e-6     | 36.580   | 18.23         | 33.21      | 6.220           |                |             |
| MinVariance     | 15.51           | 19.68           | 35.19       | 35.189      | 602931      | 32.65e-6     | 36.618   | 25.22         | 45.08      | 7.692           |                |             |
| BSplines        | 12.01           | 103.2           | 115.2       | 01:55       | 3386455     | 30.46e-6     | 37.505   | 132.2         | 147.5      | 43.20           |                |             |
| MinNorm         | 14.78           | 17.69           | 32.47       | 32.471      | 538937      | 32.83e-6     | 36.601   | 22.67         | 41.60      | 6.875           |                |             |
| MinVolume       | 13.36           | 22.91           | 36.27       | 36.272      | 725055      | 31.60e-6     | 36.702   | 29.35         | 46.47      | 9.250           |                |             |
| ApproxMinVolume | 13.12           | 22.90           | 36.02       | 36.024      | 727749      | 31.47e-6     | 36.703   | 29.34         | 46.15      | 9.284           |                |             |
| Recursive       | 15.99           | 21.04           | 37.03       | 37.027      | 680775      | 30.91e-6     | 36.580   | 26.96         | 47.44      | 8.685           |                |             |
| Recursive2      | 15.80           | 18.71           | 34.51       | 34.514      | 583135      | 32.09e-6     | 36.580   | 23.98         | 44.22      | 7.439           |                |             |
| Interval        | 30.04e-6        | 14296           | 14296       | 03:58:16    | 1253896755  | 11.40e-6     | 38.245   | 100.0         | 100.0      | 100.0           |                |             |
| Bernstein       | 95.20           | 18616           | 18711       | 05:11:51    | 353805779   | 52.62e-6     | 37.086   | 130.2         | 130.9      | 28.22           |                |             |
| MinVariance     | 98.99           | 23482           | 23581       | 06:33:01    | 434809703   | 54.01e-6     | 37.202   | 164.3         | 164.9      | 34.68           |                |             |
| BSplines        | 96.14           | 115764          | 115764      | 1-08:09:24  | 2204613390  | 52.51e-6     | 38.704   | 809.7         | 809.7      | 175.8           |                |             |
| MinNorm         | 98.85           | 21223           | 21322       | 05:55:22    | 395877979   | 53.61e-6     | 37.151   | 148.5         | 149.1      | 31.57           |                |             |
| MinVolume       | 97.84           | 27058           | 27156       | 07:32:36    | 521431577   | 51.89e-6     | 37.312   | 189.3         | 189.9      | 41.58           |                |             |
| ApproxMinVolume | 96.05           | 27740           | 27836       | 07:43:56    | 523421895   | 53.00e-6     | 37.313   | 194.0         | 194.7      | 41.74           |                |             |
| Recursive       | 94.16           | 25520           | 25614       | 07:06:54    | 478738743   | 53.31e-6     | 37.086   | 178.5         | 179.2      | 38.18           |                |             |
| Recursive2      | 94.53           | 22335           | 22430       | 06:13:50    | 417985929   | 53.44e-6     | 37.086   | 156.2         | 156.9      | 33.33           |                |             |

Table 44: Bisection of problem 2D with problem: 16, type: MaxFirst

| precision  | ndof | solver          | prep_time       | comput_time | total_time | (D:H:M:S.ms) | nb_iter      | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |       |
|------------|------|-----------------|-----------------|-------------|------------|--------------|--------------|---------------|----------|-----------------|----------------|-------------|-------|
| 0.001      | 2    | Interval        | 41.01e-6        | 39.26e-3    | 39.30e-3   | 0.039        | 6513         | 6.027e-6      | 79.256   | 100.0           | 100.0          | 100.0       |       |
|            |      | Bernstein       | 846.4e-3        | 16.35e-3    | 862.8e-3   | 0.863        | 5035         | 3.248e-6      | 79.195   | 41.66           | 2195.6         | 77.31       |       |
|            |      | MinVariance     | 938.5e-3        | 16.45e-3    | 954.9e-3   | 0.955        | 5233         | 3.143e-6      | 79.197   | 41.90           | 2430.0         | 80.35       |       |
|            |      | BSplines        | 889.8e-3        | 28.30e-3    | 918.1e-3   | 0.918        | 8647         | 3.273e-6      | 79.286   | 72.09           | 2336.4         | 132.8       |       |
|            |      | MinNorm         | 966.3e-3        | 17.23e-3    | 983.5e-3   | 0.984        | 5101         | 3.377e-6      | 79.196   | 43.89           | 2502.9         | 78.32       |       |
|            |      | MinVolume       | 914.6e-3        | 17.53e-3    | 932.2e-3   | 0.932        | 5431         | 3.228e-6      | 79.199   | 44.66           | 2372.1         | 83.39       |       |
|            |      | ApproxMinVolume | 889.6e-3        | 17.80e-3    | 907.4e-3   | 0.907        | 5435         | 3.275e-6      | 79.199   | 45.35           | 2309.1         | 83.45       |       |
|            |      | Recursive       | 871.8e-3        | 16.07e-3    | 887.9e-3   | 0.888        | 5145         | 3.123e-6      | 79.195   | 40.94           | 2259.4         | 79.00       |       |
|            |      | Recursive2      | 870.1e-3        | 17.79e-3    | 887.9e-3   | 0.888        | 5271         | 3.375e-6      | 79.195   | 45.32           | 2259.5         | 80.93       |       |
|            |      | Recursive2      | 31.95e-6        | 1.802       | 1.802      | 1.802        | 234993       | 7.670e-6      | 80.820   | 100.0           | 100.0          | 100.0       |       |
|            | 3    | Bernstein       | 3.385           | 642.8e-3    | 4.028      | 4.028        | 66595        | 9.652e-6      | 80.719   | 35.66           | 223.4          | 28.34       |       |
|            |      | MinVariance     | 4.672           | 775.2e-3    | 5.447      | 5.447        | 74839        | 10.36e-6      | 80.723   | 43.01           | 302.2          | 31.85       |       |
|            |      | BSplines        | 3.578           | 2.791       | 6.368      | 6.368        | 255429       | 10.92e-6      | 80.806   | 154.8           | 353.3          | 108.7       |       |
|            |      | MinNorm         | 4.533           | 797.0e-3    | 5.330      | 5.330        | 71603        | 11.13e-6      | 80.722   | 44.22           | 295.7          | 30.47       |       |
|            |      | MinVolume       | 3.897           | 872.2e-3    | 4.769      | 4.769        | 84159        | 10.36e-6      | 80.727   | 48.39           | 264.6          | 35.81       |       |
|            |      | ApproxMinVolume | 3.896           | 819.2e-3    | 4.716      | 4.716        | 84441        | 9.702e-6      | 80.727   | 45.45           | 261.6          | 35.93       |       |
|            |      | Recursive       | 3.825           | 689.5e-3    | 4.514      | 4.514        | 68411        | 10.08e-6      | 80.719   | 38.25           | 250.5          | 29.11       |       |
|            |      | Recursive2      | 4.362           | 725.5e-3    | 5.087      | 5.087        | 68733        | 10.55e-6      | 80.719   | 40.25           | 282.2          | 29.25       |       |
|            |      | Recursive2      | 30.99e-6        | 1999        | 1999       | 33.19        | 226343431    | 8.835e-6      | 93.403   | 100.0           | 100.0          | 100.0       |       |
|            |      | Recursive2      | 11.50           | 310.4       | 321.9      | 05:21        | 11132313     | 27.88e-6      | 93.208   | 15.52           | 16.10          | 4.918       |       |
|            | 4    | MinVariance     | 16.68           | 450.5       | 467.2      | 07:47        | 13945137     | 32.30e-6      | 93.219   | 22.53           | 23.36          | 6.161       |       |
|            |      | BSplines        | 12.51           | 2225        | 2238       | 37:18        | 83778093     | 26.57e-6      | 93.349   | 111.3           | 111.9          | 37.01       |       |
|            |      | MinNorm         | 16.18           | 390.4       | 406.6      | 06:46        | 12565245     | 31.07e-6      | 93.214   | 19.52           | 20.33          | 5.551       |       |
|            |      | MinVolume       | 13.78           | 504.7       | 518.5      | 08:38        | 16878643     | 29.90e-6      | 93.229   | 25.24           | 25.92          | 7.457       |       |
|            |      | ApproxMinVolume | 13.60           | 502.3       | 515.9      | 08:35        | 16946953     | 29.64e-6      | 93.229   | 25.12           | 25.80          | 7.487       |       |
|            |      | Recursive       | 16.49           | 370.1       | 386.6      | 06:26        | 12239785     | 30.24e-6      | 93.208   | 18.51           | 19.33          | 5.408       |       |
|            |      | Recursive2      | 16.34           | 374.7       | 391.1      | 06:31        | 12310805     | 30.44e-6      | 93.208   | 18.74           | 19.55          | 5.439       |       |
