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Sébastien Lengagne

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Detailed results of the comparison of basis functions used in interval-based optimization process

Sébastien Lengagne

December 4, 2023

1 Introduction

In this paper, we assess the results of the method proposed in [?] by addressing Optimization Problems (OP) to find the posture of a robot that optimally fits robot constraints and desired end-effector position such as :

$$\begin{aligned} & \text{Find} && q \in \mathbb{Q} \subset \mathbb{R}^n \\ & \text{such as} && \min_q \mathcal{F}(q) \\ & \text{with } \forall j \in \{1, \dots, m\} && \mathcal{G}_j(q) \in [\underline{g}_j, \bar{g}_j] \end{aligned} \quad (1)$$

Where n is called the number of degrees of freedom. This paper will have several versions since several computations will be done.

2 Presentation of the Planar robot problem

First, we assess our method on multi degrees-of-freedom 2D-robots, and considering possible constraint on the end effector, the Center of Mass position and maximum torque:

2.1 End effector position

We define the end effector by recursively computing :

$$T_i = T_{i-1} \times \begin{bmatrix} \cos(q_i) & -\sin(q_i) & l \\ \sin(q_i) & \cos(q_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

With the number of a link $l = 2/n$ and $T_0 = I_{3 \times 3}$ and the end-effector position $\{P_x, P_y\}$ and orientation θ are defined as:

$$T_n = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & P_x \\ \sin(\theta) & \cos(\theta) & P_y \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

This yields the total length of the robot is equal to 2.

2.2 Considering several targets

We assess our method on nine different constraint satisfaction problems, with a square target of size 0.2 by 0.2, a circle target with a radius of 0.1 and a ring target with an external radius of 0.15 and an interval radius of 0.05. The targets are localized at three different positions $x, y : 0.2; 1.7$ (pb 1,4,7),

We defined the following constraints :

$$\text{square: } \begin{cases} P_x \in [x - 0.1; x + 0.1] \\ P_y \in [y - 0.1; y + 0.1] \end{cases} \quad (4)$$

$$\text{circle: } (P_x - x)^2 + (P_y - y)^2 \in [0; 0.01] \quad (5)$$

$$\text{ring: } (P_x - x)^2 + (P_y - y)^2 \in [0.05^2; 0.15^2] \quad (6)$$

2.3 Center of Mass

We consider that all the links have the same length and same weight and the same Centres Of Mass (COM) located in the middle of each links. Thus the x-coordinate of the COM of each link c_i can be computed through :

$$c_i = [1 \quad 0 \quad 0] T_i \times \begin{bmatrix} l/2 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

and the global COM as :

$$COM = \frac{1}{n} \sum_{i=1}^n c_i \quad (8)$$

Finally we consider that the COM of the robot must remain in $[0.1, 0.1]$ on the horizontal plane.

2.4 Cost function

We consider two different cost functions to minimize. The first one is the sum of joint position square and the second is the sum or torque square:

$$\text{position : } \sum_{i=1}^n q_i^2 \quad (9)$$

$$\text{torque : } \sum_{i=1}^n \Gamma_i^2 \quad (10)$$

$$(11)$$

2.5 Bisection mode

In this comparison, we consider the results of two different bisection process :

- **MinFirst** : Cut the largest interval and store in a FILO (First In Last Out) stack the higher part and then the lower part. So the lower part will be the next to be evaluated.
- **MaxFirst** : Cut the largest interval and store in a FILO (First In Last Out) stack the lower part and then the higher part. So the higher part will be the next to be evaluated.

3 Detail of the basis functions

The different basis functions presented in [?] are more deeply presented here. Table 1 represents the coefficient of the matrix \mathbf{B} . Table 2 represents the coefficient of the matrix \mathbf{B}^{-1} . Eventually, Table 3 shows the plot for the different functions from order 2 to 6.

type	order	\mathbf{B}^{-1}					
all	1			1.000000	-1.000000		
				1.000000	1.000000		
Bernstein	2			0.250000	0.500000	0.250000	
				-0.500000	0.000000	0.500000	
				0.250000	-0.500000	0.250000	
	3			0.125000	0.375000	0.375000	0.125000
				-0.375000	-0.375000	0.375000	0.375000
				0.375000	-0.375000	-0.375000	0.375000
	4			0.062500	0.250000	0.375000	0.250000
				-0.250000	-0.500000	0.000000	0.500000
				0.375000	0.000000	-0.750000	0.000000
				-0.250000	0.500000	0.000000	-0.500000
				0.062500	-0.250000	0.375000	-0.250000
	5			0.031250	0.156250	0.312500	0.312500
				-0.156250	-0.468750	-0.312500	0.312500
				0.312500	0.312500	-0.625000	-0.625000
				-0.312500	0.312500	0.625000	-0.625000
				0.156250	-0.468750	0.312500	-0.468750
				-0.031250	0.156250	-0.312500	0.312500
	6			0.015625	0.093750	0.234375	0.312500
				-0.093750	-0.375000	-0.468750	0.000000
				0.234375	0.468750	-0.234375	-0.937500
				-0.312500	0.000000	0.937500	0.000000
				0.234375	-0.468750	-0.234375	0.937500
				-0.093750	0.375000	-0.468750	0.000000
				0.015625	-0.093750	0.234375	-0.312500
BSplines	2			0.125000	0.750000	0.125000	
				-0.250000	0.000000	0.250000	
				0.125000	-0.250000	0.125000	
	3			0.020833	0.479167	0.479167	0.020833
				-0.062500	-0.312500	0.312500	0.062500
				0.062500	-0.062500	-0.062500	0.062500
	4			0.002604	0.197917	0.598958	0.197917
				-0.010417	-0.229167	0.000000	0.229167
				0.015625	0.062500	-0.156250	0.062500
				-0.010417	0.020833	0.000000	-0.020833
				0.002604	-0.010417	0.015625	-0.010417
	5			0.000260	0.061719	0.438021	0.438021
				-0.001302	-0.097656	-0.200521	0.200521
				0.002604	0.054688	-0.057292	-0.057292
				-0.002604	-0.007812	0.036458	-0.036458
				0.001302	-0.003906	0.002604	0.002604
				-0.000260	0.001302	-0.002604	0.002604
	6			0.000022	0.015668	0.228798	0.511024
				-0.000130	-0.030729	-0.188151	0.000000
				0.000326	0.024089	0.025716	-0.100260
				-0.000434	-0.008681	0.018663	0.000000
				0.000326	0.000651	-0.005534	0.009115
				-0.000130	0.000521	-0.000651	0.000651
				0.000022	-0.000130	0.000326	-0.000434
MinVo	2			0.125000	0.750000	0.125000	
				-0.433013	0.000000	0.433013	
				0.375000	-0.750000	0.375000	
	3			0.000410	0.499590	0.499590	0.000410
				-0.026982	-0.792093	0.792093	0.026982
				0.456775	-0.456775	-0.456775	0.456775
	4			0.030227	0.148259	0.643029	0.148259
				0.138091	-0.810775	0.000000	0.810775
				-0.094352	0.960202	-1.731700	0.960202
				-0.575789	0.810775	0.000000	-0.810775
				0.525520	-1.108461	1.165882	-1.108461
	5			0.045894	0.002895	0.451210	0.451210
				-0.036501	-0.120977	-1.064403	1.064403
				-0.377284	1.333003	-0.955718	-0.955718
				0.330192	-1.365550	2.776543	-2.776543
				0.776885	-1.319318	0.542433	0.542433
				-0.739186	1.503106	-1.750066	1.750066
	6			0.018328	0.031209	0.152009	0.596907
				-0.136709	0.243263	-1.080915	0.000000
				0.105859	-0.084538	1.580291	-3.203224
				0.834817	-2.298575	2.468184	0.000000
				-0.736850	2.281168	-4.269993	5.451350
				-1.134019	2.055312	-1.408577	0.000000
				1.060626	-2.227839	2.589730	-2.227839
ApproxMinVo	2			0.124377	0.751246	0.124377	
				-0.432291	0.000000	0.432291	
	3			0.000649	0.499351	0.499351	0.000649
				-0.033676	-0.803260	0.803260	0.033676
				0.453110	-0.453110	-0.453110	0.453110
				-0.420083	0.849501	-0.849501	0.420083