|            |      | Recursive2      | 23.00e-6        | 2250450     | 2250450    | 26-01:07:30  | 200618424529 | 11.22e-6      | 107.82   | 100.0           | 100.0          | 100.0       |       |
|            |      | Recursive2      | 73.41           | 377027      | 377027     | 4-08:43:47   | 7407984656   | 50.89e-6      | 107.48   | 16.75           | 16.75          | 3.693       |       |
|            |      | Recursive2      | 77.73           | 491159      | 491159     | 5-16:25:59   | 9615782432   | 51.08e-6      | 107.49   | 21.82           | 21.82          | 4.793       |       |
|            | 5    | BSplines        | 73.71           | 4652710     | 4652710    | 53-20:25:10  | 94057467869  | 49.47e-6      | 107.66   | 206.7           | 206.7          | 46.88       |       |
|            |      | MinNorm         | 75.34           | 431024      | 431024     | 4-23:43:44   | 8541711916   | 50.46e-6      | 107.48   | 19.15           | 19.15          | 4.258       |       |
|            |      | MinVolume       | 75.95           | 606076      | 606076     | 7-00:21:16   | 12192893464  | 49.71e-6      | 107.50   | 26.93           | 26.93          | 6.078       |       |
|            |      | ApproxMinVolume | 75.41           | 603068      | 603068     | 6-23:31:08   | 12265852372  | 49.17e-6      | 107.50   | 26.80           | 26.80          | 6.114       |       |
|            |      | Recursive       | 74.40           | 409660      | 409660     | 4-17:47:40   | 8104307124   | 50.55e-6      | 107.48   | 18.20           | 18.20          | 4.040       |       |
|            |      | Recursive2      | 73.89           | 399327      | 399327     | 4-14:55:27   | 8070959330   | 49.48e-6      | 107.48   | 17.74           | 17.74          | 4.023       |       |
|            |      | Recursive2      | 23.84e-6        | 12.11e-3    | 12.13e-3   | 0.012        | 1441         | 8.404e-6      | 80.229   | 100.0           | 100.0          | 100.0       |       |
|            |      | Recursive2      | 833.1e-3        | 7.175e-3    | 840.3e-3   | 0.840        | 1005         | 7.139e-6      | 79.743   | 59.25           | 6925.2         | 69.74       |       |
|            |      | Recursive2      | 899.6e-3        | 8.409e-3    | 908.0e-3   | 0.908        | 1145         | 7.344e-6      | 79.762   | 69.44           | 7483.3         | 79.46       |       |
|            |      | Recursive2      | 768.6e-3        | 10.50e-3    | 779.1e-3   | 0.779        | 2341         | 4.485e-6      | 80.473   | 86.71           | 6421.1         | 162.5       |       |
|            | 0.01 | 2               | MinNorm         | 825.2e-3    | 6.332e-3   | 831.6e-3     | 0.832        | 1117          | 5.669e-6 | 79.753          | 52.29          | 6853.5      | 77.52 |
|            |      |                 | MinVolume       | 811.8e-3    | 7.004e-3   | 818.8e-3     | 0.819        | 1213          | 5.774e-6 | 79.781          | 57.84          | 6748.4      | 84.18 |
|            |      |                 | ApproxMinVolume | 788.8e-3    | 6.475e-3   | 795.3e-3     | 0.795        | 1217          | 5.321e-6 | 79.781          | 53.47          | 6554.6      | 84.46 |
|            |      |                 | Recursive       | 770.5e-3    | 8.921e-3   | 779.4e-3     | 0.779        | 1099          | 8.118e-6 | 79.743          | 73.67          | 6423.5      | 76.27 |
|            |      |                 | Recursive2      | 813.5e-3    | 7.174e-3   | 820.7e-3     | 0.821        | 1219          | 5.885e-6 | 79.743          | 59.24          | 6763.9      | 84.59 |
| Recursive2 |      |                 | 31.95e-6        | 288.9e-3    | 288.9e-3   | 0.289        | 37277        | 7.749e-6      | 82.293   | 100.0           | 100.0          | 100.0       |       |
| Recursive2 |      |                 | 3.363           | 174.7e-3    | 3.538      | 3.538        | 15637        | 11.17e-6      | 81.625   | 60.47           | 1224.6         | 41.95       |       |
| Recursive2 |      |                 | 4.653           | 190.5e-3    | 4.843      | 4.843        | 17147        | 11.11e-6      | 81.656   | 65.93           | 1676.4         | 46.00       |       |
| Recursive2 |      |                 | 3.528           | 559.8e-3    | 4.087      | 4.087        | 52413        | 10.68e-6      | 82.311   | 193.8           | 1414.8         | 140.6       |       |
| Recursive2 |      |                 | 4.446           | 172.6e-3    | 4.618      | 4.618        | 16373        | 10.54e-6      | 81.642   | 59.76           | 1598.5         | 43.92       |       |
| 3          |      | MinVolume       | 3.872           | 212.7e-3    | 4.085      | 4.085        | 18965        | 11.22e-6      | 81.685   | 73.63           | 1414.0         | 50.88       |       |
|            |      | ApproxMinVolume | 3.919           | 207.3e-3    | 4.127      | 4.127        | 19009        | 10.90e-6      | 81.686   | 71.75           | 1428.4         | 50.99       |       |
|            |      | Recursive       | 3.800           | 180.3e-3    | 3.980      | 3.980        | 17221        | 10.47e-6      | 81.625   | 62.43           | 1377.6         | 46.20       |       |
|            |      | Recursive2      | 4.255           | 189.7e-3    | 4.444      | 4.444        | 17639        | 10.75e-6      | 81.625   | 65.65           | 1538.3         | 47.32       |       |
|            |      | Recursive2      | 30.04e-6        | 189.6       | 189.6      | 03:09        | 16915625     | 11.21e-6      | 95.784   | 100.0           | 100.0          | 100.0       |       |
|            |      | Recursive2      | 11.66           | 47.38       | 59.03      | 59.031       | 1658429      | 28.57e-6      | 94.237   | 24.99           | 31.14          | 9.804       |       |
|            |      | Recursive2      | 15.51           | 65.07       | 80.58      | 01:20        | 2021301      | 32.19e-6      | 94.322   | 34.32           | 42.50          | 11.95       |       |
|            |      | Recursive2      | 11.99           | 342.8       | 354.8      | 05:54        | 11575531     | 29.61e-6      | 95.378   | 180.8           | 187.1          | 68.43       |       |
|            |      | Recursive2      | 15.40           | 58.57       | 73.97      | 01:13        | 1853443      | 31.60e-6      | 94.302   | 30.89           | 39.02          | 10.96       |       |
|            |      | Recursive2      | 13.40           | 76.48       | 89.87      | 01:29        | 2470633      | 30.95e-6      | 94.416   | 40.34           | 47.40          | 14.61       |       |
| 4          |      | ApproxMinVolume | 13.09           | 74.99       | 88.08      | 01:28        | 2482491      | 30.21e-6      | 94.417   | 39.55           | 46.46          | 14.68       |       |
|            |      | Recursive       | 15.74           | 76.11       | 91.85      | 01:31        | 2505595      | 30.38e-6      | 94.237   | 40.15           | 48.45          | 14.81       |       |
|            |      | Recursive2      | 15.85           | 78.00       | 93.86      | 01:33        | 2479921      | 31.45e-6      | 94.237   | 41.14           | 49.50          | 14.66       |       |
|            |      | Recursive2      | 31.00e-6        | 97561       | 97561      | 1-03:06:01   | 8785676254   | 11.10e-6      | 111.53   | 100.0           | 100.0          | 100.0       |       |
|            |      | Recursive2      | 93.67           | 24220       | 24313      | 06:45:13     | 466369727    | 51.93e-6      | 108.86   | 24.83           | 24.92          | 5.308       |       |
|            |      | Recursive2      | 98.97           | 32492       | 32591      | 09:03:11     | 609433693    | 53.32e-6      | 108.99   | 33.30           | 33.41          | 6.937       |       |
|            |      | Recursive2      | 94.04           | 297053      | 297054     | 3-10:30:54   | 5822558882   | 51.02e-6      | 110.49   | 304.5           | 304.5          | 66.27       |       |
|            |      | Recursive2      | 98.11           | 28047       | 28145      | 07:49:05     | 534119247    | 52.51e-6      | 108.92   | 28.75           | 28.85          | 6.079       |       |
|            |      | Recursive2      | 98.14           | 41018       | 41117      | 11:25:17     | 788900223    | 52.00e-6      | 109.10   | 42.04           | 42.14          | 8.979       |       |
|            |      | Recursive2      | 97.94           | 42189       | 42287      | 11:44:47     | 793275339    | 53.18e-6      | 109.10   | 43.24           | 43.34          | 9.029       |       |
| 5          |      | Recursive       | 95.92           | 42926       | 43022      | 11:57:02     | 808840765    | 53.07e-6      | 108.86   | 44.00           | 44.10          | 9.206       |       |
|            |      | Recursive2      | 95.36           | 37659       | 37754      | 10:29:14     | 712057779    | 52.89e-6      | 108.86   | 38.60           | 38.70          | 8.105       |       |

Table 45: Bisection of problem 2D with problem: 17, type: MaxFirst



| precision       | ndof            | solver          | prep_time   | comput_time | total_time  | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------------|-------------|-------------|-------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|
| 0.001           | 2               | Interval        | 28.85e-6    | 3.111e-3    | 3.140e-3    | 0.003        | 107      | 29.08e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 851.9e-3    | 3.150e-3    | 855.0e-3    | 0.855        | 69       | 45.60e-6      | infeasible | 101.3           | 27230.4        | 64.49       |
|                 |                 | MinVariance     | 915.9e-3    | 3.317e-3    | 919.2e-3    | 0.919        | 63       | 52.65e-6      | infeasible | 106.6           | 29275.2        | 58.88       |
|                 |                 | BSplines        | 872.1e-3    | 3.628e-3    | 875.7e-3    | 0.876        | 113      | 32.11e-6      | infeasible | 116.6           | 27889.3        | 105.6       |
|                 |                 | MinNorm         | 897.1e-3    | 3.134e-3    | 900.2e-3    | 0.900        | 61       | 51.38e-6      | infeasible | 100.7           | 28669.2        | 57.01       |
|                 |                 | MinVolume       | 878.0e-3    | 3.081e-3    | 881.1e-3    | 0.881        | 69       | 44.65e-6      | infeasible | 99.03           | 28060.5        | 64.49       |
|                 |                 | ApproxMinVolume | 879.4e-3    | 3.101e-3    | 882.5e-3    | 0.883        | 69       | 44.94e-6      | infeasible | 99.67           | 28106.2        | 64.49       |
|                 |                 | Recursive       | 876.5e-3    | 3.492e-3    | 880.0e-3    | 0.880        | 93       | 37.55e-6      | infeasible | 112.2           | 28024.6        | 86.92       |
|                 |                 | Recursive2      | 871.7e-3    | 3.210e-3    | 874.9e-3    | 0.875        | 123      | 26.10e-6      | infeasible | 103.2           | 27864.4        | 115.0       |
|                 |                 | Interval        | 30.99e-6    | 559.4e-3    | 559.4e-3    | 0.559        | 67829    | 8.247e-6      | 44.363     | 100.0           | 100.0          | 100.0       |
|                 |                 | Bernstein       | 3.377       | 347.7e-3    | 3.725       | 3.725        | 59033    | 5.889e-6      | 44.358     | 62.15           | 665.8          | 87.03       |
|                 |                 | MinVariance     | 4.619       | 377.0e-3    | 4.996       | 4.996        | 62027    | 6.078e-6      | 44.363     | 67.40           | 893.0          | 91.45       |
|                 | BSplines        | 3.554           | 1.133       | 4.687       | 4.687       | 171549       | 6.605e-6 | 44.451        | 202.5      | 837.9           | 252.9          |             |
|                 | MinNorm         | 4.515           | 366.8e-3    | 4.882       | 4.882       | 60287        | 6.083e-6 | 44.361        | 65.56      | 872.8           | 88.88          |             |
|                 | MinVolume       | 3.927           | 407.0e-3    | 4.334       | 4.334       | 67867        | 5.996e-6 | 44.367        | 72.75      | 774.8           | 100.1          |             |
|                 | ApproxMinVolume | 3.917           | 416.6e-3    | 4.334       | 4.334       | 67997        | 6.127e-6 | 44.367        | 74.47      | 774.7           | 100.2          |             |
|                 | Recursive       | 3.843           | 356.2e-3    | 4.199       | 4.199       | 59535        | 5.983e-6 | 44.358        | 63.68      | 750.6           | 87.77          |             |
|                 | Recursive2      | 4.294           | 364.3e-3    | 4.658       | 4.658       | 60313        | 6.040e-6 | 44.358        | 65.12      | 832.7           | 88.92          |             |
|                 | Interval        | 25.99e-6        | 106.3       | 106.3       | 01:46       | 11321541     | 9.390e-6 | 33.203        | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 12.15           | 51.76       | 63.91       | 01:03       | 1976463      | 26.19e-6 | 33.151        | 48.69      | 60.12           | 17.46          |             |
|                 | MinVariance     | 16.69           | 73.04       | 89.73       | 01:29       | 2438639      | 29.95e-6 | 33.159        | 68.70      | 84.40           | 21.54          |             |
|                 | BSplines        | 12.50           | 416.4       | 428.9       | 07:08       | 14862617     | 28.02e-6 | 33.250        | 391.7      | 403.4           | 131.3          |             |
|                 | MinNorm         | 16.06           | 64.21       | 80.27       | 01:20       | 2223893      | 28.87e-6 | 33.157        | 60.39      | 75.50           | 19.64          |             |
|                 | MinVolume       | 13.87           | 83.68       | 97.55       | 01:37       | 2981505      | 28.07e-6 | 33.166        | 78.71      | 91.76           | 26.33          |             |
|                 | ApproxMinVolume | 13.60           | 82.09       | 95.69       | 01:35       | 2994203      | 27.42e-6 | 33.166        | 77.21      | 90.01           | 26.45          |             |
|                 | Recursive       | 16.46           | 56.27       | 72.73       | 01:12       | 2085287      | 26.98e-6 | 33.151        | 52.93      | 68.41           | 18.42          |             |
|                 | Recursive2      | 13.97           | 56.77       | 70.74       | 01:10       | 2050371      | 27.69e-6 | 33.151        | 53.40      | 66.54           | 18.11          |             |
|                 | Interval        | 31.00e-6        | 1169670     | 1169670     | 13-12:54:30 | 109246741762 | 10.71e-6 | 31.568        | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 73.21           | 1355690     | 1355690     | 15-16:34:50 | 26612339096  | 50.94e-6 | 31.442        | 115.9      | 115.9           | 24.36          |             |
|                 | MinVariance     | 77.95           | 1776650     | 1776650     | 20-13:30:50 | 34539615227  | 51.44e-6 | 31.458        | 151.9      | 151.9           | 31.62          |             |
|                 | BSplines        | 73.65           | in progress | > 60 days   |             |              |          |               |            |                 |                |             |
|                 | MinNorm         | 77.24           | 1562000     | 1562000     | 18-01:53:20 | 30651698964  | 50.96e-6 | 31.450        | 133.5      | 133.5           | 28.06          |             |
|                 | MinVolume       | 76.02           | 2174030     | 2174030     | 25-03:53:50 | 42898379793  | 50.68e-6 | 31.472        | 185.9      | 185.9           | 39.27          |             |
|                 | ApproxMinVolume | 75.06           | 2199990     | 2199990     | 25-11:06:30 | 43133314536  | 51.00e-6 | 31.472        | 188.1      | 188.1           | 39.48          |             |
|                 | Recursive       | 74.22           | 1402990     | 1403000     | 16-05:43:20 | 27599489698  | 50.83e-6 | 31.442        | 119.9      | 119.9           | 25.26          |             |
|                 | Recursive2      | 73.27           | 1387000     | 1387000     | 16-01:16:40 | 27257663280  | 50.88e-6 | 31.442        | 118.6      | 118.6           | 24.95          |             |
|                 | Interval        | 24.08e-6        | 2.947e-3    | 2.971e-3    | 0.003       | 107          | 27.54e-6 | infeasible    | 100.0      | 100.0           | 100.0          |             |
|                 | Bernstein       | 854.4e-3        | 3.240e-3    | 857.6e-3    | 0.858       | 69           | 46.96e-6 | infeasible    | 110.0      | 28868.1         | 64.49          |             |
|                 | MinVariance     | 913.6e-3        | 3.120e-3    | 916.7e-3    | 0.917       | 63           | 49.53e-6 | infeasible    | 105.9      | 30855.3         | 58.88          |             |
|                 | BSplines        | 851.4e-3        | 3.482e-3    | 854.9e-3    | 0.855       | 113          | 30.81e-6 | infeasible    | 118.2      | 28776.6         | 105.6          |             |
|                 | MinNorm         | 892.4e-3        | 3.096e-3    | 895.5e-3    | 0.896       | 61           | 50.76e-6 | infeasible    | 105.1      | 30143.8         | 57.01          |             |
|                 | MinVolume       | 878.1e-3        | 3.513e-3    | 881.6e-3    | 0.882       | 69           | 50.91e-6 | infeasible    | 119.2      | 29674.7         | 64.49          |             |
|                 | ApproxMinVolume | 885.0e-3        | 3.499e-3    | 888.5e-3    | 0.888       | 69           | 50.71e-6 | infeasible    | 118.7      | 29905.9         | 64.49          |             |
|                 | Recursive       | 873.6e-3        | 3.398e-3    | 877.0e-3    | 0.877       | 93           | 36.54e-6 | infeasible    | 115.3      | 29518.4         | 86.92          |             |
|                 | Recursive2      | 859.9e-3        | 3.243e-3    | 863.2e-3    | 0.863       | 123          | 26.37e-6 | infeasible    | 110.0      | 29053.8         | 115.0          |             |
| Interval        | 34.09e-6        | 112.8e-3        | 112.9e-3    | 0.113       | 13441       | 8.394e-6     | 45.695   | 100.0         | 100.0      | 100.0           |                |             |
| Bernstein       | 3.290           | 69.99e-3        | 3.360       | 3.360       | 7541        | 9.281e-6     | 45.365   | 62.04         | 2977.1     | 56.10           |                |             |
| MinVariance     | 4.656           | 77.27e-3        | 4.733       | 4.733       | 8139        | 9.494e-6     | 45.400   | 68.49         | 4194.1     | 60.55           |                |             |
| BSplines        | 3.374           | 279.3e-3        | 3.653       | 3.653       | 28201       | 9.904e-6     | 46.398   | 247.6         | 3237.0     | 209.8           |                |             |
| MinNorm         | 4.334           | 73.13e-3        | 4.407       | 4.407       | 7831        | 9.339e-6     | 45.384   | 64.82         | 3904.7     | 58.26           |                |             |
| MinVolume       | 3.849           | 84.50e-3        | 3.933       | 3.933       | 9197        | 9.187e-6     | 45.434   | 74.89         | 3485.2     | 68.42           |                |             |
| ApproxMinVolume | 3.919           | 92.34e-3        | 4.011       | 4.011       | 9207        | 10.03e-6     | 45.435   | 81.85         | 3554.2     | 68.50           |                |             |
| Recursive       | 3.764           | 80.81e-3        | 3.845       | 3.845       | 8099        | 9.978e-6     | 45.365   | 71.63         | 3407.2     | 60.26           |                |             |
| Recursive2      | 4.217           | 84.07e-3        | 4.301       | 4.301       | 8813        | 9.539e-6     | 45.365   | 74.52         | 3811.3     | 65.57           |                |             |
| Interval        | 22.89e-6        | 11.13           | 11.13       | 11.133      | 1183541     | 9.407e-6     | 34.408   | 100.0         | 100.0      | 100.0           |                |             |
| Bernstein       | 11.64           | 8.347           | 19.99       | 19.988      | 290217      | 28.76e-6     | 33.973   | 74.97         | 179.5      | 24.52           |                |             |
| MinVariance     | 15.74           | 11.91           | 27.66       | 27.655      | 362485      | 32.86e-6     | 34.010   | 107.0         | 248.4      | 30.63           |                |             |
| BSplines        | 12.05           | 54.73           | 66.78       | 01:06       | 1845259     | 29.60e-6     | 34.818   | 491.6         | 599.8      | 155.9           |                |             |
| MinNorm         | 15.27           | 10.62           | 25.89       | 25.893      | 335011      | 31.71e-6     | 33.994   | 95.43         | 232.6      | 28.31           |                |             |
| MinVolume       | 13.25           | 12.34           | 25.59       | 25.587      | 431479      | 28.59e-6     | 34.071   | 110.8         | 229.8      | 36.46           |                |             |
| ApproxMinVolume | 13.13           | 13.46           | 26.60       | 26.596      | 433717      | 31.04e-6     | 34.072   | 120.9         | 238.9      | 36.65           |                |             |
| Recursive       | 15.73           | 11.34           | 27.07       | 27.066      | 368149      | 30.80e-6     | 33.973   | 101.9         | 243.1      | 31.11           |                |             |
| Recursive2      | 15.69           | 11.20           | 26.89       | 26.892      | 362575      | 30.89e-6     | 33.973   | 100.6         | 241.5      | 30.63           |                |             |
| Interval        | 26.94e-6        | 11199           | 11199       | 03:06:39    | 995655319   | 11.25e-6     | 33.874   | 100.0         | 100.0      | 100.0           |                |             |
| Bernstein       | 98.18           | 15392           | 15490       | 04:18:10    | 294803321   | 52.21e-6     | 32.826   | 137.4         | 138.3      | 29.61           |                |             |
| MinVariance     | 103.3           | 19323           | 19427       | 05:23:47    | 361755921   | 53.42e-6     | 32.947   | 172.5         | 173.5      | 36.33           |                |             |
| BSplines        | 97.45           | 96097           | 96097       | 1-02:41:37  | 1843777424  | 52.12e-6     | 34.374   | 858.0         | 858.0      | 185.2           |                |             |
| MinNorm         | 101.4           | 17883           | 17985       | 04:59:45    | 332115815   | 53.85e-6     | 32.900   | 159.7         | 160.6      | 33.36           |                |             |
| MinVolume       | 99.49           | 22987           | 23086       | 06:24:46    | 434072849   | 52.96e-6     | 33.059   | 205.2         | 206.1      | 43.60           |                |             |
| ApproxMinVolume | 99.28           | 22737           | 22837       | 06:20:37    | 435725303   | 52.18e-6     | 33.061   | 203.0         | 203.9      | 43.76           |                |             |
| Recursive       | 97.30           | 18791           | 18888       | 05:14:48    | 357482911   | 52.57e-6     | 32.826   | 167.8         | 168.7      | 35.90           |                |             |
| Recursive2      | 97.49           | 17439           | 17536       | 04:52:16    | 334202881   | 52.18e-6     | 32.826   | 155.7         | 156.6      | 33.57           |                |             |