4		0.035652	0.146138	0.636420	0.146138	0.035652
		0.149222	-0.808622	0.000000	0.808622	-0.149222
		-0.125047	0.972447	-1.694798	0.972447	-0.125047
5		-0.588448	0.808622	0.000000	-0.808622	0.588448
		0.554424	-1.118584	1.128320	-1.118584	0.554424
		0.046170	0.000790	0.453040	0.453040	0.000790
6		-0.030551	-0.064281	-1.047189	1.047189	0.064281
		-0.376843	1.346724	-0.969881	-0.969881	1.346724
		0.299901	-1.536987	2.717358	-2.717358	1.536987
7		0.773030	-1.326775	0.553745	0.553745	-1.326775
		-0.711707	1.622007	-1.707074	1.707074	-1.622007
		0.018148	0.034239	0.143952	0.143952	0.034239
8		-0.137219	0.252439	-1.058872	1.058872	-0.252439
		0.112414	-0.122731	1.632940	-1.632940	0.122731
		0.836717	-2.293984	2.375355	0.000000	-2.375355
9		-0.765241	2.327842	-4.313797	5.502391	-4.313797
		-1.138248	2.041545	-1.331193	0.000000	1.331193
		1.088642	-2.239350	2.582942	-2.864469	2.582942
MinNo	2		0.178806	0.642387	0.178806	
			-0.479297	0.000000	0.479297	
			0.321194	-0.642387	0.321194	
3			0.028323	0.471677	0.471677	0.028323
			-0.208384	-0.572950	0.572950	0.208384
			0.466241	-0.466241	-0.466241	0.466241
4			-0.286180	0.578386	-0.578386	0.286180
			0.001808	0.232346	0.531691	0.232346
			-0.040769	-0.719547	0.000000	0.719547
5			0.270116	0.324740	-1.189712	0.324740
			-0.454129	0.719547	0.000000	-0.719547
			0.224324	-0.557086	0.665525	-0.557086
6			0.000548	0.087040	0.412412	0.412412
			-0.013185	-0.484330	-0.611879	0.611879
			0.104674	0.774368	-0.879042	-0.879042
7			-0.313820	0.006407	1.345160	-1.345160
			0.390724	-0.861075	0.470351	0.470351
			-0.168943	0.478256	-0.737001	0.737001
8			0.000246	0.039541	0.234007	0.452411
			-0.004996	-0.286016	-0.769529	0.000000
			0.040290	0.715108	0.132833	-1.776461
9			-0.162385	-0.572706	1.643712	0.000000
			0.336335	-0.398220	-1.084518	2.292808
			-0.328442	0.858721	-0.877742	0.000000
MinVariance	2		0.249987	0.500025	0.249987	
			-0.500000	0.000000	0.500000	
			0.250013	-0.500025	0.250013	
3			0.032931	0.467069	0.467069	0.032931
			-0.223402	-0.513908	0.513908	0.223402
			0.465895	-0.465895	-0.465895	0.465895
4			-0.275423	0.515083	-0.515083	0.275423
			0.015689	0.231421	0.505779	0.231421
			-0.123291	-0.674161	0.000000	0.674161
5			0.336046	0.259560	-1.191211	0.259560
			-0.368674	0.674161	0.000000	-0.674161
			0.140289	-0.490981	0.701385	-0.490981
6			0.001608	0.061734	0.436657	0.436657
			0.023353	-0.429784	-0.577850	0.577850
			0.037024	0.869002	-0.906026	-0.906026
7			-0.332514	-0.160883	1.218372	-1.218372
			0.459445	-0.929928	0.470483	0.470483
			-0.188917	0.591475	-0.641637	0.641637
8			0.000355	0.045384	0.247471	0.413579
			-0.009474	-0.312039	-0.738798	0.000000
			0.081233	0.720551	0.003182	-1.609932
9			-0.249225	-0.477190	1.628534	0.000000
			0.350633	-0.475605	-0.905787	2.061518
			-0.233281	0.789229	-0.897423	0.000000
Recursive1	2		0.500000	0.250000	0.500000	
			-0.500000	0.000000	0.500000	
			0.000000	-0.250000	0.250000	
3			0.500000	0.250000	0.125000	0.125000
			-0.500000	0.000000	0.125000	0.375000
			0.000000	0.000000	-0.125000	0.125000
4			0.500000	0.250000	0.125000	0.062500
			-0.500000	0.000000	0.125000	0.250000
			0.000000	-0.250000	-0.125000	0.000000
5			0.000000	0.000000	-0.125000	0.375000
			0.000000	0.000000	-0.125000	-0.125000
			0.000000	0.000000	0.000000	-0.062500
6			0.500000	0.250000	0.125000	0.062500
			-0.500000	0.000000	0.125000	0.031250
			0.000000	-0.250000	-0.125000	0.093750
7			0.000000	0.000000	-0.125000	0.156250
			0.000000	0.000000	-0.125000	-0.062500
			0.000000	0.000000	0.000000	0.312500
8			0.000000	0.000000	0.000000	-0.062500
			0.000000	0.000000	0.000000	-0.093750
			0.000000	0.000000	0.000000	0.156250
9			0.000000	0.000000	0.000000	-0.031250
			0.000000	0.000000	0.000000	0.031250
			0.500000	0.250000	0.125000	0.015625
Recursive2	2		0.250000	0.250000	0.500000	
			-0.500000	0.000000	0.500000	
			0.250000	-0.250000	0.000000	
3			0.125000	0.125000	0.250000	0.500000
			-0.375000	-0.125000	0.000000	0.500000
			0.375000	-0.125000	-0.250000	0.000000
			-0.125000	0.125000	0.000000	0.000000

4		0.062500	0.062500	0.125000	0.250000	0.500000		
		-0.250000	-0.125000	-0.125000	0.000000	0.500000		
5		0.031250	0.031250	0.062500	0.125000	0.250000	0.500000	
		-0.156250	-0.093750	-0.125000	-0.125000	0.000000	0.500000	
		0.312500	0.062500	0.000000	-0.125000	-0.250000	0.000000	
		-0.312500	0.062500	0.125000	0.125000	0.000000	0.000000	
6		0.015625	0.015625	0.031250	0.062500	0.125000	0.250000	0.500000
		-0.093750	-0.062500	-0.093750	-0.125000	-0.125000	0.000000	0.500000
		0.234375	0.078125	0.062500	0.000000	-0.125000	-0.250000	0.000000
		-0.312500	0.000000	0.062500	0.125000	0.125000	0.000000	0.000000
		0.234375	-0.078125	-0.093750	-0.062500	0.000000	0.000000	0.000000
		-0.093750	0.062500	0.031250	0.000000	0.000000	0.000000	0.000000

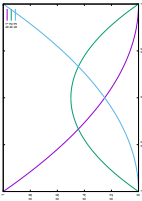
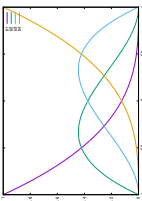
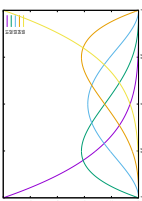
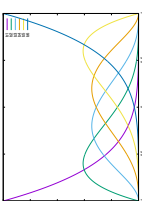
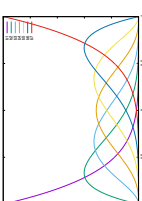
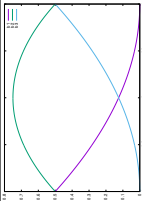
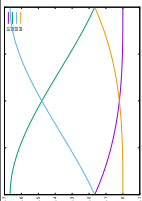

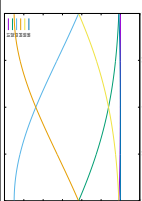
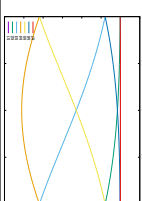
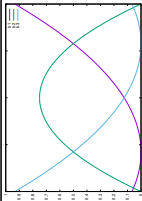

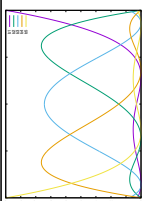
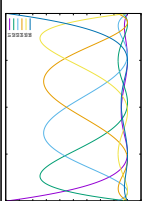
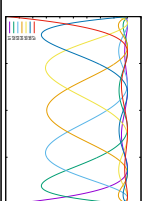
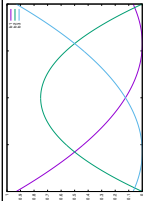
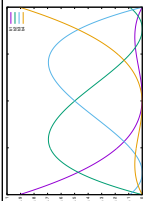
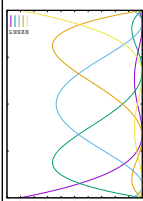
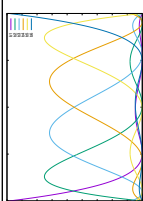
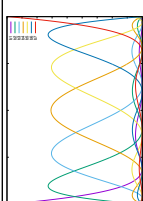
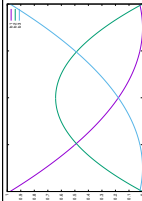
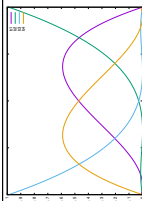
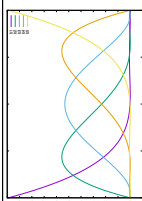
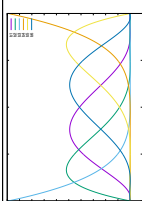
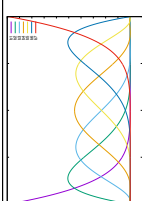
Table 1: Values of the matrices for the different basis functions \mathbf{B} for order 1 to 6.

type	order	\mathbf{B}^{-1}								
all	1			1.000000	-1.000000					
				1.000000	1.000000					
	Bernstein	2			1.000000	-1.000000	1.000000			
					1.000000	0.000000	-1.000000			
					1.000000	1.000000	1.000000			
					1.000000	-1.000000	1.000000	-1.000000		
3			1.000000	-1.000000	1.000000	-1.000000				
				1.000000	-0.333333	-0.333333	1.000000			
				1.000000	0.333333	-0.333333	-1.000000			
				1.000000	1.000000	1.000000	1.000000			
4				1.000000	-1.000000	1.000000	-1.000000	1.000000		
				1.000000	-0.500000	-0.000000	0.500000	-1.000000		
				1.000000	-0.000000	-0.333333	-0.000000	1.000000		
				1.000000	0.500000	0.000000	-0.500000	-1.000000		
				1.000000	1.000000	1.000000	1.000000	1.000000		
5				1.000000	-1.000000	1.000000	-1.000000	1.000000	-1.000000	
				1.000000	-0.600000	0.200000	0.200000	-0.600000	1.000000	
				1.000000	-0.200000	-0.200000	0.200000	0.200000	-1.000000	
				1.000000	0.200000	-0.200000	-0.200000	0.200000	1.000000	
				1.000000	0.600000	0.200000	-0.200000	-0.600000	-1.000000	
				1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	
6				1.000000	-1.000000	1.000000	-1.000000	1.000000	1.000000	
				1.000000	-0.666667	0.333333	-0.000000	-0.333333	0.666667	-1.000000
				1.000000	-0.333333	-0.066667	0.200000	-0.066667	-0.333333	1.000000
				1.000000	0.000000	-0.200000	-0.000000	0.200000	0.000000	-1.000000
				1.000000	0.333333	-0.066667	-0.200000	-0.066667	0.333333	1.000000
				1.000000	0.666667	0.333333	0.000000	-0.333333	-0.666667	-1.000000
				1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
BSplines	2			1.000000	-2.000000	3.000000				
				1.000000	0.000000	-1.000000				
				1.000000	2.000000	3.000000				
	3			1.000000	-3.000000	7.666667	-15.000000			
				1.000000	-1.000000	-0.333333	3.000000			
				1.000000	1.000000	-0.333333	-3.000000			
4			1.000000	-4.000000	14.333333	-44.000000	105.000000			
			1.000000	-2.000000	2.333333	2.000000	-15.000000			
			1.000000	0.000000	-1.666667	0.000000	9.000000			
			1.000000	2.000000	2.333333	-2.000000	-15.000000			
			1.000000	4.000000	14.333333	44.000000	105.000000			
5			1.000000	-5.000000	23.000000	-95.000000	337.800000	-945.000000		
			1.000000	-3.000000	7.000000	-9.000000	-14.200000	105.000000		
			1.000000	-1.000000	-1.000000	5.000000	1.800000	-45.000000		
			1.000000	1.000000	-1.000000	-5.000000	1.800000	45.000000		
			1.000000	3.000000	7.000000	9.000000	-14.200000	-105.000000		
			1.000000	5.000000	23.000000	95.000000	337.800000	945.000000		
6			1.000000	-6.000000	33.666667	-174.000000	809.266667	-3254.000000	10395.000000	
			1.000000	-4.000000	13.666667	-36.000000	49.266667	124.000000	-945.000000	
			1.000000	-2.000000	1.666667	6.000000	-22.733333	-18.000000	315.000000	
			1.000000	0.000000	-2.333333	0.000000	17.266667	0.000000	-225.000000	
			1.000000	2.000000	1.666667	-6.000000	-22.733333	18.000000	315.000000	
			1.000000	4.000000	13.666667	36.000000	49.266667	-124.000000	-945.000000	
			1.000000	6.000000	33.666667	174.000000	809.266667	3254.000000	10395.000000	
MinVo	2			1.000000	-1.154701	1.000000				
				1.000000	0.000000	-0.333333				
				1.000000	1.154701	1.000000				
	3			1.000000	-1.149096	1.093732	-1.090170			
				1.000000	-0.592096	-0.000898	0.037136			
			1.000000	0.592096	-0.000898	-0.037136				
			1.000000	1.149096	1.093732	1.090170				
4			1.000000	-1.142338	1.106977	-1.142338	1.092669			
			1.000000	-0.811256	0.378336	-0.194562	0.010409			
			1.000000	-0.000000	-0.278531	-0.000000	-0.107525			
			1.000000	0.811256	0.378336	0.194562	0.010409			
			1.000000	1.142338	1.106977	1.142338	1.092669			
5			1.000000	-1.129437	1.109754	-1.147162	1.123459	-1.133082		
			1.000000	-0.933700	0.605476	-0.457764	0.234967	-0.158376		
			1.000000	-0.324894	-0.116762	0.091367	-0.115779	0.056856		
			1.000000	0.324894	-0.116762	-0.091367	-0.115779	-0.056856		
			1.000000	0.933700	0.605476	0.457764	0.234967	0.158376		
			1.000000	1.129437	1.109754	1.147162	1.123459	1.133082		

	6	<table border="1"> <tbody> <tr><td>1.000000</td><td>-1.120437</td><td>1.105897</td><td>-1.147175</td><td>1.129137</td><td>-1.150340</td><td>1.128210</td></tr> <tr><td>1.000000</td><td>-0.990939</td><td>0.751557</td><td>-0.630814</td><td>0.445470</td><td>-0.344918</td><td>0.217191</td></tr> <tr><td>1.000000</td><td>-0.543877</td><td>0.088407</td><td>0.003123</td><td>-0.111657</td><td>0.067865</td><td>-0.103675</td></tr> <tr><td>1.000000</td><td>0.000000</td><td>-0.191530</td><td>0.000000</td><td>-0.059053</td><td>0.000000</td><td>-0.039190</td></tr> <tr><td>1.000000</td><td>0.543877</td><td>0.088407</td><td>-0.003123</td><td>-0.111657</td><td>-0.067865</td><td>-0.103675</td></tr> <tr><td>1.000000</td><td>0.990939</td><td>0.751557</td><td>0.630814</td><td>0.445470</td><td>0.344918</td><td>0.217191</td></tr> <tr><td>1.000000</td><td>1.120437</td><td>1.105897</td><td>1.147175</td><td>1.129137</td><td>1.150340</td><td>1.128210</td></tr> </tbody> </table>	1.000000	-1.120437	1.105897	-1.147175	1.129137	-1.150340	1.128210	1.000000	-0.990939	0.751557	-0.630814	0.445470	-0.344918	0.217191	1.000000	-0.543877	0.088407	0.003123	-0.111657	0.067865	-0.103675	1.000000	0.000000	-0.191530	0.000000	-0.059053	0.000000	-0.039190	1.000000	0.543877	0.088407	-0.003123	-0.111657	-0.067865	-0.103675	1.000000	0.990939	0.751557	0.630814	0.445470	0.344918	0.217191	1.000000	1.120437	1.105897	1.147175	1.129137	1.150340	1.128210
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MinNo	2	<table border="1"> <tbody> <tr><td>1.000000</td><td>-1.043194</td><td>1.000000</td><td></td><td></td><td></td><td></td></tr> <tr><td>1.000000</td><td>0.000000</td><td>-0.556693</td><td></td><td></td><td></td><td></td></tr> <tr><td>1.000000</td><td>1.043194</td><td>1.000000</td><td></td><td></td><td></td><td></td></tr> </tbody> </table>	1.000000	-1.043194	1.000000					1.000000	0.000000	-0.556693					1.000000	1.043194	1.000000																																
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Recursive2	2	$\begin{bmatrix} 1.000000 & -1.000000 & 1.000000 \\ 1.000000 & -1.000000 & -3.000000 \\ 1.000000 & 1.000000 & 1.000000 \end{bmatrix}$
	3	$\begin{bmatrix} 1.000000 & -1.000000 & 1.000000 & -1.000000 \\ 1.000000 & -1.000000 & 1.000000 & 7.000000 \\ 1.000000 & -1.000000 & -3.000000 & -5.000000 \\ 1.000000 & 1.000000 & 1.000000 & 1.000000 \end{bmatrix}$
	4	$\begin{bmatrix} 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 \\ 1.000000 & -1.000000 & 1.000000 & 1.000000 & -15.000000 \\ 1.000000 & -1.000000 & 1.000000 & 7.000000 & 17.000000 \\ 1.000000 & -1.000000 & -3.000000 & -5.000000 & -7.000000 \\ 1.000000 & 1.000000 & 1.000000 & 1.000000 & 1.000000 \end{bmatrix}$
	5	$\begin{bmatrix} 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 & -1.000000 \\ 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 & 31.000000 \\ 1.000000 & -1.000000 & 1.000000 & -1.000000 & -15.000000 & -49.000000 \\ 1.000000 & -1.000000 & 1.000000 & 7.000000 & 17.000000 & 31.000000 \\ 1.000000 & -1.000000 & -3.000000 & -5.000000 & -7.000000 & -9.000000 \\ 1.000000 & 1.000000 & 1.000000 & 1.000000 & 1.000000 & 1.000000 \end{bmatrix}$
	6	$\begin{bmatrix} 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 \\ 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 & -1.000000 & -63.000000 \\ 1.000000 & -1.000000 & 1.000000 & -1.000000 & 1.000000 & 31.000000 & 129.000000 \\ 1.000000 & -1.000000 & 1.000000 & -1.000000 & -15.000000 & -49.000000 & -111.000000 \\ 1.000000 & -1.000000 & 1.000000 & 7.000000 & 17.000000 & 31.000000 & 49.000000 \\ 1.000000 & -1.000000 & -3.000000 & -5.000000 & -7.000000 & -9.000000 & -11.000000 \\ 1.000000 & 1.000000 & 1.000000 & 1.000000 & 1.000000 & 1.000000 & 1.000000 \end{bmatrix}$

Table 2: Values of the matrices for the different basis functions \mathbf{B}^{-1} for order 1 to 6.

order :	2	3	4	5	6
Bernstein					
BSplines					
MinvVo					
ApproxMinvo					
MinNo					

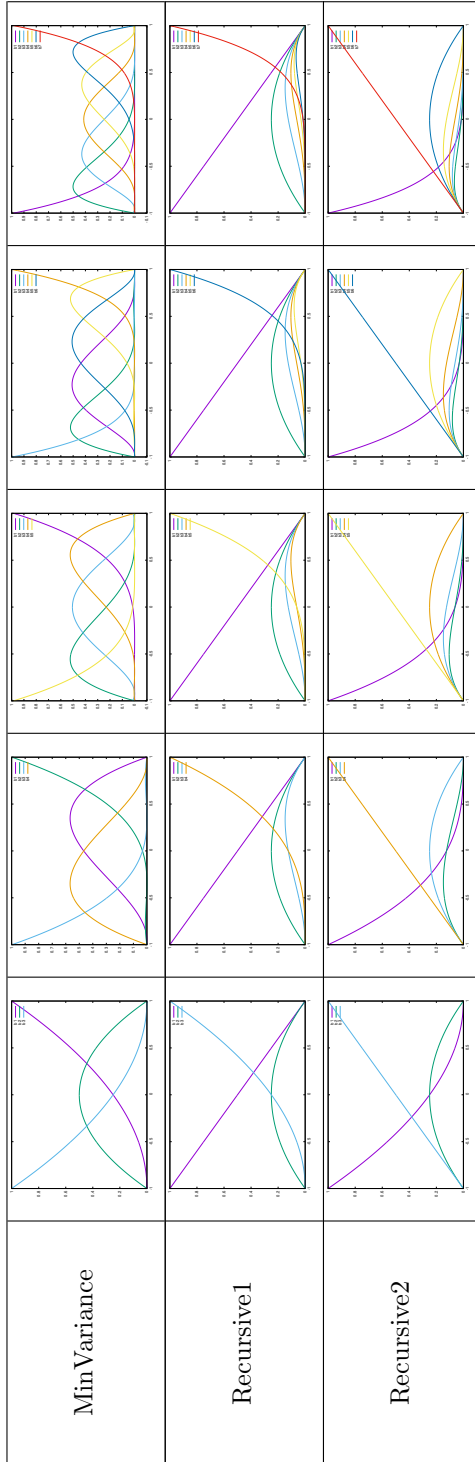


Table 3: Representation of the basis functions for the order 2 to 6

4 Results of the Planar robot problem

We assess our contribution with 18 different constraint satisfaction problems, with the two bisection processes (MinFirst and MaxFirst) such as presented in Table 4.

problem	center of the target	shape of the target	criteria
1	{0.2; 1.7}	square	$\sum_i d_i^2$
2	{0.4; 1.4}	square	$\sum_i d_i^2$
3	{0.6; 1.1}	square	$\sum_i d_i^2$
4	{0.2; 1.7}	circle	$\sum_i d_i^2$
5	{0.4; 1.4}	circle	$\sum_i d_i^2$
6	{0.6; 1.1}	circle	$\sum_i d_i^2$
7	{0.2; 1.7}	ring	$\sum_i d_i^2$
8	{0.4; 1.4}	ring	$\sum_i d_i^2$
9	{0.6; 1.1}	ring	$\sum_i d_i^2$
11	{0.2; 1.7}	square	$\sum_i \Gamma_i^2$
12	{0.4; 1.4}	square	$\sum_i \Gamma_i^2$
13	{0.6; 1.1}	square	$\sum_i \Gamma_i^2$
14	{0.2; 1.7}	circle	$\sum_i \Gamma_i^2$
15	{0.4; 1.4}	circle	$\sum_i \Gamma_i^2$
16	{0.6; 1.1}	circle	$\sum_i \Gamma_i^2$
17	{0.2; 1.7}	ring	$\sum_i \Gamma_i^2$
18	{0.4; 1.4}	ring	$\sum_i \Gamma_i^2$
19	{0.6; 1.1}	ring	$\sum_i \Gamma_i^2$

Table 4: Problem list and their specificities.

The results are summarized in the following tables.