Table 47: Bisection of problem 2D with problem: 19, type: MaxFirst

| precision       | ndof | solver          | prep_time       | comput_time | total_time | (D-H:M:S.ms) | nb_iter     | time_per_iter | comput_time (%) | total_time (%) | nb_iter (%) |       |
|-----------------|------|-----------------|-----------------|-------------|------------|--------------|-------------|---------------|-----------------|----------------|-------------|-------|
| 0.001           | 2    | Interval        | 31.00e-6        | 15.88e-3    | 15.91e-3   | 0.016        | 2044        | 20.00e-6      | 100.0           | 100.0          | 100.0       |       |
|                 |      | Bernstein       | 798.9e-3        | 6.706e-3    | 805.6e-3   | 0.806        | 1456        | 25.00e-6      | 65.22           | 12254.6        | 67.61       |       |
|                 |      | MinVariance     | 859.9e-3        | 6.746e-3    | 866.7e-3   | 0.867        | 1492        | 25.00e-6      | 64.99           | 13107.2        | 68.26       |       |
|                 |      | BSplines        | 813.7e-3        | 9.669e-3    | 823.3e-3   | 0.823        | 2399        | 19.00e-6      | 79.15           | 12437.2        | 111.8       |       |
|                 |      | MinNorm         | 854.9e-3        | 6.771e-3    | 861.7e-3   | 0.862        | 1471        | 26.00e-6      | 64.27           | 13003.7        | 67.05       |       |
|                 |      | MinVolume       | 841.9e-3        | 7.052e-3    | 849.0e-3   | 0.849        | 1553        | 24.00e-6      | 67.03           | 12855.1        | 72.45       |       |
|                 |      | ApproxMinVolume | 830.0e-3        | 7.001e-3    | 837.0e-3   | 0.837        | 1555        | 24.00e-6      | 66.17           | 12690.6        | 72.52       |       |
|                 |      | Recursive       | 814.0e-3        | 6.820e-3    | 820.8e-3   | 0.821        | 1509        | 21.00e-6      | 65.77           | 12430.7        | 75.80       |       |
|                 |      | Recursive2      | 816.9e-3        | 7.051e-3    | 824.0e-3   | 0.824        | 1534        | 20.00e-6      | 67.21           | 12479.4        | 82.56       |       |
|                 | 3    | Interval        | 29.00e-6        | 2.137       | 2.137      | 2.137        | 291396      | 8.000e-6      | 100.0           | 100.0          | 100.0       |       |
|                 |      | Bernstein       | 2.571           | 750.4e-3    | 3.322      | 3.322        | 80668       | 9.000e-6      | 42.63           | 387.5          | 39.15       |       |
|                 |      | MinVariance     | 3.504           | 921.9e-3    | 4.426      | 4.426        | 91282       | 9.000e-6      | 51.98           | 499.6          | 43.83       |       |
|                 |      | BSplines        | 2.689           | 2.724       | 5.413      | 5.413        | 275442      | 9.000e-6      | 149.5           | 510.5          | 126.2       |       |
|                 |      | MinNorm         | 3.379           | 861.2e-3    | 4.240      | 4.240        | 86279       | 9.000e-6      | 48.12           | 482.7          | 41.45       |       |
|                 |      | MinVolume       | 2.970           | 987.8e-3    | 3.958      | 3.958        | 102430      | 9.000e-6      | 55.53           | 445.8          | 49.04       |       |
|                 |      | ApproxMinVolume | 2.956           | 994.2e-3    | 3.950      | 3.950        | 102740      | 9.000e-6      | 55.87           | 445.8          | 49.16       |       |
|                 |      | Recursive       | 2.900           | 801.6e-3    | 3.702      | 3.702        | 84015       | 9.000e-6      | 45.27           | 428.7          | 40.46       |       |
|                 |      | Recursive2      | 3.144           | 811.4e-3    | 3.956      | 3.956        | 83342       | 9.000e-6      | 46.00           | 463.4          | 40.20       |       |
|                 | 4    | Interval        | 29.00e-6        | 1036        | 1036       | 17.16        | 118990242   | 9.000e-6      | 100.0           | 100.0          | 100.0       |       |
|                 |      | Bernstein       | 7.841           | 303.4       | 311.2      | 05:11        | 15509297    | 21.00e-6      | 34.22           | 36.98          | 15.11       |       |
|                 |      | MinVariance     | 10.39           | 385.3       | 395.7      | 06:35        | 18418458    | 23.00e-6      | 43.90           | 47.55          | 17.88       |       |
|                 |      | BSplines        | 8.086           | 1609        | 1617       | 26:57        | 80660031    | 21.00e-6      | 188.1           | 191.0          | 79.21       |       |
|                 |      | MinNorm         | 10.15           | 352.6       | 362.8      | 06:02        | 17058387    | 23.00e-6      | 40.10           | 43.67          | 16.58       |       |
|                 |      | MinVolume       | 8.907           | 438.5       | 447.4      | 07:27        | 21516318    | 22.00e-6      | 49.61           | 52.72          | 20.81       |       |
|                 |      | ApproxMinVolume | 8.863           | 439.4       | 448.3      | 07:28        | 21595419    | 22.00e-6      | 49.63           | 52.71          | 20.89       |       |
|                 |      | Recursive       | 9.988           | 323.6       | 333.5      | 05:33        | 15973579    | 22.00e-6      | 36.66           | 40.26          | 15.56       |       |
|                 |      | Recursive2      | 10.01           | 339.2       | 349.2      | 05:49        | 16750712    | 22.00e-6      | 38.19           | 41.78          | 16.17       |       |
|                 | 5    | Interval        | 28.00e-6        | 1085503     | 1085503    | 12-13:31:43  | 97765664157 | 11.00e-6      | 100.0           | 100.0          | 100.0       |       |
|                 |      | Bernstein       | 64.88           | 441722      | 441734     | 5-02:42:14   | 10498059595 | 42.00e-6      | 64.95           | 64.97          | 16.14       |       |
|                 |      | MinVariance     | 69.19           | 555135      | 555145     | 6-10:12:25   | 12863110512 | 43.00e-6      | 81.36           | 81.38          | 19.72       |       |
|                 |      | BSplines        | 63.24           | 2112760     | 2112768    | 24-10:52:48  | 50975345972 | 39.00e-6      | 445.4           | 445.4          | 109.3       |       |
|                 |      | MinNorm         | 69.24           | 501404      | 501417     | 5-19:16:57   | 11726379813 | 43.00e-6      | 73.85           | 73.87          | 18.01       |       |
|                 |      | MinVolume       | 67.31           | 661765      | 661776     | 7-15:49:36   | 15459329658 | 42.00e-6      | 96.40           | 96.42          | 23.64       |       |
|                 |      | ApproxMinVolume | 67.59           | 667536      | 667547     | 7-17:25:47   | 15541638589 | 42.00e-6      | 97.28           | 97.30          | 23.76       |       |
|                 |      | Recursive       | 65.69           | 460923      | 460936     | 5-08:02:16   | 10847674989 | 42.00e-6      | 67.02           | 67.04          | 16.51       |       |
|                 |      | Recursive2      | 65.48           | 466391      | 466403     | 5-09:33:23   | 11028926772 | 42.00e-6      | 69.76           | 69.79          | 17.25       |       |
|                 | 0.01 | 2               | Interval        | 30.00e-6    | 18.67e-3   | 18.70e-3     | 0.019       | 533           | 93.00e-6        | 100.0          | 100.0       | 100.0 |
|                 |      |                 | Bernstein       | 1.127       | 5.497e-3   | 1.133        | 1.133       | 337           | 39.00e-6        | 81.11          | 16178.4     | 66.84 |
|                 |      |                 | MinVariance     | 1.117       | 6.323e-3   | 1.123        | 1.123       | 349           | 40.00e-6        | 87.87          | 16325.4     | 67.43 |
|                 |      |                 | BSplines        | 1.085       | 7.050e-3   | 1.092        | 1.092       | 640           | 29.00e-6        | 98.95          | 15836.6     | 115.6 |
|                 |      |                 | MinNorm         | 1.151       | 5.957e-3   | 1.157        | 1.157       | 341           | 46.00e-6        | 89.40          | 17260.4     | 65.80 |
|                 |      |                 | MinVolume       | 1.149       | 6.395e-3   | 1.155        | 1.155       | 373           | 39.00e-6        | 86.95          | 17252.4     | 72.15 |
|                 |      |                 | ApproxMinVolume | 1.181       | 6.322e-3   | 1.188        | 1.188       | 374           | 38.00e-6        | 89.31          | 18001.4     | 72.25 |
|                 |      |                 | Recursive       | 1.166       | 6.361e-3   | 1.172        | 1.172       | 387           | 34.00e-6        | 93.90          | 17908.2     | 79.35 |
|                 |      |                 | Recursive2      | 1.184       | 6.123e-3   | 1.190        | 1.190       | 411           | 32.00e-6        | 91.33          | 18228.4     | 86.47 |
|                 |      | 3               | Interval        | 29.00e-6    | 367.9e-3   | 368.0e-3     | 0.368       | 49165         | 8.000e-6        | 100.0          | 100.0       | 100.0 |
|                 |      |                 | Bernstein       | 2.553       | 152.8e-3   | 2.706        | 2.706       | 15480         | 10.00e-6        | 47.31          | 1468.6      | 38.71 |
|                 |      |                 | MinVariance     | 3.490       | 182.9e-3   | 3.673        | 3.673       | 17422         | 10.00e-6        | 55.98          | 1936.4      | 43.47 |
| BSplines        |      |                 | 2.689           | 542.0e-3    | 3.231      | 3.231        | 51919       | 9.000e-6      | 166.3           | 1656.3         | 129.4       |       |
| MinNorm         |      |                 | 3.389           | 170.4e-3    | 3.559      | 3.559        | 16450       | 10.00e-6      | 52.35           | 1888.0         | 41.06       |       |
| MinVolume       |      |                 | 2.977           | 200.0e-3    | 3.177      | 3.177        | 19466       | 10.00e-6      | 61.59           | 1692.6         | 48.67       |       |
| ApproxMinVolume |      |                 | 2.958           | 199.8e-3    | 3.158      | 3.158        | 19527       | 10.00e-6      | 61.66           | 1681.7         | 48.83       |       |
| Recursive       |      |                 | 2.907           | 186.4e-3    | 3.093      | 3.093        | 18115       | 10.00e-6      | 56.72           | 1660.0         | 44.56       |       |
| Recursive2      |      |                 | 3.129           | 187.0e-3    | 3.316      | 3.316        | 17985       | 10.00e-6      | 56.76           | 1794.3         | 44.29       |       |
| 4               |      | Interval        | 30.00e-6        | 85.06       | 85.06      | 01:25        | 9264411     | 9.000e-6      | 100.0           | 100.0          | 100.0       |       |
|                 |      | Bernstein       | 7.742           | 25.74       | 33.48      | 33.485       | 1211176     | 22.00e-6      | 38.40           | 56.41          | 16.70       |       |
|                 |      | MinVariance     | 10.15           | 32.53       | 42.68      | 42.680       | 1412576     | 24.00e-6      | 48.99           | 72.70          | 19.49       |       |
|                 |      | BSplines        | 7.986           | 138.8       | 146.8      | 02:26        | 6197189     | 22.00e-6      | 205.7           | 224.3          | 83.96       |       |
|                 |      | MinNorm         | 9.913           | 29.68       | 39.59      | 39.593       | 1310372     | 23.00e-6      | 44.66           | 67.81          | 18.11       |       |
|                 |      | MinVolume       | 8.757           | 36.84       | 45.59      | 45.594       | 1652798     | 22.00e-6      | 54.88           | 75.23          | 22.70       |       |
|                 |      | ApproxMinVolume | 8.756           | 36.87       | 45.63      | 45.631       | 1659003     | 23.00e-6      | 55.36           | 75.65          | 22.79       |       |
|                 |      | Recursive       | 9.835           | 34.69       | 44.53      | 44.529       | 1544402     | 23.00e-6      | 49.96           | 73.02          | 20.80       |       |
|                 |      | Recursive2      | 9.924           | 39.96       | 49.88      | 49.884       | 1797626     | 23.00e-6      | 58.13           | 81.44          | 24.29       |       |
| 5               |      | Interval        | 30.00e-6        | 32645       | 32645      | 09:04:05     | 2939694658  | 10.00e-6      | 100.0           | 100.0          | 100.0       |       |
|                 |      | Bernstein       | 78.12           | 14397       | 14475      | 04:01:15     | 310835714   | 44.00e-6      | 79.14           | 80.32          | 18.93       |       |
|                 |      | MinVariance     | 81.80           | 17777       | 17859      | 04:57:39     | 376104619   | 45.00e-6      | 97.11           | 98.36          | 22.69       |       |
|                 |      | BSplines        | 79.02           | 115252      | 115298     | 1-08:01:38   | 2369248363  | 43.00e-6      | 551.4           | 552.5          | 128.0       |       |
|                 |      | MinNorm         | 82.16           | 16109       | 16191      | 04:29:51     | 343377734   | 45.00e-6      | 88.30           | 89.54          | 20.81       |       |
|                 |      | MinVolume       | 80.34           | 21399       | 21479      | 05:57:59     | 455149620   | 44.00e-6      | 115.0           | 116.3          | 27.12       |       |
|                 |      | ApproxMinVolume | 80.43           | 21584       | 21665      | 06:01:05     | 457327925   | 44.00e-6      | 115.5           | 116.7          | 27.25       |       |
|                 |      | Recursive       | 78.34           | 20539       | 20618      | 05:43:38     | 428065054   | 44.00e-6      | 99.42           | 100.6          | 23.37       |       |
|                 |      | Recursive2      | 78.50           | 20441       | 20520      | 05:42:00     | 433903019   | 44.00e-6      | 125.1           | 126.3          | 29.37       |       |

Table 48: Average of Bisection for 2D problems







| problem | $n$ | center of the target | $\alpha$ | criteria            |
|---------|-----|----------------------|----------|---------------------|
| 1       | 4   | {0.4; 0.4; 0.7}      | 1.0      | $\sum_i q_i^2$      |
| 2       | 4   | {0.6; 0.6; 0.6}      | 1.0      | $\sum_i q_i^2$      |
| 3       | 4   | {0.4; 0.4; 0.7}      | 0.1      | $\sum_i q_i^2$      |
| 4       | 4   | {0.6; 0.6; 0.6}      | 0.1      | $\sum_i q_i^2$      |
| 5       | 6   | {0.4; 0.4; 0.7}      | 1.0      | $\sum_i q_i^2$      |
| 6       | 6   | {0.6; 0.6; 0.6}      | 1.0      | $\sum_i q_i^2$      |
| 7       | 6   | {0.4; 0.4; 0.7}      | 0.1      | $\sum_i q_i^2$      |
| 8       | 6   | {0.6; 0.6; 0.6}      | 0.1      | $\sum_i q_i^2$      |
| 11      | 4   | {0.4; 0.4; 0.7}      | 1.0      | $\sum_i \Gamma_i^2$ |
| 12      | 4   | {0.6; 0.6; 0.6}      | 1.0      | $\sum_i \Gamma_i^2$ |
| 13      | 4   | {0.4; 0.4; 0.7}      | 0.1      | $\sum_i \Gamma_i^2$ |
| 14      | 4   | {0.6; 0.6; 0.6}      | 0.1      | $\sum_i \Gamma_i^2$ |

Table 51: 3D Problem list and their specificities.

| precision | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------|-----------------|-----------|-------------|------------|--------------|---------|---------------|----------|-----------------|----------------|-------------|
| 0.001     | Interval        | 55.07e-6  | 268.4       | 268.4      | 04:28        | 4272203 | 62.83e-6      | 2.6120   | 100.0           | 100.0          | 100.0       |
|           | Bernstein       | 89.93     | 27.52       | 117.4      | 01:57        | 1268249 | 21.70e-6      | 2.6084   | 10.25           | 43.75          | 29.69       |
|           | MinVariance     | 153.1     | 28.86       | 181.9      | 03:01        | 1379659 | 20.92e-6      | 2.6086   | 10.75           | 67.77          | 32.29       |
|           | BSplines        | 95.29     | 58.47       | 153.8      | 02:33        | 3196075 | 18.29e-6      | 2.6100   | 21.78           | 57.28          | 74.81       |
|           | MinNorm         | 154.2     | 27.15       | 181.3      | 03:01        | 1327313 | 20.46e-6      | 2.6085   | 10.12           | 67.56          | 31.07       |
|           | MinVolume       | 142.5     | 29.66       | 172.2      | 02:52        | 1489937 | 19.91e-6      | 2.6087   | 11.05           | 64.14          | 34.88       |
|           | ApproxMinVolume | 146.7     | 29.69       | 176.4      | 02:56        | 1492819 | 19.89e-6      | 2.6087   | 11.06           | 65.70          | 34.94       |
|           | Recursive       | 104.9     | 26.95       | 131.9      | 02:11        | 1270375 | 21.22e-6      | 2.6084   | 10.04           | 49.13          | 29.74       |
|           | Recursive2      | 120.3     | 26.88       | 147.2      | 02:27        | 1270105 | 21.16e-6      | 2.6084   | 10.01           | 54.84          | 29.73       |
|           | Interval        | 30.99e-6  | 24.53       | 24.53      | 24:53        | 401723  | 61.06e-6      | 2.7144   | 100.0           | 100.0          | 100.0       |
| 0.01      | Bernstein       | 89.60     | 3.151       | 92.75      | 01:32        | 116209  | 27.11e-6      | 2.6766   | 12.84           | 37.81          | 28.93       |
|           | MinVariance     | 158.4     | 3.188       | 161.6      | 02:41        | 123359  | 25.84e-6      | 2.6783   | 12.99           | 65.86          | 30.71       |
|           | BSplines        | 95.63     | 5.464       | 101.1      | 01:41        | 208225  | 26.24e-6      | 2.6975   | 22.27           | 412.1          | 51.83       |
|           | MinNorm         | 153.3     | 3.139       | 156.5      | 02:36        | 119935  | 26.18e-6      | 2.6775   | 12.80           | 637.9          | 29.86       |
|           | MinVolume       | 148.2     | 3.440       | 151.6      | 02:31        | 129969  | 26.47e-6      | 2.6799   | 14.02           | 618.1          | 32.35       |
|           | ApproxMinVolume | 145.7     | 3.541       | 149.2      | 02:29        | 130143  | 27.21e-6      | 2.6799   | 14.43           | 608.3          | 32.40       |
|           | Recursive       | 104.7     | 3.246       | 108.0      | 01:47        | 117323  | 27.67e-6      | 2.6766   | 13.23           | 440.1          | 29.20       |
|           | Recursive2      | 119.8     | 3.284       | 123.1      | 02:03        | 117539  | 27.94e-6      | 2.6766   | 13.39           | 501.6          | 29.26       |

Table 52: Bisection of problem 3D with problem: 1, type: MinFirst

| precision | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------|-----------------|-----------|-------------|------------|--------------|---------|---------------|------------|-----------------|----------------|-------------|
| 0.001     | Interval        | 32.90e-6  | 1.563       | 1.563      | 1:563        | 24393   | 64.06e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|           | Bernstein       | 89.48     | 188.7e-3    | 89.67      | 01:29        | 4157    | 45.38e-6      | infeasible | 12.07           | 5738.0         | 17.04       |
|           | MinVariance     | 153.3     | 179.7e-3    | 156.5      | 02:36        | 4157    | 43.24e-6      | infeasible | 11.50           | 10014.1        | 17.04       |
|           | BSplines        | 96.50     | 203.5e-3    | 96.70      | 01:36        | 4157    | 48.97e-6      | infeasible | 13.03           | 6188.1         | 17.04       |
|           | MinNorm         | 150.7     | 184.7e-3    | 150.9      | 02:30        | 4157    | 44.42e-6      | infeasible | 11.82           | 9656.6         | 17.04       |
|           | MinVolume       | 144.6     | 192.7e-3    | 144.8      | 02:24        | 4157    | 46.35e-6      | infeasible | 12.33           | 9263.6         | 17.04       |
|           | ApproxMinVolume | 140.2     | 192.4e-3    | 140.4      | 02:20        | 4157    | 46.28e-6      | infeasible | 12.31           | 8985.3         | 17.04       |
|           | Recursive       | 105.1     | 189.3e-3    | 105.3      | 01:45        | 4157    | 45.54e-6      | infeasible | 12.11           | 6736.3         | 17.04       |
|           | Recursive2      | 119.9     | 189.6e-3    | 120.0      | 02:00        | 4157    | 45.60e-6      | infeasible | 12.13           | 7681.7         | 17.04       |
|           | Interval        | 45.06e-6  | 1.672       | 1.672      | 1:672        | 24393   | 68.54e-6      | infeasible | 100.0           | 100.0          | 100.0       |
| 0.01      | Bernstein       | 89.86     | 191.4e-3    | 90.05      | 01:30        | 4157    | 46.05e-6      | infeasible | 11.45           | 5385.7         | 17.04       |
|           | MinVariance     | 162.0     | 185.7e-3    | 162.2      | 02:42        | 4157    | 44.68e-6      | infeasible | 11.11           | 9703.2         | 17.04       |
|           | BSplines        | 96.26     | 199.5e-3    | 96.46      | 01:36        | 4157    | 47.98e-6      | infeasible | 11.93           | 5769.4         | 17.04       |
|           | MinNorm         | 156.8     | 192.8e-3    | 157.0      | 02:36        | 4157    | 46.37e-6      | infeasible | 11.53           | 9389.9         | 17.04       |
|           | MinVolume       | 150.3     | 196.2e-3    | 150.5      | 02:30        | 4157    | 47.21e-6      | infeasible | 11.74           | 8999.9         | 17.04       |
|           | ApproxMinVolume | 147.5     | 197.2e-3    | 147.7      | 02:27        | 4157    | 47.43e-6      | infeasible | 11.79           | 8832.9         | 17.04       |
|           | Recursive       | 105.1     | 188.4e-3    | 105.3      | 01:45        | 4157    | 45.33e-6      | infeasible | 11.27           | 6299.8         | 17.04       |
|           | Recursive2      | 120.2     | 189.1e-3    | 120.4      | 02:00        | 4157    | 45.49e-6      | infeasible | 11.31           | 7200.2         | 17.04       |

Table 53: Bisection of problem 3D with problem: 2, type: MinFirst

| precision | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------|-----------------|-----------|-------------|------------|--------------|---------|---------------|----------|-----------------|----------------|-------------|
| 0.001     | Interval        | 36.95e-6  | 283.0       | 283.0      | 04:42        | 4461159 | 63.43e-6      | 2.6120   | 100.0           | 100.0          | 100.0       |
|           | Bernstein       | 89.20     | 29.68       | 118.9      | 01:58        | 1268485 | 23.39e-6      | 2.6084   | 10.49           | 42.01          | 28.43       |
|           | MinVariance     | 160.2     | 32.76       | 193.0      | 03:12        | 1383413 | 23.68e-6      | 2.6086   | 11.58           | 68.20          | 31.01       |
|           | BSplines        | 95.90     | 70.43       | 166.3      | 02:46        | 3265633 | 21.57e-6      | 2.6100   | 24.89           | 58.78          | 73.20       |
|           | MinNorm         | 155.6     | 32.50       | 188.1      | 03:08        | 1329327 | 24.45e-6      | 2.6085   | 11.48           | 66.48          | 29.80       |
|           | MinVolume       | 149.2     | 35.02       | 184.2      | 03:04        | 1496455 | 23.40e-6      | 2.6087   | 12.38           | 65.08          | 33.54       |
|           | ApproxMinVolume | 146.4     | 35.73       | 182.1      | 03:02        | 1499327 | 23.83e-6      | 2.6087   | 12.63           | 64.36          | 33.61       |
|           | Recursive       | 104.8     | 30.58       | 135.4      | 02:15        | 1270659 | 24.07e-6      | 2.6084   | 10.81           | 47.83          | 28.48       |
|           | Recursive2      | 119.8     | 30.02       | 149.8      | 02:29        | 1270449 | 23.63e-6      | 2.6084   | 10.61           | 52.94          | 28.48       |
|           | Interval        | 41.96e-6  | 25.23       | 25.23      | 25:23        | 396967  | 63.56e-6      | 2.7144   | 100.0           | 100.0          | 100.0       |
| 0.01      | Bernstein       | 89.69     | 4.003       | 93.69      | 01:33        | 114233  | 35.04e-6      | 2.6766   | 15.87           | 371.3          | 28.78       |
|           | MinVariance     | 164.7     | 4.307       | 169.0      | 02:48        | 121209  | 35.53e-6      | 2.6783   | 17.07           | 669.6          | 30.53       |
|           | BSplines        | 95.95     | 7.112       | 103.1      | 01:43        | 201303  | 35.33e-6      | 2.6975   | 28.19           | 408.5          | 50.71       |
|           | MinNorm         | 154.1     | 3.939       | 158.1      | 02:38        | 117841  | 33.43e-6      | 2.6775   | 15.61           | 626.5          | 29.69       |
|           | MinVolume       | 148.6     | 4.414       | 153.0      | 02:32        | 127467  | 34.63e-6      | 2.6799   | 17.50           | 606.3          | 32.11       |
|           | ApproxMinVolume | 146.6     | 4.462       | 151.0      | 02:31        | 127643  | 34.96e-6      | 2.6799   | 17.69           | 598.5          | 32.15       |
|           | Recursive       | 105.3     | 4.045       | 109.3      | 01:49        | 115373  | 35.06e-6      | 2.6766   | 16.03           | 433.2          | 29.06       |
|           | Recursive2      | 119.4     | 4.075       | 123.5      | 02:03        | 115651  | 35.23e-6      | 2.6766   | 16.15           | 489.6          | 29.13       |