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
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Table 5: problem 2D : bisection: problem:

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BisectionInterval	2.19345e-05	0.018388	0.018388	0.018	3367	5.46125e-06	0.955684
	Bis_Bernstein	0.650644	0.00606608	0.656728	0.657	991	6.12117e-06	0.951923
	Bis_MinVariance	0.833582	0.0107419	0.844362	0.844	991	1.08395e-05	0.951923
	Bis_BSplines	0.940621	0.00904894	0.949688	0.950	1535	5.89507e-06	0.952862
	Bis_MinNo	0.831009	0.00961685	0.840665	0.841	1017	9.4561e-06	0.951964
	Bis_ApproxMinVo	0.632826	0.00634003	0.639185	0.639	1049	6.04388e-06	0.95207
	Bis_MinVo	0.809529	0.010143	0.819719	0.820	1047	9.68772e-06	0.952068
	Bis_RecursiveBSplines	0.822043	0.0108702	0.832933	0.833	1005	1.08161e-05	0.951923
	Bis_RecursiveBSplines2	0.627969	0.00613809	0.634126	0.634	1007	6.09542e-06	0.951923
	3	BisectionInterval	1.81198e-05	4.18923	4.18923	4.189	703327	5.95631e-06
Bis_Bernstein		0.929225	1.39605	2.32529	2.325	116043	1.20304e-05	0.653307
Bis_MinVariance		1.22921	1.42219	2.65144	2.651	116043	1.22557e-05	0.653307
Bis_BSplines		1.19529	2.83233	4.02763	4.028	238083	1.18964e-05	0.654294
Bis_MinNo		0.952715	1.4345	2.38723	2.387	120347	1.19197e-05	0.65335
Bis_ApproxMinVo		0.940284	1.57562	2.51592	2.516	132393	1.19011e-05	0.653462
Bis_MinVo		1.19123	1.60007	2.79134	2.791	132195	1.21039e-05	0.65346
Bis_RecursiveBSplines		1.20946	1.44704	2.65654	2.657	116609	1.24093e-05	0.653307
Bis_RecursiveBSplines2		0.938426	1.40253	2.34098	2.341	116689	1.20194e-05	0.653307
4		BisectionInterval	1.71661e-05	715.984	715.984	11:55	98257105	7.28684e-06
	Bis_Bernstein	2.13551	312.74	314.876	05:14	9778767	3.19816e-05	0.543019
	Bis_MinVariance	2.14114	314.288	316.429	05:16	9778767	3.21398e-05	0.543019
	Bis_BSplines	2.42715	706.55	708.977	11:48	22193961	3.18352e-05	0.544027
	Bis_MinNo	2.39658	338.232	340.629	05:40	10195003	3.31763e-05	0.543062
	Bis_ApproxMinVo	2.52491	364.021	366.546	06:06	11343171	3.20917e-05	0.543176
	Bis_MinVo	2.32766	364.342	366.67	06:06	11323297	3.21763e-05	0.543174
	Bis_RecursiveBSplines	2.21628	315.275	317.492	05:17	9800097	3.21706e-05	0.543019
	Bis_RecursiveBSplines2	2.18749	319.34	321.528	05:21	9803071	3.25756e-05	0.543019
	5	BisectionInterval	4.1008e-05					
Bis_Bernstein		17.4989						
Bis_MinVariance		17.5847						

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
	Bis_BSplines	17.6574						
	Bis_MinNo	17.6442						
	Bis_ApproxMinVo	17.5045						
	Bis_MinVo	17.7666						
	Bis_RecursiveBSplines	17.9255						

Table 6: problem 2D : bisection:MinFirst problem:1

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BisectionInterval	2.00272e-05	0.00499701	0.00499701	0.005	857	5.83082e-06	2.70904
	Bis_Bernstein	0.624614	0.00283909	0.627471	0.627	451	6.2951e-06	2.70567
	Bis_MinVariance	0.826382	0.00529408	0.831715	0.832	451	1.17385e-05	2.70567
	Bis_BSplines	0.627269	0.00427699	0.631564	0.632	703	6.08391e-06	2.70735
	Bis_MinNo	0.821613	0.00508595	0.826718	0.827	459	1.10805e-05	2.70574
	Bis_ApproxMinVo	0.82831	0.00570607	0.834035	0.834	489	1.16689e-05	2.70593
	Bis_MinVo	0.852161	0.00549984	0.85777	0.858	489	1.12471e-05	2.70593
3	Bis_RecursiveBSplines	0.840583	0.00243807	0.84305	0.843	451	5.40592e-06	2.70567
	Bis_RecursiveBSplines2	0.653001	0.00288796	0.655908	0.656	451	6.40347e-06	2.70567
	BisectionInterval	1.5974e-05	1.93473	1.93473	1.935	302195	6.40227e-06	2.43691
	Bis_Bernstein	0.931433	0.547586	1.47903	1.479	44913	1.21921e-05	2.43104
	Bis_MinVariance	1.22713	0.588139	1.81531	1.815	44913	1.30951e-05	2.43104
	Bis_BSplines	0.937821	1.20013	2.13797	2.138	99953	1.20069e-05	2.43287
	Bis_MinNo	1.17861	0.600898	1.77955	1.780	46669	1.28757e-05	2.43111
4	Bis_ApproxMinVo	1.22325	0.68057	1.90386	1.904	51749	1.31514e-05	2.43132
	Bis_MinVo	1.16622	0.667371	1.83361	1.834	51647	1.29218e-05	2.43132
	Bis_RecursiveBSplines	0.936742	0.546592	1.48335	1.483	45017	1.21419e-05	2.43104
	Bis_RecursiveBSplines2	0.937513	0.556269	1.4938	1.494	45173	1.23142e-05	2.43104
	BisectionInterval	1.88351e-05	616.353	616.353	10.16	83653481	7.36793e-06	1.27747
	Bis_Bernstein	2.12747	294.679	296.806	04.56	9140611	3.22384e-05	1.27308
	Bis_MinVariance	2.12986	291.592	293.722	04.53	9140611	3.19007e-05	1.27308
4	Bis_BSplines	2.42012	718.258	720.678	12.00	21844851	3.288e-05	1.27463
	Bis_MinNo	2.13687	308.651	310.788	05.10	9542235	3.23458e-05	1.27314
	Bis_ApproxMinVo	2.40985	341.138	343.548	05.43	10649131	3.20344e-05	1.27332
	Bis_MinVo	2.35682	351.634	353.991	05.53	10629605	3.30807e-05	1.27332
	Bis_RecursiveBSplines	2.4685	293.592	296.061	04.56	9159035	3.20549e-05	1.27308
	Bis_RecursiveBSplines2	2.44924	295.978	298.427	04.58	9155725	3.23271e-05	1.27308

Table 7: problem 2D : bisection:MinFirst problem:2

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BisectionInterval	1.90735e-05	0.000422001	0.000422001	0.000	59	7.15256e-06	-1
	Bis_Bernstein	0.626396	0.000423908	0.626839	0.627	41	1.03392e-05	-1
	Bis_MinVariance	0.646325	0.000379086	0.646719	0.647	41	9.24599e-06	-1
	Bis_BSplines	0.819064	0.000800133	0.819886	0.820	41	1.95154e-05	-1
	Bis_MinNo	0.832943	0.000741959	0.83373	0.834	41	1.80966e-05	-1
	Bis_ApproxMinVo	0.857936	0.000703096	0.858682	0.859	41	1.71487e-05	-1
	Bis_MinVo	0.856494	0.000583887	0.857103	0.857	41	1.42411e-05	-1
3	Bis_RecursiveBSplines	0.629654	0.000386	0.630058	0.630	41	9.41463e-06	-1
	Bis_RecursiveBSplines2	0.626482	0.000407934	0.626908	0.627	41	9.94961e-06	-1
	BisectionInterval	1.78814e-05	0.454381	0.454381	0.454	74295	6.1159e-06	3.9157
	Bis_Bernstein	0.931798	0.227882	1.1597	1.160	18541	1.22907e-05	3.91076
	Bis_MinVariance	0.945114	0.230429	1.17556	1.176	18541	1.24281e-05	3.91076
	Bis_BSplines	0.940479	0.464474	1.40498	1.405	38723	1.19948e-05	3.91305
	Bis_MinNo	1.21286	0.263057	1.47593	1.476	19189	1.37087e-05	3.91086
4	Bis_ApproxMinVo	1.21809	0.289409	1.50754	1.508	21033	1.37598e-05	3.91112
	Bis_MinVo	1.17704	0.305535	1.48259	1.483	21011	1.45417e-05	3.91112
	Bis_RecursiveBSplines	1.20971	0.256602	1.46633	1.466	18541	1.38397e-05	3.91076
	Bis_RecursiveBSplines2	0.976628	0.227445	1.20409	1.204	18611	1.2221e-05	3.91076
	BisectionInterval	2.09808e-05	700.411	700.411	11.40	96586019	7.25168e-06	2.22706
	Bis_Bernstein	2.4143	338.749	341.163	05.41	10067619	3.36474e-05	1.77375
	Bis_MinVariance	2.45402	328.318	330.772	05.30	10067619	3.26113e-05	1.77375
4	Bis_BSplines	2.14316	776.842	778.985	12.58	24283325	3.19908e-05	1.77557
	Bis_MinNo	2.42511	343.298	345.723	05.45	10516385	3.26441e-05	1.77383
	Bis_ApproxMinVo	2.1209	380.314	382.435	06.22	11766887	3.23207e-05	1.77404
	Bis_MinVo	2.05253	379.47	381.523	06.21	11744841	3.23095e-05	1.77403
	Bis_RecursiveBSplines	2.49215	330.45	332.942	05.32	10083143	3.27725e-05	1.77375
	Bis_RecursiveBSplines2	2.57007	334.044	336.615	05.36	10079265	3.31418e-05	1.77375

Table 8: problem 2D : bisection:MinFirst problem:3

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BisectionInterval	1.5974e-05	0.012625	0.012625	0.013	2383	5.29793e-06	1.07551
	Bis_Bernstein	0.915912	0.011596	0.927549	0.928	817	1.41933e-05	1.07165
	Bis_MinVariance	0.726617	0.005723	0.732358	0.732	817	7.0049e-06	1.07165
	Bis_BSplines	0.917048	0.0250661	0.942153	0.942	2039	1.22933e-05	1.0735
	Bis_MinNo	0.699906	0.00692105	0.706854	0.707	853	8.11378e-06	1.0717
	Bis_ApproxMinVo	0.911776	0.0132151	0.92503	0.925	957	1.38088e-05	1.07182
	Bis_MinVo	0.914044	0.013061	0.92715	0.927	955	1.36765e-05	1.07181
3	Bis_RecursiveBSplines	0.701498	0.00664186	0.708155	0.708	817	8.12958e-06	1.07165
	Bis_RecursiveBSplines2	0.696149	0.00837803	0.704558	0.705	1033	8.11039e-06	1.07165
	BisectionInterval	1.88351e-05	4.31025	4.31025	4.310	711195	6.06057e-06	0.74167
	Bis_Bernstein	1.44165	3.45618	4.89785	4.898	193081	1.79001e-05	0.738333
	Bis_MinVariance	2.0758	3.61587	5.6917	5.692	193079	1.87274e-05	0.738333
	Bis_BSplines	1.75309	13.7486	15.5018	15.502	775983	1.77177e-05	0.742236
	Bis_MinNo	1.94129	4.01878	5.96011	5.960	208239	1.92989e-05	0.738474
3	Bis_ApproxMinVo	2.03694	4.72464	6.76162	6.762	256251	1.84375e-05	0.738939
	Bis_MinVo	1.51092	4.54974	6.06068	6.061	255471	1.78092e-05	0.738937

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
4	Bis_RecursiveBSplines	1.68729	3.73468	5.42199	5.422	209177	1.78542e-05	0.738333
	Bis_RecursiveBSplines2	1.71238	3.67227	5.38467	5.385	203103	1.80808e-05	0.738333
	BissectionInterval	1.81198e-05	888.611	888.611	14:48	127570319	6.96565e-06	0.621318
	Bis_Bernstein	2.14381	830.927	833.071	13:53	21728517	3.82413e-05	0.617742
	Bis_MinVariance	2.1578	834.13	836.288	13:56	21728495	3.83888e-05	0.617742
	Bis_BSplines	2.44535	4687.59	4690.04	01:18:10	123124171	3.80721e-05	0.621908
	Bis_MinNo	2.44062	936.149	938.59	15:38	24001889	3.90032e-05	0.617913
	Bis_ApproxMinVo	2.42647	1186.37	1188.8	19:48	31063759	3.81915e-05	0.618381
	Bis_MinVo	2.36265	1204.09	1206.46	20:06	30955169	3.8898e-05	0.618375
	Bis_RecursiveBSplines	2.18896	897.212	899.401	14:59	23408037	3.83292e-05	0.617742
	Bis_RecursiveBSplines2	2.1952	1000.43	1002.62	16:42	25972947	3.8518e-05	0.617742

Table 9: problem 2D : bissection:MinFirst problem:4

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	2.00272e-05	0.018337	0.018337	0.018	2661	6.89102e-06	2.75656
	Bis_Bernstein	0.903234	0.016484	0.919755	0.920	1229	1.34125e-05	2.75333
	Bis_MinVariance	0.694352	0.0100842	0.704466	0.704	1229	8.20517e-06	2.75333
	Bis_BSplines	0.907831	0.025882	0.933731	0.934	3251	7.96124e-06	2.75641
	Bis_MinNo	0.71481	0.0106111	0.725437	0.725	1295	8.19387e-06	2.75341
	Bis_ApproxMinVo	0.912922	0.0244889	0.937431	0.937	1857	1.31874e-05	2.7536
	Bis_MinVo	0.898349	0.0139439	0.912313	0.912	1851	7.53318e-06	2.75359
	Bis_RecursiveBSplines	0.695488	0.0103638	0.705869	0.706	1377	7.52637e-06	2.75333
Bis_RecursiveBSplines2	0.917948	0.0180509	0.936018	0.936	1351	1.33611e-05	2.75333	
3	BissectionInterval	1.90735e-05	2.94033	2.94033	2.940	493957	5.95261e-06	2.4706
	Bis_Bernstein	1.42178	2.03476	3.45658	3.457	115093	1.76793e-05	2.46464
	Bis_MinVariance	1.79501	2.12882	3.92386	3.924	115093	1.84966e-05	2.46464
	Bis_BSplines	1.46365	7.80876	9.27243	9.272	439045	1.77858e-05	2.46849
	Bis_MinNo	1.93454	2.34187	4.27645	4.276	125473	1.86644e-05	2.46472
	Bis_ApproxMinVo	2.07657	2.83003	4.90662	4.907	151917	1.86288e-05	2.46517
	Bis_MinVo	1.79002	2.7875	4.57754	4.578	151441	1.84065e-05	2.46516
	Bis_RecursiveBSplines	1.66371	2.21204	3.87577	3.876	122989	1.79857e-05	2.46464
Bis_RecursiveBSplines2	1.6687	2.20323	3.87196	3.872	122605	1.79702e-05	2.46464	
4	BissectionInterval	2.00272e-05	1324.65	1324.65	22:04	189085251	7.00558e-06	1.40029
	Bis_Bernstein	2.13928	1580.28	1582.42	26:22	41354831	3.82126e-05	1.39545
	Bis_MinVariance	2.15623	1572.98	1575.14	26:15	41354813	3.80363e-05	1.39545
	Bis_BSplines	2.45004	9435.15	9437.6	02:37:17	236599277	3.98782e-05	1.40174
	Bis_MinNo	2.1495	1769.42	1771.57	29:31	46079317	3.83995e-05	1.39572
	Bis_ApproxMinVo	2.44382	2305.82	2308.27	38:28	60064813	3.83889e-05	1.39643
	Bis_MinVo	2.36537	2342.54	2344.91	39:04	59830667	3.91529e-05	1.39641
	Bis_RecursiveBSplines	2.19448	1615.8	1618	26:58	42162345	3.83234e-05	1.39545
Bis_RecursiveBSplines2	2.18998	1764.13	1766.32	29:26	45870041	3.84593e-05	1.39545	

Table 10: problem 2D : bissection:MinFirst problem:5

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	2.7895e-05	0.000934839	0.000934839	0.001	89	1.05038e-05	-1
	Bis_Bernstein	0.899543	0.00119996	0.900762	0.901	65	1.84609e-05	-1
	Bis_MinVariance	0.686552	0.000693083	0.687261	0.687	65	1.06628e-05	-1
	Bis_BSplines	0.95347	0.00188398	0.955394	0.955	111	1.69728e-05	-1
	Bis_MinNo	0.685577	0.000631094	0.686226	0.686	59	1.06965e-05	-1
	Bis_ApproxMinVo	0.69934	0.000725031	0.700083	0.700	69	1.05077e-05	-1
	Bis_MinVo	0.677593	0.000699043	0.678312	0.678	69	1.01311e-05	-1
	Bis_RecursiveBSplines	0.68535	0.000856876	0.686226	0.686	91	9.41622e-06	-1
Bis_RecursiveBSplines2	0.682905	0.00107718	0.683998	0.684	119	9.05189e-06	-1	
3	BissectionInterval	4.1008e-05	1.63658	1.63658	1.637	229895	7.11883e-06	4.01459
	Bis_Bernstein	1.69876	1.22124	2.92002	2.920	67617	1.80612e-05	4.00919
	Bis_MinVariance	2.07255	1.28434	3.35693	3.357	67617	1.89943e-05	4.00919
	Bis_BSplines	1.73464	4.85794	6.59262	6.593	273987	1.77306e-05	4.01481
	Bis_MinNo	1.63768	1.33592	2.97362	2.974	72515	1.84227e-05	4.0093
	Bis_ApproxMinVo	1.79649	1.64301	3.43951	3.440	89173	1.8425e-05	4.01023
	Bis_MinVo	1.72996	1.62429	3.35429	3.354	88901	1.82708e-05	4.01023
	Bis_RecursiveBSplines	1.66426	1.2708	2.93507	2.935	71131	1.78656e-05	4.00919
Bis_RecursiveBSplines2	1.67295	1.27528	2.94824	2.948	70305	1.81392e-05	4.00919	
4	BissectionInterval	2.00272e-05	1800.46	1800.46	30:00	247582197	7.27217e-06	2.3843
	Bis_Bernstein	2.14995	1751.01	1753.16	29:13	45814331	3.82197e-05	1.91959
	Bis_MinVariance	2.45225	1781.57	1784.02	29:44	45814325	3.88686e-05	1.91959
	Bis_BSplines	2.45352	9921.82	9924.27	02:45:24	256211333	3.87251e-05	1.92629
	Bis_MinNo	2.1363	1998.09	2000.22	33:20	50942427	3.92225e-05	1.91989
	Bis_ApproxMinVo	2.45776	2618.44	2620.9	43:40	65798501	3.97949e-05	1.92064
	Bis_MinVo	2.07288	2527.89	2529.96	42:09	65529011	3.85767e-05	1.92062
	Bis_RecursiveBSplines	2.20326	1802.99	1805.19	30:05	46289775	3.89501e-05	1.91959
Bis_RecursiveBSplines2	2.19676	1932.66	1934.86	32:14	49657837	3.89195e-05	1.91959	

Table 11: problem 2D : bissection:MinFirst problem:6

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	1.81198e-05	0.019803	0.019803	0.020	3901	5.0764e-06	0.755101
	Bis_Bernstein	0.677333	0.00667	0.684019	0.684	839	7.94994e-06	0.746136
	Bis_MinVariance	0.701226	0.00683689	0.708081	0.708	839	8.14886e-06	0.746136
	Bis_BSplines	0.673873	0.0137122	0.687627	0.688	1861	7.36817e-06	0.75593
	Bis_MinNo	0.689415	0.00680685	0.696237	0.696	851	7.99865e-06	0.750647
	Bis_ApproxMinVo	0.68443	0.00794697	0.692394	0.692	983	8.0844e-06	0.750741
	Bis_MinVo	0.71822	0.00784421	0.726084	0.726	979	8.01247e-06	0.750739
	Bis_RecursiveBSplines	0.920451	0.0132661	0.933755	0.934	989	1.34136e-05	0.746136
	Bis_RecursiveBSplines2	0.9378	0.0135188	0.951359	0.951	1021	1.32408e-05	0.746136
	BissectionInterval	3.8147e-05	5.67826	5.67826	5.678	881625	6.44067e-06	0.531586

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
	Bis_Bernstein	1.42218	4.25377	5.67597	5.676	243299	1.74837e-05	0.528778
	Bis_MinVariance	2.12434	4.59674	6.7211	6.721	243299	1.88934e-05	0.528778
	Bis_BSplines	1.45599	16.6461	18.1021	18.102	939967	1.77093e-05	0.532308
	Bis_MinNo	1.9297	4.84474	6.77446	6.774	265135	1.82727e-05	0.528946
	Bis_ApproxMinVo	2.08749	5.95021	8.03777	8.038	323159	1.84127e-05	0.52934
	Bis_MinVo	1.5053	5.78434	7.28966	7.290	322223	1.79514e-05	0.529323
	Bis_RecursiveBSplines	1.67395	4.62832	6.30229	6.302	258469	1.79067e-05	0.528778
	Bis_RecursiveBSplines2	1.95283	4.55418	6.50703	6.507	251577	1.81025e-05	0.528778
	BissectionInterval	4.41074e-05	950.711	950.711	15:50	131412411	7.23456e-06	0.444317
	Bis_Bernstein	2.13983	847.748	849.888	14:09	22173377	3.82327e-05	0.441412
Bis_MinVariance	2.14821	843.855	846.003	14:06	22173357	3.80571e-05	0.441412	
Bis_BSplines	2.16559	4941.49	4943.66	01:22:23	129190081	3.82498e-05	0.445183	
Bis_MinNo	2.14625	944.567	946.713	15:46	24612095	3.83781e-05	0.441577	
Bis_ApproxMinVo	2.42205	1230.15	1232.57	20:32	32129977	3.82697e-05	0.441991	
Bis_MinVo	2.26212	1204.95	1207.21	20:07	31996259	3.76589e-05	0.441987	
Bis_RecursiveBSplines	2.48412	915.931	918.415	15:18	23363409	3.92036e-05	0.441412	
Bis_RecursiveBSplines2	2.51359	969.159	971.673	16:11	25614033	3.7837e-05	0.441412	