Table 54: Bisection of problem 3D with problem: 3, type: MinFirst





| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|
| 0.001           | Interval        | 32.90e-6  | 974.3e-3    | 974.3e-3   | 0:974        | 15151    | 64.30e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 95.81     | 206.5e-3    | 96.02      | 01:36        | 2207     | 93.56e-6      | infeasible | 21.19           | 9854.8         | 14.57       |
|                 | MinVariance     | 165.0     | 192.6e-3    | 165.2      | 02:45        | 2207     | 87.26e-6      | infeasible | 19.77           | 16953.6        | 14.57       |
|                 | BSplines        | 101.7     | 244.6e-3    | 101.9      | 01:41        | 2637     | 92.76e-6      | infeasible | 25.11           | 10462.9        | 17.40       |
|                 | MinNorm         | 159.1     | 195.2e-3    | 159.3      | 02:39        | 2205     | 88.52e-6      | infeasible | 20.03           | 16345.5        | 14.55       |
|                 | MinVolume       | 156.8     | 204.9e-3    | 157.0      | 02:36        | 2257     | 90.78e-6      | infeasible | 21.03           | 16111.2        | 14.90       |
|                 | ApproxMinVolume | 154.6     | 211.2e-3    | 154.8      | 02:34        | 2257     | 93.58e-6      | infeasible | 21.68           | 15890.1        | 14.90       |
|                 | Recursive       | 111.4     | 213.3e-3    | 111.6      | 01:51        | 2285     | 93.35e-6      | infeasible | 21.89           | 11453.0        | 15.08       |
|                 | Recursive2      | 127.0     | 215.6e-3    | 127.2      | 02:07        | 2309     | 93.38e-6      | infeasible | 22.13           | 13057.4        | 15.24       |
|                 | Interval        | 37.19e-6  | 898.6e-3    | 898.6e-3   | 0:899        | 15151    | 59.31e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 95.74     | 205.7e-3    | 95.94      | 01:35        | 2207     | 93.18e-6      | infeasible | 22.89           | 10676.9        | 14.57       |
|                 | MinVariance     | 168.1     | 199.1e-3    | 168.3      | 02:48        | 2207     | 90.23e-6      | infeasible | 22.16           | 18726.3        | 14.57       |
|                 | BSplines        | 101.7     | 244.8e-3    | 101.9      | 01:41        | 2637     | 92.83e-6      | infeasible | 27.24           | 11344.3        | 17.40       |
|                 | MinNorm         | 161.7     | 203.3e-3    | 161.9      | 02:41        | 2205     | 92.21e-6      | infeasible | 22.63           | 18015.7        | 14.55       |
| MinVolume       | 159.1           | 212.3e-3  | 159.3       | 02:39      | 2257         | 94.08e-6 | infeasible    | 23.63      | 17726.8         | 14.90          |             |
| ApproxMinVolume | 156.0           | 218.1e-3  | 156.2       | 02:36      | 2257         | 96.64e-6 | infeasible    | 24.27      | 17386.0         | 14.90          |             |
| Recursive       | 110.7           | 214.1e-3  | 110.9       | 01:50      | 2285         | 93.70e-6 | infeasible    | 23.83      | 12343.3         | 15.08          |             |
| Recursive2      | 127.3           | 217.6e-3  | 127.6       | 02:07      | 2309         | 94.26e-6 | infeasible    | 24.22      | 14195.0         | 15.24          |             |

Table 63: Bisection of problem 3D with problem: 14, type: MinFirst

| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | Interval        | 32.90e-6  | 83.36       | 83.36      | 01:23        | 1316157  | 63.34e-6      | 2.6120   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 88.95     | 7.566       | 96.51      | 01:36        | 385193   | 19.64e-6      | 2.6084   | 9.077           | 115.8          | 29.27       |
|                 | MinVariance     | 163.1     | 7.736       | 170.9      | 02:50        | 427167   | 18.11e-6      | 2.6086   | 9.280           | 205.0          | 32.46       |
|                 | BSplines        | 96.59     | 18.74       | 115.3      | 01:55        | 1112923  | 16.84e-6      | 2.6100   | 22.48           | 138.3          | 84.56       |
|                 | MinNorm         | 158.1     | 7.797       | 165.9      | 02:45        | 407805   | 19.12e-6      | 2.6085   | 9.353           | 199.1          | 30.98       |
|                 | MinVolume       | 151.4     | 9.076       | 160.5      | 02:40        | 471811   | 19.24e-6      | 2.6087   | 10.89           | 192.5          | 35.85       |
|                 | ApproxMinVolume | 148.4     | 9.325       | 157.7      | 02:37        | 472951   | 19.72e-6      | 2.6087   | 11.19           | 189.2          | 35.93       |
|                 | Recursive       | 105.7     | 7.580       | 113.2      | 01:53        | 385813   | 19.65e-6      | 2.6084   | 9.093           | 135.8          | 29.31       |
|                 | Recursive2      | 121.0     | 7.656       | 128.6      | 02:08        | 386967   | 19.79e-6      | 2.6084   | 9.184           | 154.3          | 29.40       |
|                 | Interval        | 40.05e-6  | 9.641       | 9.641      | 9:641        | 154829   | 62.27e-6      | 2.7144   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 89.73     | 1.042       | 90.77      | 01:30        | 37321    | 27.91e-6      | 2.6766   | 10.80           | 941.5          | 24.10       |
|                 | MinVariance     | 158.9     | 1.079       | 159.9      | 02:39        | 39779    | 27.12e-6      | 2.6783   | 11.19           | 1659.0         | 25.69       |
|                 | BSplines        | 95.82     | 1.864       | 97.69      | 01:37        | 68765    | 27.11e-6      | 2.6975   | 19.33           | 1013.3         | 44.41       |
|                 | MinNorm         | 153.8     | 1.050       | 154.8      | 02:34        | 38621    | 27.18e-6      | 2.6775   | 10.89           | 1605.8         | 24.94       |
| MinVolume       | 148.3           | 1.140     | 149.4       | 02:29      | 42283        | 26.96e-6 | 2.6799        | 11.82    | 1549.6          | 27.31          |             |
| ApproxMinVolume | 145.8           | 1.200     | 147.0       | 02:26      | 42337        | 28.34e-6 | 2.6799        | 12.45    | 1524.4          | 27.34          |             |
| Recursive       | 104.8           | 1.065     | 105.9       | 01:45      | 37933        | 28.07e-6 | 2.6766        | 11.04    | 1098.5          | 24.50          |             |
| Recursive2      | 119.5           | 1.102     | 120.6       | 02:00      | 38543        | 28.59e-6 | 2.6766        | 11.43    | 1251.2          | 24.89          |             |

Table 64: Bisection of problem 3D with problem: 1, type: MaxFirst

| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|
| 0.001           | Interval        | 50.07e-6  | 1.566       | 1.566      | 1:566        | 24393    | 64.20e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 89.68     | 191.4e-3    | 89.87      | 01:29        | 4157     | 46.03e-6      | infeasible | 12.22           | 5738.2         | 17.04       |
|                 | MinVariance     | 162.6     | 181.7e-3    | 162.8      | 02:42        | 4157     | 43.72e-6      | infeasible | 11.60           | 10395.7        | 17.04       |
|                 | BSplines        | 95.47     | 200.5e-3    | 95.67      | 01:35        | 4157     | 48.24e-6      | infeasible | 12.81           | 6108.6         | 17.04       |
|                 | MinNorm         | 154.5     | 182.3e-3    | 154.7      | 02:34        | 4157     | 43.85e-6      | infeasible | 11.64           | 9878.3         | 17.04       |
|                 | MinVolume       | 148.7     | 186.8e-3    | 148.8      | 02:28        | 4157     | 44.93e-6      | infeasible | 11.93           | 9504.4         | 17.04       |
|                 | ApproxMinVolume | 146.0     | 196.3e-3    | 146.2      | 02:26        | 4157     | 47.23e-6      | infeasible | 12.54           | 9336.1         | 17.04       |
|                 | Recursive       | 105.2     | 189.2e-3    | 105.4      | 01:45        | 4157     | 45.52e-6      | infeasible | 12.08           | 6730.9         | 17.04       |
|                 | Recursive2      | 119.4     | 188.7e-3    | 119.6      | 01:59        | 4157     | 45.39e-6      | infeasible | 12.05           | 7634.1         | 17.04       |
|                 | Interval        | 27.89e-6  | 1.520       | 1.521      | 1:521        | 24393    | 62.33e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 89.70     | 191.9e-3    | 89.89      | 01:29        | 4157     | 46.17e-6      | infeasible | 12.62           | 5911.6         | 17.04       |
|                 | MinVariance     | 164.2     | 198.6e-3    | 164.4      | 02:44        | 4157     | 47.78e-6      | infeasible | 13.06           | 10815.0        | 17.04       |
|                 | BSplines        | 95.28     | 197.9e-3    | 95.48      | 01:35        | 4157     | 47.61e-6      | infeasible | 13.02           | 6279.4         | 17.04       |
|                 | MinNorm         | 158.9     | 195.1e-3    | 159.1      | 02:39        | 4157     | 46.94e-6      | infeasible | 12.83           | 10460.8        | 17.04       |
| MinVolume       | 151.0           | 196.3e-3  | 151.2       | 02:31      | 4157         | 47.22e-6 | infeasible    | 12.91      | 9943.9          | 17.04          |             |
| ApproxMinVolume | 148.0           | 194.5e-3  | 148.2       | 02:28      | 4157         | 46.79e-6 | infeasible    | 12.79      | 9745.0          | 17.04          |             |
| Recursive       | 104.8           | 188.0e-3  | 105.0       | 01:44      | 4157         | 45.23e-6 | infeasible    | 12.37      | 6902.6          | 17.04          |             |
| Recursive2      | 119.9           | 188.3e-3  | 120.1       | 02:00      | 4157         | 45.30e-6 | infeasible    | 12.39      | 7898.2          | 17.04          |             |

Table 65: Bisection of problem 3D with problem: 2, type: MaxFirst

| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | Interval        | 39.10e-6  | 114.9       | 114.9      | 01:54        | 1858809  | 61.81e-6      | 2.6120   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 66.59     | 12.65       | 79.24      | 01:19        | 442385   | 28.60e-6      | 2.6084   | 11.01           | 68.97          | 23.80       |
|                 | MinVariance     | 126.5     | 15.14       | 141.7      | 02:21        | 493407   | 30.69e-6      | 2.6086   | 13.18           | 123.3          | 26.54       |
|                 | BSplines        | 72.20     | 33.80       | 106.0      | 01:46        | 1317809  | 25.65e-6      | 2.6100   | 29.42           | 92.26          | 70.90       |
|                 | MinNorm         | 112.9     | 14.06       | 127.0      | 02:06        | 469353   | 29.95e-6      | 2.6085   | 12.23           | 110.5          | 25.25       |
|                 | MinVolume       | 115.0     | 15.95       | 130.9      | 02:10        | 545359   | 29.26e-6      | 2.6087   | 13.89           | 113.9          | 29.34       |
|                 | ApproxMinVolume | 108.2     | 15.99       | 124.2      | 02:04        | 546659   | 29.26e-6      | 2.6087   | 13.92           | 108.1          | 29.41       |
|                 | Recursive       | 77.84     | 12.87       | 90.71      | 01:30        | 443105   | 29.04e-6      | 2.6084   | 11.20           | 78.94          | 23.84       |
|                 | Recursive2      | 92.83     | 12.90       | 105.7      | 01:45        | 444233   | 29.03e-6      | 2.6084   | 11.23           | 92.02          | 23.90       |
|                 | Interval        | 36.00e-6  | 10.15       | 10.15      | 10:154       | 159279   | 63.75e-6      | 2.7144   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 89.18     | 1.503       | 90.68      | 01:30        | 38673    | 38.87e-6      | 2.6766   | 14.80           | 893.1          | 24.28       |
|                 | MinVariance     | 161.7     | 1.551       | 163.3      | 02:43        | 41101    | 37.74e-6      | 2.6783   | 15.27           | 1608.0         | 25.80       |
|                 | BSplines        | 96.06     | 2.680       | 98.74      | 01:38        | 68621    | 39.05e-6      | 2.6975   | 26.39           | 972.4          | 43.08       |
|                 | MinNorm         | 157.3     | 1.554       | 158.9      | 02:38        | 39963    | 38.89e-6      | 2.6775   | 15.31           | 1564.4         | 25.09       |
| MinVolume       | 150.5           | 1.713     | 152.2       | 02:32      | 43655        | 39.23e-6 | 2.6799        | 16.87    | 1498.7          | 27.41          |             |
| ApproxMinVolume | 147.5           | 1.783     | 149.3       | 02:29      | 43715        | 40.78e-6 | 2.6799        | 17.56    | 1470.4          | 27.45          |             |
| Recursive       | 105.1           | 1.563     | 106.7       | 01:46      | 39353        | 39.72e-6 | 2.6766        | 15.39    | 1050.8          | 24.71          |             |
| Recursive2      | 120.3           | 1.577     | 121.9       | 02:01      | 39897        | 39.53e-6 | 2.6766        | 15.53    | 1200.4          | 25.05          |             |