Table 12: problem 2D : bisseccion:MinFirst problem:7

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
	BissectionInterval	4.19617e-05	0.033371	0.033371	0.033	2865	1.16478e-05	2.46064
	Bis_Bernstein	0.942911	0.015357	0.958294	0.958	1363	1.12671e-05	2.45631
	Bis_MinVariance	0.684097	0.0109892	0.695104	0.695	1363	8.0625e-06	2.45631
	Bis_BSplines	0.935694	0.035547	0.97126	0.971	2817	1.26187e-05	2.45979
	Bis_MinNo	0.683557	0.0113232	0.694897	0.695	1415	8.00227e-06	2.45638
	Bis_ApproxMinVo	0.683268	0.0125952	0.69588	0.696	1623	7.76043e-06	2.45712
	Bis_MinVo	0.678467	0.0112891	0.689776	0.690	1623	6.95571e-06	2.45712
	Bis_RecursiveBSplines	0.920079	0.0190129	0.93913	0.939	1467	1.29604e-05	2.45631
	Bis_RecursiveBSplines2	0.91818	0.02001	0.938236	0.938	1479	1.35294e-05	2.45631
	BissectionInterval	1.78814e-05	3.80107	3.80107	3.801	624303	6.08851e-06	2.20833
Bis_Bernstein	1.71263	2.44754	4.16021	4.160	135837	1.80182e-05	2.20224	
Bis_MinVariance	1.78957	2.49529	4.28488	4.285	135837	1.83698e-05	2.20224	
Bis_BSplines	1.46932	9.54719	11.0165	11.017	541987	1.76152e-05	2.206	
Bis_MinNo	1.63703	2.72543	4.36248	4.362	148295	1.83785e-05	2.2024	
Bis_ApproxMinVo	2.07958	3.40484	5.48443	5.484	182933	1.86125e-05	2.20279	
Bis_MinVo	1.82186	3.28786	5.10974	5.110	182409	1.80246e-05	2.20279	
Bis_RecursiveBSplines	1.96354	2.58286	4.54642	4.546	142967	1.80661e-05	2.20224	
Bis_RecursiveBSplines2	1.95608	2.63586	4.59196	4.592	142927	1.8442e-05	2.20224	
	BissectionInterval	2.19345e-05	1497.79	1497.79	24:57	213360751	7.01997e-06	2.12987
	Bis_Bernstein	2.15231	1922.51	1924.66	32:04	50688057	3.79283e-05	1.21547
	Bis_MinVariance	2.19133	1947.46	1949.65	32:29	50688049	3.84204e-05	1.21547
	Bis_BSplines	2.16326	10870.6	10872.8	03:01:12	280478631	3.87574e-05	1.22159
	Bis_MinNo	2.15859	2165.3	2167.46	36:07	56281413	3.84728e-05	1.21573
	Bis_ApproxMinVo	2.44274	2774.01	2776.45	46:16	72804275	3.81023e-05	1.21642
	Bis_MinVo	2.27545	2750.48	2752.76	45:52	72572839	3.78996e-05	1.21642
	Bis_RecursiveBSplines	2.52871	1978.3	1980.83	33:00	51306187	3.85588e-05	1.21547
	Bis_RecursiveBSplines2	2.50276	2108.43	2110.93	35:10	54514207	3.86767e-05	1.21547

Table 13: problem 2D : bisseccion:MinFirst problem:8

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
	BissectionInterval	4.1008e-05	0.0014019	0.0014019	0.001	107	1.31019e-05	-1
	Bis_Bernstein	0.924698	0.00121212	0.925951	0.926	69	1.7567e-05	-1
	Bis_MinVariance	0.687985	0.000745058	0.688745	0.689	69	1.07979e-05	-1
	Bis_BSplines	0.676752	0.00106788	0.677836	0.678	113	9.45024e-06	-1
	Bis_MinNo	0.684596	0.000642061	0.685256	0.685	61	1.05256e-05	-1
	Bis_ApproxMinVo	0.917721	0.00111985	0.918883	0.919	69	1.62297e-05	-1
	Bis_MinVo	0.670599	0.000726938	0.671355	0.671	69	1.05353e-05	-1
	Bis_RecursiveBSplines	0.921652	0.00151682	0.923232	0.923	93	1.63099e-05	-1
	Bis_RecursiveBSplines2	0.911091	0.00167799	0.912809	0.913	123	1.36422e-05	-1
	BissectionInterval	1.69277e-05	1.93525	1.93525	1.935	320711	6.03424e-06	3.72703
Bis_Bernstein	1.43162	1.88967	3.32132	3.321	106597	1.77272e-05	3.72169	
Bis_MinVariance	1.78893	1.96199	3.75093	3.751	106597	1.84057e-05	3.72169	
Bis_BSplines	1.78525	6.39865	8.18393	8.184	350145	1.82743e-05	3.72703	
Bis_MinNo	1.92647	2.12924	4.05572	4.056	114461	1.86023e-05	3.72196	
Bis_ApproxMinVo	2.07544	2.56158	4.63706	4.637	136049	1.88284e-05	3.72254	
Bis_MinVo	1.51712	2.46351	3.98065	3.981	135677	1.81571e-05	3.72254	
Bis_RecursiveBSplines	1.65209	1.98022	3.63232	3.632	109979	1.80054e-05	3.72169	
Bis_RecursiveBSplines2	1.91749	2.02904	3.94656	3.947	109183	1.85838e-05	3.72169	
	BissectionInterval	2.19345e-05	1873.19	1873.19	31:13	258043271	7.2592e-06	2.19467
	Bis_Bernstein	2.41941	1636.75	1639.17	27:19	42826931	3.82177e-05	1.74098
	Bis_MinVariance	2.45027	1650.43	1652.88	27:32	42826929	3.85371e-05	1.74098
	Bis_BSplines	2.17481	9250.75	9252.92	02:34:12	241476161	3.83092e-05	1.74758
	Bis_MinNo	2.44646	1927.55	1929.99	32:09	47505203	4.05755e-05	1.74126
	Bis_ApproxMinVo	2.17456	2386.2	2388.38	39:48	61510141	3.87936e-05	1.74199
	Bis_MinVo	2.37625	2376.25	2378.62	39:38	61134839	3.88689e-05	1.74198
	Bis_RecursiveBSplines	2.21778	1675.35	1677.56	27:57	42993399	3.89675e-05	1.74098
	Bis_RecursiveBSplines2	2.49369	1744.16	1746.66	29:06	45272967	3.85255e-05	1.74098

Table 14: problem 2D : bisseccion:MinFirst problem:9

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
	BissectionInterval	2.00272e-05	0.00483394	0.00483394	0.005	671	7.20408e-06	80.7137
	Bis_Bernstein	0.84929	0.00651002	0.85584	0.856	583	1.11664e-05	80.5925
	Bis_MinVariance	0.611745	0.00367498	0.615438	0.615	583	6.30357e-06	80.5925
	Bis_BSplines	0.616019	0.00473189	0.62077	0.621	777	6.08995e-06	80.6228

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
	Bis_MinNo	0.60859	0.00362301	0.61293	0.612	583	6.21442e-06	80.5938
	Bis_ApproxMinVo	0.611682	0.00371814	0.615416	0.615	597	6.22804e-06	80.5973
	Bis_MinVo	0.83972	0.00686812	0.846608	0.847	597	1.15044e-05	80.5972
	Bis_RecursiveBSplines	0.826797	0.00606108	0.832879	0.833	591	1.02556e-05	80.5925
	Bis_RecursiveBSplines2	0.8266	0.00673485	0.833354	0.833	589	1.14344e-05	80.5925
3	BissectionInterval	1.81198e-05	2.78182	2.78182	2.782	393723	7.06541e-06	85.3108
	Bis_Bernstein	1.29219	1.06234	2.35457	2.355	69973	1.51821e-05	85.1681
	Bis_MinVariance	1.02652	1.03171	2.05825	2.058	69973	1.47444e-05	85.1681
	Bis_BSplines	1.01933	2.2535	3.27285	3.273	154947	1.45437e-05	85.2152
	Bis_MinNo	1.02368	1.0584	2.0821	2.082	72049	1.469e-05	85.1701
	Bis_ApproxMinVo	1.27708	1.21009	2.48719	2.487	81135	1.49146e-05	85.1755
	Bis_MinVo	1.0096	1.20106	2.21068	2.211	80979	1.48318e-05	85.1754
	Bis_RecursiveBSplines	1.32397	1.06108	2.38508	2.385	70077	1.51416e-05	85.1681
	Bis_RecursiveBSplines2	1.30886	1.06612	2.37502	2.375	70347	1.51552e-05	85.1681
	BissectionInterval	3.29018e-05	1691.55	1691.55	28.11	198196587	8.53472e-06	100.236
Bis_Bernstein	4.09664	527.098	531.195	08:51	11033365	4.77731e-05	99.7902	
Bis_MinVariance	5.12449	551.488	556.612	09:16	11033365	4.99836e-05	99.7902	
Bis_BSplines	3.8135	1817.17	1820.98	30:20	38208365	4.75594e-05	99.8517	
Bis_MinNo	4.58103	582.966	587.547	09:47	11779765	4.94887e-05	99.7933	
Bis_ApproxMinVo	5.18579	722.442	727.628	12:07	13904943	5.19558e-05	99.8014	
Bis_MinVo	4.26069	685.662	689.923	11:29	13888647	4.94397e-05	99.8013	
Bis_RecursiveBSplines	5.14062	557.032	562.173	09:22	11034227	5.04822e-05	99.7902	
Bis_RecursiveBSplines2	4.93829	563.653	568.591	09:28	11046097	5.10273e-05	99.7902	

Table 15: problem 2D : bissection:MinFirst problem:11

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	2.09808e-05	0.00217199	0.00217199	0.002	299	7.26419e-06	56.0172
	Bis_Bernstein	0.612977	0.00194192	0.614937	0.615	291	6.67326e-06	56.0172
	Bis_MinVariance	0.612408	0.001616	0.614039	0.614	291	5.55327e-06	56.0172
	Bis_BSplines	0.827877	0.0041461	0.832063	0.832	353	1.17453e-05	56.0472
	Bis_MinNo	0.827248	0.00362206	0.83091	0.831	291	1.24469e-05	56.0185
	Bis_ApproxMinVo	0.614677	0.00162411	0.616315	0.616	291	5.58112e-06	56.0219
	Bis_MinVo	0.613273	0.001966	0.615257	0.615	291	6.75601e-06	56.0218
	Bis_RecursiveBSplines	0.827866	0.00342011	0.831306	0.831	291	1.1753e-05	56.0172
	Bis_RecursiveBSplines2	0.819744	0.00365806	0.823421	0.823	291	1.25706e-05	56.0172
3	BissectionInterval	3.91006e-05	0.422159	0.422159	0.422	43383	9.73098e-06	66.1907
	Bis_Bernstein	1.31341	0.382036	1.69549	1.695	24033	1.58963e-05	66.1836
	Bis_MinVariance	1.02216	0.352092	1.37427	1.374	24033	1.46504e-05	66.1836
	Bis_BSplines	1.38876	0.969552	2.35835	2.358	50105	1.93504e-05	66.2352
	Bis_MinNo	1.02799	0.368716	1.39673	1.397	24875	1.48228e-05	66.1859
	Bis_ApproxMinVo	1.02696	0.401171	1.42815	1.428	27383	1.46504e-05	66.1917
	Bis_MinVo	1.01371	0.401963	1.41569	1.416	27379	1.46814e-05	66.1916
	Bis_RecursiveBSplines	1.31188	0.382913	1.69484	1.695	24047	1.59235e-05	66.1836
	Bis_RecursiveBSplines2	1.27351	0.391732	1.66528	1.665	24181	1.62e-05	66.1836
4	BissectionInterval	5.10216e-05	851.868	851.868	14:11	101228629	8.41528e-06	60.5978
	Bis_Bernstein	4.07837	521.273	525.351	08:45	10706321	4.86883e-05	60.4108
	Bis_MinVariance	5.13345	542.049	547.183	09:07	10706321	5.06289e-05	60.4108
	Bis_BSplines	3.79144	1711.31	1715.1	28:35	35104821	4.87486e-05	60.4934
	Bis_MinNo	4.56263	579.795	584.358	09:44	11401901	5.08508e-05	60.4143
	Bis_ApproxMinVo	5.27704	698.217	703.494	11:43	13375771	5.22001e-05	60.4237
	Bis_MinVo	4.28076	669.87	674.151	11:14	13339051	5.02187e-05	60.4236
	Bis_RecursiveBSplines	4.61553	528.497	533.113	08:53	10706749	4.93611e-05	60.4108
	Bis_RecursiveBSplines2	5.0283	539.791	544.82	09:04	10723269	5.03383e-05	60.4108

Table 16: problem 2D : bissection:MinFirst problem:12

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	2.00272e-05	0.000449896	0.000449896	0.000	59	7.62535e-06	-1
	Bis_Bernstein	0.612858	0.000427961	0.613305	0.613	41	1.04381e-05	-1
	Bis_MinVariance	0.835297	0.000770092	0.836101	0.836	41	1.87827e-05	-1
	Bis_BSplines	0.611469	0.000432014	0.61192	0.612	41	1.05369e-05	-1
	Bis_MinNo	0.617839	0.000407934	0.618268	0.618	41	9.94961e-06	-1
	Bis_ApproxMinVo	0.825781	0.000674963	0.826496	0.826	41	1.64625e-05	-1
	Bis_MinVo	0.611902	0.000427008	0.612348	0.612	41	1.04148e-05	-1
	Bis_RecursiveBSplines	0.825959	0.000766039	0.826744	0.827	41	1.86839e-05	-1
	Bis_RecursiveBSplines2	0.612692	0.000388145	0.61311	0.613	41	9.46696e-06	-1
3	BissectionInterval	4.00543e-05	0.107694	0.107694	0.108	8983	1.19886e-05	45.7424
	Bis_Bernstein	1.02218	0.101237	1.12344	1.123	6787	1.49163e-05	45.7376
	Bis_MinVariance	1.02465	0.10219	1.12686	1.127	6787	1.50567e-05	45.7376
	Bis_BSplines	1.36799	0.269187	1.63722	1.637	12745	2.1121e-05	45.7938
	Bis_MinNo	1.02485	0.106197	1.13106	1.131	7075	1.50102e-05	45.7401
	Bis_ApproxMinVo	1.02257	0.120012	1.1426	1.143	8021	1.49622e-05	45.7464
	Bis_MinVo	1.29173	0.139767	1.43151	1.432	8017	1.74338e-05	45.7463
	Bis_RecursiveBSplines	1.03749	0.101483	1.13899	1.139	6787	1.49526e-05	45.7376
	Bis_RecursiveBSplines2	1.32453	0.143606	1.46818	1.468	6867	2.09125e-05	45.7376
4	BissectionInterval	4.50611e-05	35.2245	35.2245	35.224	4047819	8.70209e-06	35.4184
	Bis_Bernstein	4.10826	64.9369	69.0453	01:09	1320723	4.91677e-05	34.561
	Bis_MinVariance	5.25263	66.5813	71.834	01:11	1320723	5.04128e-05	34.561
	Bis_BSplines	4.09958	178.412	182.511	03:02	3677357	4.85163e-05	34.6173
	Bis_MinNo	4.61204	70.3334	74.9455	01:14	1389879	5.0604e-05	34.5634
	Bis_ApproxMinVo	5.16821	82.5588	87.727	01:27	1576051	5.23833e-05	34.5698
	Bis_MinVo	4.29477	77.8277	82.1225	01:22	1572553	4.94913e-05	34.5697
	Bis_RecursiveBSplines	4.95848	65.2	70.1585	01:10	1320813	4.93635e-05	34.561
	Bis_RecursiveBSplines2	4.62821	66.4563	71.0846	01:11	1324925	5.01586e-05	34.561

Table 17: problem 2D : bissection:MinFirst problem:13

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	4.00543e-05	0.02301	0.02301	0.023	1759	1.30813e-05	83.6008
	Bis_Bernstein	0.6766	0.00742698	0.684046	0.684	935	7.94329e-06	83.5411
	Bis_MinVariance	0.913825	0.0114231	0.925265	0.925	935	1.22172e-05	83.5411
	Bis_BSplines	1.0509	0.0334659	1.08442	1.084	2145	1.56018e-05	83.571
	Bis_MinNo	0.679996	0.00763798	0.687664	0.688	965	7.915e-06	83.5424
	Bis_ApproxMinVo	0.676605	0.00830603	0.684929	0.685	1035	8.02515e-06	83.5458
	Bis_MinVo	0.920596	0.0134492	0.934083	0.934	1035	1.29944e-05	83.5457
	Bis_RecursiveBSplines	0.918649	0.012476	0.931175	0.931	1053	1.1848e-05	83.5411
Bis_RecursiveBSplines2	0.68441	0.00789809	0.692327	0.692	989	7.98594e-06	83.5411	
3	BissectionInterval	4.00543e-05	3.04856	3.04856	3.049	397169	7.67573e-06	86.0247
	Bis_Bernstein	1.55636	1.94418	3.50056	3.501	92125	2.11037e-05	85.877
	Bis_MinVariance	1.89637	2.00441	3.9008	3.901	92125	2.17575e-05	85.877
	Bis_BSplines	1.91908	7.97151	9.89061	9.891	373795	2.13259e-05	85.9756
	Bis_MinNo	2.01402	2.15303	4.16708	4.167	99959	2.15391e-05	85.879
	Bis_ApproxMinVo	1.88682	2.60276	4.48959	4.490	120115	2.16689e-05	85.884
	Bis_MinVo	1.60255	2.52078	4.12335	4.123	119887	2.10263e-05	85.8839
	Bis_RecursiveBSplines	2.11818	2.12998	4.24817	4.248	98935	2.15291e-05	85.877
Bis_RecursiveBSplines2	2.08232	2.03323	4.11556	4.116	93995	2.16313e-05	85.877	
4	BissectionInterval	4.1008e-05	2494.64	2494.64	41:34	279946417	8.91115e-06	100.931
	Bis_Bernstein	4.09104	962.063	966.154	16:06	17347765	5.54575e-05	100.309
	Bis_MinVariance	5.26952	1012.5	1017.77	16:57	17347739	5.83647e-05	100.309
	Bis_BSplines	4.09909	7557.45	7561.55	02:06:01	131218955	5.75942e-05	100.443
	Bis_MinNo	4.22797	1117.87	1122.09	18:42	19349831	5.77714e-05	100.313
	Bis_ApproxMinVo	4.83948	1522.71	1527.55	25:27	26453523	5.75618e-05	100.332
	Bis_MinVo	4.25255	1549.01	1553.26	25:53	26336389	5.88164e-05	100.332
	Bis_RecursiveBSplines	4.63956	1124.03	1128.67	18:48	19802109	5.67633e-05	100.309
Bis_RecursiveBSplines2	4.61194	1206.23	1210.84	20:10	20831905	5.7903e-05	100.309	

Table 18: problem 2D : bissection:MinFirst problem:14

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	1.88351e-05	0.00200605	0.00200605	0.002	309	6.49208e-06	56.1972
	Bis_Bernstein	0.674723	0.00246596	0.677216	0.677	285	8.6525e-06	56.1972
	Bis_MinVariance	0.921837	0.0038898	0.925778	0.926	285	1.36484e-05	56.1972
	Bis_BSplines	0.670822	0.00477099	0.675611	0.676	581	8.21169e-06	56.2873
	Bis_MinNo	0.927209	0.00459504	0.931843	0.932	297	1.54715e-05	56.1985
	Bis_ApproxMinVo	0.915067	0.00496507	0.92007	0.920	319	1.55645e-05	56.2019
	Bis_MinVo	0.680618	0.00276089	0.683397	0.683	319	8.65482e-06	56.2019
	Bis_RecursiveBSplines	0.682484	0.00244212	0.684942	0.685	347	7.03781e-06	56.1972
Bis_RecursiveBSplines2	0.678853	0.00259209	0.681469	0.681	301	8.61158e-06	56.1972	
3	BissectionInterval	1.90735e-05	0.407414	0.407414	0.407	57467	7.08953e-06	66.964
	Bis_Bernstein	1.54495	0.568683	2.11366	2.114	27483	2.06922e-05	66.9562
	Bis_MinVariance	2.19427	0.622766	2.81705	2.817	27483	2.266e-05	66.9562
	Bis_BSplines	1.55198	2.75712	4.30912	4.309	131623	2.09471e-05	67.0559
	Bis_MinNo	1.73277	0.642918	2.37571	2.376	29939	2.14743e-05	66.9584
	Bis_ApproxMinVo	2.17711	0.974583	3.15173	3.152	43481	2.2414e-05	66.9643
	Bis_MinVo	1.60423	0.905128	2.50939	2.509	43345	2.08819e-05	66.9642
	Bis_RecursiveBSplines	1.80733	0.642185	2.44953	2.450	30669	2.09392e-05	66.9562
Bis_RecursiveBSplines2	2.20408	0.803443	3.00754	3.008	28249	2.84415e-05	66.9562	
4	BissectionInterval	2.47955e-05	1161.22	1161.22	19:21	138635525	8.37606e-06	61.3482
	Bis_Bernstein	4.08194	916.048	920.13	15:20	15838737	5.78359e-05	61.169
	Bis_MinVariance	4.80931	921.525	926.334	15:26	15838723	5.81818e-05	61.169
	Bis_BSplines	4.12488	6353.99	6358.11	01:45:58	112655261	5.64021e-05	61.3206
	Bis_MinNo	4.24809	1053.41	1057.66	17:37	18099539	5.82008e-05	61.1752
	Bis_ApproxMinVo	5.17886	1399.82	1405	23:25	24205219	5.78312e-05	61.1923
	Bis_MinVo	4.25229	1423.36	1427.61	23:47	24109187	5.90381e-05	61.1921
	Bis_RecursiveBSplines	4.64611	961.981	966.627	16:06	17006481	5.65655e-05	61.169
Bis_RecursiveBSplines2	4.62496	980.256	984.881	16:24	17136967	5.72013e-05	61.169	