Table 66: Bisection of problem 3D with problem: 3, type: MaxFirst



| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter   | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|-----------|---------------|------------|-----------------|----------------|-------------|
| 0.001           | Interval        | 43.87e-6  | 20654       | 20654      | 05:44:14     | 229302217 | 90.08e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 370.0     | 931.2       | 1301       | 21:41        | 4457773   | 208.9e-6      | infeasible | 4.509           | 6.300          | 1.944       |
|                 | MinVariance     | 404.8     | 950.4       | 1355       | 22:35        | 4457773   | 213.2e-6      | infeasible | 4.601           | 6.561          | 1.944       |
|                 | BSplines        | 378.2     | 940.0       | 1318       | 21:58        | 4457773   | 210.9e-6      | infeasible | 4.551           | 6.382          | 1.944       |
|                 | MinNorm         | 397.4     | 934.3       | 1331       | 22:11        | 4457773   | 209.6e-6      | infeasible | 4.524           | 6.448          | 1.944       |
|                 | MinVolume       | 308.0     | 937.2       | 1245       | 20:45        | 4457773   | 210.2e-6      | infeasible | 4.537           | 6.029          | 1.944       |
|                 | ApproxMinVolume | 302.5     | 922.0       | 1224       | 20:24        | 4457773   | 206.8e-6      | infeasible | 4.464           | 5.928          | 1.944       |
|                 | Recursive       | 315.9     | 921.4       | 1237       | 20:37        | 4457773   | 206.7e-6      | infeasible | 4.461           | 5.991          | 1.944       |
|                 | Recursive2      | 333.5     | 925.1       | 1258       | 20:58        | 4457773   | 207.5e-6      | infeasible | 4.479           | 6.093          | 1.944       |
|                 | Interval        | 29.09e-6  | 20779       | 20779      | 05:46:19     | 229302217 | 90.62e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 275.3     | 936.4       | 1211       | 20:11        | 4457773   | 210.1e-6      | infeasible | 4.506           | 5.831          | 1.944       |
|                 | MinVariance     | 315.0     | 950.2       | 1265       | 21:05        | 4457773   | 213.2e-6      | infeasible | 4.573           | 6.089          | 1.944       |
|                 | BSplines        | 287.2     | 933.6       | 1220       | 20:20        | 4457773   | 209.4e-6      | infeasible | 4.493           | 5.875          | 1.944       |
|                 | MinNorm         | 343.0     | 920.8       | 1263       | 21:03        | 4457773   | 206.6e-6      | infeasible | 4.431           | 6.082          | 1.944       |
| MinVolume       | 312.2           | 930.7     | 1242        | 20:42      | 4457773      | 208.8e-6  | infeasible    | 4.479      | 5.981           | 1.944          |             |
| ApproxMinVolume | 369.8           | 952.2     | 1322        | 22:02      | 4457773      | 213.6e-6  | infeasible    | 4.583      | 6.362           | 1.944          |             |
| Recursive       | 332.2           | 939.8     | 1271        | 21:11      | 4457773      | 210.8e-6  | infeasible    | 4.523      | 6.121           | 1.944          |             |
| Recursive2      | 283.1           | 926.5     | 1209        | 20:09      | 4457773      | 207.8e-6  | infeasible    | 4.459      | 5.821           | 1.944          |             |

Table 71: Bisection of problem 3D with problem: 8, type: MaxFirst

| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | Interval        | 40.05e-6  | 1998        | 1998       | 33:18        | 36483067 | 54.78e-6      | 386.24   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 96.19     | 747.1       | 843.3      | 14:03        | 6347179  | 117.7e-6      | 385.26   | 37.38           | 42.19          | 17.40       |
|                 | MinVariance     | 168.0     | 947.0       | 1114       | 18:34        | 7229953  | 131.0e-6      | 385.34   | 47.38           | 55.78          | 19.82       |
|                 | BSplines        | 101.6     | 4231        | 4333       | 01:12:13     | 40300001 | 105.0e-6      | 386.51   | 211.7           | 216.8          | 110.5       |
|                 | MinNorm         | 160.1     | 880.5       | 1040       | 17:20        | 6826491  | 129.0e-6      | 385.30   | 44.05           | 52.06          | 18.71       |
|                 | MinVolume       | 158.2     | 1043        | 1201       | 20:01        | 8309215  | 125.6e-6      | 385.41   | 52.21           | 60.12          | 22.78       |
|                 | ApproxMinVolume | 155.1     | 1054        | 1209       | 20:09        | 8349359  | 126.3e-6      | 385.41   | 52.74           | 60.50          | 22.89       |
|                 | Recursive       | 110.5     | 768.4       | 878.9      | 14:38        | 6389281  | 120.3e-6      | 385.26   | 38.45           | 43.97          | 17.51       |
|                 | Recursive2      | 127.0     | 777.8       | 904.7      | 15:04        | 6365803  | 122.2e-6      | 385.26   | 38.91           | 45.27          | 17.45       |
|                 | Interval        | 36.00e-6  | 56.05       | 56.05      | 56.049       | 910361   | 61.57e-6      | 414.99   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 95.58     | 25.91       | 121.5      | 02:01        | 202943   | 127.7e-6      | 404.04   | 46.23           | 216.8          | 22.29       |
|                 | MinVariance     | 167.5     | 29.96       | 197.5      | 03:17        | 221987   | 134.9e-6      | 405.30   | 53.44           | 352.3          | 24.38       |
|                 | BSplines        | 101.0     | 61.35       | 162.3      | 02:42        | 475893   | 128.9e-6      | 425.62   | 109.4           | 289.6          | 52.28       |
|                 | MinNorm         | 159.4     | 29.43       | 188.8      | 03:08        | 213119   | 138.1e-6      | 404.71   | 52.51           | 336.9          | 23.41       |
| MinVolume       | 157.4           | 33.40     | 190.8       | 03:10      | 245427       | 136.1e-6 | 406.55        | 59.59    | 340.4           | 26.96          |             |
| ApproxMinVolume | 155.4           | 33.96     | 189.4       | 03:09      | 246173       | 137.9e-6 | 406.58        | 60.59    | 337.9           | 27.04          |             |
| Recursive       | 111.2           | 27.46     | 138.6       | 02:18      | 208803       | 131.5e-6 | 404.04        | 48.98    | 247.4           | 22.94          |             |
| Recursive2      | 126.7           | 27.39     | 154.1       | 02:34      | 208691       | 131.3e-6 | 404.04        | 48.88    | 274.9           | 22.92          |             |

Table 72: Bisection of problem 3D with problem: 11, type: MaxFirst

| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|
| 0.001           | Interval        | 36.95e-6  | 1.532       | 1.532      | 1.532        | 24393    | 62.79e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 96.12     | 195.6e-3    | 96.31      | 01:36        | 4157     | 47.05e-6      | infeasible | 12.77           | 6288.6         | 17.04       |
|                 | MinVariance     | 166.2     | 175.3e-3    | 166.4      | 02:46        | 4157     | 42.18e-6      | infeasible | 11.45           | 10866.2        | 17.04       |
|                 | BSplines        | 102.4     | 201.3e-3    | 102.6      | 01:42        | 4157     | 48.43e-6      | infeasible | 13.14           | 6698.4         | 17.04       |
|                 | MinNorm         | 159.9     | 176.0e-3    | 160.0      | 02:40        | 4157     | 42.33e-6      | infeasible | 11.49           | 10448.8        | 17.04       |
|                 | MinVolume       | 158.0     | 187.5e-3    | 158.2      | 02:38        | 4157     | 45.10e-6      | infeasible | 12.24           | 10328.2        | 17.04       |
|                 | ApproxMinVolume | 155.2     | 194.9e-3    | 155.4      | 02:35        | 4157     | 46.89e-6      | infeasible | 12.73           | 10147.4        | 17.04       |
|                 | Recursive       | 111.9     | 190.8e-3    | 112.1      | 01:52        | 4157     | 45.90e-6      | infeasible | 12.46           | 7319.8         | 17.04       |
|                 | Recursive2      | 127.9     | 192.5e-3    | 128.1      | 02:08        | 4157     | 46.31e-6      | infeasible | 12.57           | 8361.0         | 17.04       |
|                 | Interval        | 32.90e-6  | 1.512       | 1.512      | 1.512        | 24393    | 61.99e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 95.99     | 193.1e-3    | 96.19      | 01:36        | 4157     | 46.46e-6      | infeasible | 12.77           | 6361.4         | 17.04       |
|                 | MinVariance     | 165.6     | 175.3e-3    | 165.8      | 02:45        | 4157     | 42.18e-6      | infeasible | 11.60           | 10964.1        | 17.04       |
|                 | BSplines        | 102.4     | 201.8e-3    | 102.6      | 01:42        | 4157     | 48.55e-6      | infeasible | 13.35           | 6785.3         | 17.04       |
|                 | MinNorm         | 159.1     | 177.8e-3    | 159.3      | 02:39        | 4157     | 42.76e-6      | infeasible | 11.76           | 10534.5        | 17.04       |
| MinVolume       | 157.8           | 189.9e-3  | 158.0       | 02:38      | 4157         | 45.68e-6 | infeasible    | 12.56      | 10451.3         | 17.04          |             |
| ApproxMinVolume | 154.5           | 191.2e-3  | 154.7       | 02:34      | 4157         | 46.00e-6 | infeasible    | 12.65      | 10232.4         | 17.04          |             |
| Recursive       | 111.2           | 191.4e-3  | 111.4       | 01:51      | 4157         | 46.04e-6 | infeasible    | 12.66      | 7369.6          | 17.04          |             |
| Recursive2      | 127.2           | 191.5e-3  | 127.4       | 02:07      | 4157         | 46.08e-6 | infeasible    | 12.67      | 8424.1          | 17.04          |             |

Table 73: Bisection of problem 3D with problem: 12, type: MaxFirst

| precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria | comput_time (%) | total_time (%) | nb_iter (%) |
|-----------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|----------|-----------------|----------------|-------------|
| 0.001           | Interval        | 35.05e-6  | 2173        | 2173       | 36:13        | 36386983 | 59.72e-6      | 386.24   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 69.88     | 751.6       | 821.5      | 13:41        | 6324949  | 118.8e-6      | 385.26   | 34.59           | 37.80          | 17.38       |
|                 | MinVariance     | 131.3     | 939.4       | 1070       | 17:50        | 7206457  | 130.4e-6      | 385.34   | 43.23           | 49.27          | 19.81       |
|                 | BSplines        | 75.13     | 4610        | 4685       | 01:18:05     | 40254141 | 114.5e-6      | 386.51   | 212.1           | 215.6          | 110.6       |
|                 | MinNorm         | 116.0     | 863.9       | 979.9      | 16:19        | 6803387  | 127.0e-6      | 385.30   | 39.75           | 45.09          | 18.70       |
|                 | MinVolume       | 122.0     | 1051        | 1173       | 19:33        | 8284391  | 127.0e-6      | 385.41   | 48.40           | 54.01          | 22.77       |
|                 | ApproxMinVolume | 113.7     | 1041        | 1155       | 19:15        | 8324463  | 125.1e-6      | 385.41   | 47.92           | 53.16          | 22.88       |
|                 | Recursive       | 81.51     | 768.0       | 849.5      | 14:09        | 6366677  | 120.6e-6      | 385.26   | 35.34           | 39.09          | 17.50       |
|                 | Recursive2      | 94.84     | 789.5       | 884.3      | 14:44        | 6343087  | 124.5e-6      | 385.26   | 36.33           | 40.69          | 17.43       |
|                 | Interval        | 30.99e-6  | 56.81       | 56.81      | 56.814       | 908209   | 62.56e-6      | 414.99   | 100.0           | 100.0          | 100.0       |
|                 | Bernstein       | 96.23     | 25.90       | 122.1      | 02:02        | 201961   | 128.3e-6      | 404.04   | 45.59           | 215.0          | 22.24       |
|                 | MinVariance     | 167.2     | 30.31       | 197.5      | 03:17        | 220827   | 137.3e-6      | 405.30   | 53.36           | 347.6          | 24.31       |
|                 | BSplines        | 101.7     | 60.92       | 162.7      | 02:42        | 469759   | 129.7e-6      | 425.62   | 107.2           | 286.3          | 51.72       |
|                 | MinNorm         | 161.0     | 29.29       | 190.3      | 03:10        | 212063   | 138.1e-6      | 404.71   | 51.56           | 334.9          | 23.35       |
| MinVolume       | 158.1           | 33.04     | 191.1       | 03:11      | 244065       | 135.4e-6 | 406.55        | 58.15    | 336.4           | 26.87          |             |
| ApproxMinVolume | 155.0           | 33.35     | 188.4       | 03:08      | 244805       | 136.2e-6 | 406.58        | 58.70    | 331.5           | 26.95          |             |
| Recursive       | 110.9           | 26.70     | 137.6       | 02:17      | 207469       | 128.7e-6 | 404.04        | 47.00    | 242.3           | 22.84          |             |
| Recursive2      | 127.1           | 27.15     | 154.2       | 02:34      | 207231       | 131.0e-6 | 404.04        | 47.79    | 271.4           | 22.82          |             |

Table 74: Bisection of problem 3D with problem: 13, type: MaxFirst

| precision  | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter  | time_per_iter | criteria   | comput_time (%) | total_time (%) | nb_iter (%) |
|------------|-----------------|-----------|-------------|------------|--------------|----------|---------------|------------|-----------------|----------------|-------------|
| 0.001      | Interval        | 36.00e-6  | 952.0e-3    | 952.0e-3   | 0.952        | 15151    | 62.83e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|            | Bernstein       | 70.20     | 204.0e-3    | 70.40      | 01:10        | 2207     | 92.44e-6      | infeasible | 21.43           | 7395.1         | 14.57       |
|            | MinVariance     | 133.3     | 213.7e-3    | 133.5      | 02:13        | 2207     | 96.84e-6      | infeasible | 22.45           | 14023.1        | 14.57       |
|            | BSplines        | 75.89     | 239.2e-3    | 76.13      | 01:16        | 2637     | 90.72e-6      | infeasible | 25.13           | 7996.8         | 17.40       |
|            | MinNorm         | 147.6     | 214.4e-3    | 147.9      | 02:27        | 2205     | 97.25e-6      | infeasible | 22.53           | 15530.8        | 14.55       |
|            | MinVolume       | 145.2     | 218.6e-3    | 145.4      | 02:25        | 2257     | 96.84e-6      | infeasible | 22.96           | 15275.6        | 14.90       |
|            | ApproxMinVolume | 141.3     | 218.1e-3    | 141.5      | 02:21        | 2257     | 96.64e-6      | infeasible | 22.91           | 14867.8        | 14.90       |
|            | Recursive       | 92.43     | 187.8e-3    | 92.61      | 01:32        | 2285     | 82.20e-6      | infeasible | 19.73           | 9728.4         | 15.08       |
| Recursive2 | 104.1           | 191.9e-3  | 104.3       | 01:44      | 2309         | 83.10e-6 | infeasible    | 20.16      | 10952.7         | 15.24          |             |
| 0.01       | Interval        | 56.98e-6  | 943.5e-3    | 943.6e-3   | 0.944        | 15151    | 62.27e-6      | infeasible | 100.0           | 100.0          | 100.0       |
|            | Bernstein       | 95.47     | 205.0e-3    | 95.68      | 01:35        | 2207     | 92.89e-6      | infeasible | 21.73           | 10139.9        | 14.57       |
|            | MinVariance     | 165.5     | 192.0e-3    | 165.7      | 02:45        | 2207     | 87.01e-6      | infeasible | 20.35           | 17564.0        | 14.57       |
|            | BSplines        | 102.1     | 245.6e-3    | 102.4      | 01:42        | 2637     | 93.12e-6      | infeasible | 26.03           | 10848.4        | 17.40       |
|            | MinNorm         | 159.0     | 195.4e-3    | 159.2      | 02:39        | 2205     | 88.62e-6      | infeasible | 20.71           | 16875.9        | 14.55       |
|            | MinVolume       | 157.1     | 204.6e-3    | 157.3      | 02:37        | 2257     | 90.67e-6      | infeasible | 21.69           | 16667.5        | 14.90       |
|            | ApproxMinVolume | 154.7     | 211.3e-3    | 154.9      | 02:34        | 2257     | 93.62e-6      | infeasible | 22.40           | 16413.2        | 14.90       |
|            | Recursive       | 110.9     | 213.3e-3    | 111.1      | 01:51        | 2285     | 93.34e-6      | infeasible | 22.61           | 11778.3        | 15.08       |
|            | Recursive2      | 127.1     | 215.7e-3    | 127.3      | 02:07        | 2309     | 93.42e-6      | infeasible | 22.86           | 13494.4        | 15.24       |

Table 75: Bisection of problem 3D with problem: 14, type: MaxFirst

| robot                     | precision       | solver          | prep_time | comput_time | total_time | (D-H:M:S.ms) | nb_iter     | time_per_iter | comput_time (%) | total_time (%) | nb_iter (%) |       |
|---------------------------|-----------------|-----------------|-----------|-------------|------------|--------------|-------------|---------------|-----------------|----------------|-------------|-------|
| ../././model/kuka_lwr.xml | 0.001           | Interval        | 39.00e-6  | 598958      | 598958     | 6-22:22.38   | 6676040964  | 89.00e-6      | 100.0           | 100.0          | 100.0       |       |
|                           |                 | Bernstein       | 378.3     | 643161      | 643444     | 7-10:44:04   | 10807857179 | 111.0e-6      | 6.361           | 7.065          | 6.837       |       |
|                           |                 | MinVariance     | 414.4     | 733199      | 733508     | 8-11:45:08   | 12258260822 | 112.0e-6      | 6.781           | 7.546          | 7.339       |       |
|                           |                 | BSplines        | 375.8     | 283355      | 283609     | 3-06:46:49   | 4568299416  | 126.0e-6      | 16.28           | 17.22          | 21.61       |       |
|                           |                 | MinNorm         | 410.4     | 694917      | 695224     | 8-01:07:04   | 11577934452 | 110.0e-6      | 6.504           | 7.263          | 7.106       |       |
|                           |                 | MinVolume       | 396.9     | 794907      | 795201     | 9-04:53:21   | 13774006396 | 110.0e-6      | 6.993           | 7.691          | 7.847       |       |
|                           |                 | ApproxMinVolume | 382.0     | 805262      | 805544     | 9-07:45:44   | 13813523254 | 110.0e-6      | 7.004           | 7.683          | 7.860       |       |
|                           |                 | Recursive       | 373.3     | 641137      | 641414     | 7-10:10:14   | 10821224833 | 110.0e-6      | 6.239           | 6.913          | 6.837       |       |
|                           | Recursive2      | 377.2           | 648204    | 648486      | 7-12:08:06 | 10813911461  | 110.0e-6    | 6.341         | 7.025           | 6.846          |             |       |
|                           | 0.01            | Interval        | 32.00e-6  | 332676      | 332676     | 3-20:24:36   | 3624925264  | 91.00e-6      | 100.0           | 100.0          | 100.0       |       |
|                           |                 | Bernstein       | 346.8     | 22166       | 22513      | 06:15:13     | 242652648   | 126.0e-6      | 9.706           | 10.42          | 7.982       |       |
|                           |                 | MinVariance     | 384.1     | 24294       | 24678      | 06:51:18     | 264227566   | 127.0e-6      | 10.41           | 11.21          | 8.475       |       |
|                           |                 | BSplines        | 363.4     | 49644       | 49908      | 13:51:48     | 619888324   | 122.0e-6      | 16.82           | 17.57          | 16.45       |       |
|                           |                 | MinNorm         | 388.0     | 23320       | 23708      | 06:35:08     | 254207507   | 126.0e-6      | 10.08           | 10.90          | 8.246       |       |
|                           |                 | MinVolume       | 375.2     | 25425       | 25800      | 07:10:00     | 285991552   | 125.0e-6      | 10.65           | 11.43          | 8.969       |       |
|                           |                 | ApproxMinVolume | 373.8     | 25545       | 25919      | 07:11:59     | 286566441   | 126.0e-6      | 10.70           | 11.50          | 8.982       |       |
|                           |                 | Recursive       | 341.3     | 22390       | 22732      | 06:18:52     | 244304039   | 126.0e-6      | 9.869           | 10.60          | 8.001       |       |
|                           |                 | Recursive2      | 336.6     | 22535       | 22871      | 06:21:11     | 244106457   | 126.0e-6      | 9.935           | 10.64          | 8.049       |       |
|                           |                 | 0.001           | Interval  | 38.00e-6    | 642.5      | 642.5        | 10:42       | 11047784      | 62.00e-6        | 100.0          | 100.0       | 100.0 |
|                           |                 |                 | Bernstein | 38.50       | 212.5      | 297.6        | 04:57       | 2079581       | 61.00e-6        | 18.94          | 2995.8      | 20.74 |
| MinVariance               |                 |                 | 68.98     | 268.7       | 422.2      | 07:02        | 2379840     | 68.00e-6      | 22.55           | 5349.6         | 22.32       |       |
| BSplines                  | 41.17           |                 | 1075      | 1167        | 19:27      | 10671601     | 60.00e-6    | 60.90         | 3238.0          | 57.08          |             |       |
| MinNorm                   | 66.53           |                 | 250.1     | 400.1       | 06:40      | 2243746      | 64.00e-6    | 20.62         | 5262.3          | 21.59          |             |       |
| MinVolume                 | 64.72           |                 | 297.5     | 443.0       | 07:23      | 2703713      | 63.00e-6    | 23.17         | 5135.8          | 24.03          |             |       |
| ApproxMinVolume           | 63.48           |                 | 298.2     | 438.4       | 07:18      | 2714059      | 64.00e-6    | 23.35         | 5055.6          | 24.08          |             |       |
| Recursive                 | 45.54           |                 | 217.5     | 318.2       | 05:18      | 2087414      | 62.00e-6    | 19.24         | 3590.0          | 20.86          |             |       |
| Recursive2                | 52.07           |                 | 222.0     | 335.6       | 05:35      | 2087932      | 62.00e-6    | 19.41         | 4029.8          | 20.90          |             |       |
| 0.01                      | Interval        |                 | 38.00e-6  | 19.14       | 19.14      | 19.138       | 306036      | 63.00e-6      | 100.0           | 100.0          | 100.0       |       |
|                           | Bernstein       |                 | 41.20     | 6.705       | 94.26      | 01:34        | 72424       | 67.00e-6      | 21.81           | 3335.1         | 21.01       |       |
|                           | MinVariance     |                 | 72.95     | 7.779       | 168.4      | 02:48        | 78200       | 69.00e-6      | 23.82           | 6160.4         | 22.02       |       |
|                           | BSplines        | 43.92           | 15.29     | 112.4       | 01:52      | 152933       | 69.00e-6    | 40.28         | 3780.0          | 35.14          |             |       |
|                           | MinNorm         | 70.18           | 7.492     | 164.3       | 02:44      | 75509        | 70.00e-6    | 23.27         | 6046.8          | 21.54          |             |       |
|                           | MinVolume       | 68.33           | 8.484     | 161.3       | 02:41      | 84924        | 70.00e-6    | 25.57         | 5953.2          | 23.19          |             |       |
|                           | ApproxMinVolume | 67.21           | 8.511     | 158.2       | 02:38      | 85144        | 70.00e-6    | 25.75         | 5842.8          | 23.23          |             |       |
|                           | Recursive       | 48.00           | 6.977     | 112.7       | 01:52      | 73986        | 68.00e-6    | 22.48         | 4125.0          | 21.35          |             |       |
|                           | Recursive2      | 54.91           | 7.215     | 129.0       | 02:09      | 74118        | 69.00e-6    | 23.04         | 4744.4          | 21.44          |             |       |

Table 76: Average of Bisection for 3D problems