Table 19: problem 2D : bissection:MinFirst problem:15

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	2.28882e-05	0.000638962	0.000638962	0.001	89	7.17935e-06	-1
	Bis_Bernstein	0.973228	0.00124097	0.974509	0.975	65	1.90918e-05	-1
	Bis_MinVariance	0.916031	0.00116897	0.917238	0.917	65	1.79841e-05	-1
	Bis_BSplines	0.670066	0.00102806	0.671119	0.671	111	9.26181e-06	-1
	Bis_MinNo	0.978288	0.000798941	0.979111	0.979	59	1.35414e-05	-1
	Bis_ApproxMinVo	0.682684	0.00072813	0.683427	0.683	69	1.05526e-05	-1
	Bis_MinVo	0.904707	0.00132489	0.906072	0.906	69	1.92013e-05	-1
	Bis_RecursiveBSplines	0.683147	0.000865936	0.684032	0.684	91	9.51578e-06	-1
Bis_RecursiveBSplines2	0.679114	0.00113416	0.680266	0.680	119	9.53073e-06	-1	
3	BissectionInterval	1.81198e-05	0.382704	0.382704	0.383	50577	7.56676e-06	47.6076
	Bis_Bernstein	1.82862	0.67852	2.50715	2.507	31069	2.18391e-05	47.5669
	Bis_MinVariance	2.1904	0.696698	2.88714	2.887	31069	2.24242e-05	47.5669
	Bis_BSplines	1.57939	2.25968	3.83909	3.839	109921	2.05573e-05	47.694
	Bis_MinNo	2.01195	0.746464	2.75843	2.758	33345	2.23861e-05	47.5693
	Bis_ApproxMinVo	1.8867	0.850567	2.73728	2.737	40003	2.12626e-05	47.5756
	Bis_MinVo	1.60702	0.831574	2.43862	2.439	39941	2.08201e-05	47.5755
	Bis_RecursiveBSplines	1.79963	0.688678	2.48832	2.488	32383	2.12667e-05	47.5669
Bis_RecursiveBSplines2	2.04253	0.706306	2.74887	2.749	31859	2.21697e-05	47.5669	
4	BissectionInterval	2.09808e-05	97.8205	97.8205	01:37	11486469	8.51615e-06	35.729
	Bis_Bernstein	3.7643	143.06	146.825	02:26	2566309	5.57456e-05	34.8896
	Bis_MinVariance	4.80825	151.155	155.963	02:35	2566309	5.88996e-05	34.8896
	Bis_BSplines	3.79253	1028.25	1032.04	17:12	18342789	5.60574e-05	34.9977
	Bis_MinNo	4.24201	168.432	172.674	02:52	2865573	5.87778e-05	34.8979
	Bis_ApproxMinVo	4.83039	224.709	229.539	03:49	3835237	5.85906e-05	34.9065
	Bis_MinVo	4.25484	218.202	222.457	03:42	3821863	5.7093e-05	34.9064
	Bis_RecursiveBSplines	4.64134	154.064	158.705	02:38	2691655	5.72376e-05	34.8896
Bis_RecursiveBSplines2	5.03049	161.821	166.852	02:46	2799389	5.7806e-05	34.8896	

Table 20: problem 2D : bissection:MinFirst problem:16

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	2.00272e-05	0.012336	0.012336	0.012	1977	6.23977e-06	79.2556
	Bis_Bernstein	0.909412	0.014173	0.923605	0.924	1193	1.18802e-05	79.1946
	Bis_MinVariance	0.685791	0.00953698	0.695353	0.695	1193	7.99412e-06	79.1946
	Bis_BSplines	0.67186	0.0191638	0.691044	0.691	2731	7.01715e-06	79.286
	Bis_MinNo	0.903002	0.0124331	0.915524	0.916	1219	1.01994e-05	79.1959
	Bis_ApproxMinVo	0.674087	0.0106118	0.684717	0.685	1361	7.79704e-06	79.1994
	Bis_MinVo	0.902251	0.0177231	0.920013	0.920	1359	1.30413e-05	79.1993
	Bis_RecursiveBSplines	0.911773	0.0151021	0.926915	0.927	1295	1.16619e-05	79.1946
Bis_RecursiveBSplines2	0.677678	0.00901198	0.686709	0.687	1227	7.34473e-06	79.1946	
3	BissectionInterval	3.88622e-05	1.8271	1.8271	1.827	205881	8.87453e-06	80.8196
	Bis_Bernstein	1.55359	1.59931	3.15293	3.153	76769	2.08327e-05	80.7194
	Bis_MinVariance	2.20359	1.68149	3.88511	3.885	76769	2.19032e-05	80.7194
	Bis_BSplines	1.56158	6.40752	7.96911	7.969	309183	2.0724e-05	80.8061
	Bis_MinNo	1.72969	1.82172	3.55144	3.551	84561	2.15433e-05	80.7215
	Bis_ApproxMinVo	1.89254	2.23002	4.12258	4.123	104255	2.13901e-05	80.7272
	Bis_MinVo	1.92625	2.37218	4.29846	4.298	104051	2.27983e-05	80.7269
	Bis_RecursiveBSplines	2.09245	1.73481	3.82732	3.827	80601	2.15234e-05	80.7194
Bis_RecursiveBSplines2	2.09451	1.69318	3.78771	3.788	77765	2.1773e-05	80.7194	
4	BissectionInterval	3.69549e-05	2501	2501	41:41	279256333	8.95594e-06	93.4034
	Bis_Bernstein	3.79094	882.611	886.402	14:46	15777307	5.59418e-05	92.9911
	Bis_MinVariance	4.8216	918.709	923.531	15:23	15777287	5.82298e-05	92.9911
	Bis_BSplines	3.81229	6653.42	6657.24	01:50:57	118427177	5.61816e-05	93.1259
	Bis_MinNo	4.2146	1037.01	1041.22	17:21	17868739	5.80348e-05	92.9954
	Bis_ApproxMinVo	5.30136	1449.84	1455.14	24:15	24035421	6.03209e-05	93.0091
	Bis_MinVo	3.93661	1355.29	1359.22	22:39	23935829	5.66217e-05	93.009
	Bis_RecursiveBSplines	5.08848	987.968	993.056	16:33	17075581	5.78585e-05	92.9911
Bis_RecursiveBSplines2	4.61128	1012.08	1016.69	16:56	17633283	5.7396e-05	92.9911	

Table 21: problem 2D : bissection:MinFirst problem:17

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	1.78814e-05	0.00108004	0.00108004	0.001	159	6.79268e-06	53.3972
	Bis_Bernstein	0.67041	0.00145793	0.671887	0.672	159	9.16937e-06	53.3972
	Bis_MinVariance	0.683169	0.00144696	0.684633	0.685	159	9.10039e-06	53.3972
	Bis_BSplines	0.670424	0.00322199	0.673664	0.674	391	8.24038e-06	53.4859
	Bis_MinNo	0.916378	0.002913	0.919345	0.919	179	1.02297e-05	53.3985
	Bis_ApproxMinVo	0.910594	0.00394893	0.914579	0.915	207	1.90769e-05	53.4019
	Bis_MinVo	0.908424	0.00325513	0.911698	0.912	207	1.57253e-05	53.4018
	Bis_RecursiveBSplines	0.677372	0.00156116	0.678949	0.679	175	8.92094e-06	53.3972
Bis_RecursiveBSplines2	0.982506	0.00305319	0.985585	0.986	179	1.70569e-05	53.3972	
3	BissectionInterval	1.81198e-05	0.635681	0.635681	0.636	83677	7.59684e-06	64.0418
	Bis_Bernstein	1.55774	0.938971	2.49674	2.497	44791	2.09634e-05	64.0367
	Bis_MinVariance	2.1748	0.998315	3.17316	3.173	44791	2.22883e-05	64.0367
	Bis_BSplines	1.62116	4.07719	5.69837	5.698	197089	2.06871e-05	64.1372
	Bis_MinNo	2.015	1.07972	3.09473	3.095	49279	2.19103e-05	64.0389
	Bis_ApproxMinVo	1.92726	1.33915	3.26642	3.266	62071	2.15745e-05	64.0447
	Bis_MinVo	1.98084	1.53999	3.52087	3.521	61811	2.49144e-05	64.0446
	Bis_RecursiveBSplines	2.09171	1.02846	3.12018	3.120	47183	2.17972e-05	64.0367
Bis_RecursiveBSplines2	1.80066	0.958328	2.759	2.759	45353	2.11304e-05	64.0367	
4	BissectionInterval	1.90735e-05	912.353	912.353	15:12	104660775	8.71724e-06	55.9972
	Bis_Bernstein	3.75696	764.716	768.473	12:48	13651795	5.60158e-05	55.8464
	Bis_MinVariance	4.802	798.284	803.086	13:23	13651793	5.84746e-05	55.8464
	Bis_BSplines	4.12017	5248.46	5252.58	01:27:32	92998833	5.64358e-05	55.9964
	Bis_MinNo	4.23798	888.112	892.35	14:52	15391555	5.77012e-05	55.8531
	Bis_ApproxMinVo	5.28782	1241.65	1246.93	20:46	20496721	6.05778e-05	55.8703
	Bis_MinVo	3.92867	1151.48	1155.41	19:15	20380319	5.64998e-05	55.8694
	Bis_RecursiveBSplines	5.04822	828.965	834.014	13:54	14234693	5.82356e-05	55.8464
Bis_RecursiveBSplines2	4.61013	826.702	831.312	13:51	14365987	5.75458e-05	55.8464	

Table 22: problem 2D : bissection:MinFirst problem:18

ndof	solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
2	BissectionInterval	1.81198e-05	0.00072813	0.00072813	0.001	107	6.80496e-06	-1
	Bis_Bernstein	0.673632	0.000738144	0.674386	0.674	69	1.06977e-05	-1
	Bis_MinVariance	0.680295	0.000725985	0.681051	0.681	69	1.05215e-05	-1
	Bis_BSplines	0.672533	0.00105	0.673602	0.674	113	9.29199e-06	-1
	Bis_MinNo	0.677878	0.000664949	0.678561	0.679	61	1.09008e-05	-1
	Bis_ApproxMinVo	0.916093	0.0011971	0.917309	0.917	69	1.73493e-05	-1
	Bis_MinVo	0.895963	0.00111485	0.897095	0.897	69	1.61572e-05	-1
	Bis_RecursiveBSplines	0.906789	0.00130296	0.908132	0.908	93	1.40103e-05	-1
Bis_RecursiveBSplines2	0.941749	0.00221705	0.943993	0.944	123	1.80248e-05	-1	
3	BissectionInterval	1.81198e-05	0.1733	0.1733	0.173	24121	7.18461e-06	44.3629
	Bis_Bernstein	1.52929	0.302642	1.83195	1.832	14567	2.07759e-05	44.3583
	Bis_MinVariance	2.18681	0.342953	2.5298	2.530	14567	2.35431e-05	44.3583
	Bis_BSplines	1.57609	1.28352	2.85963	2.860	62615	2.04986e-05	44.4509
	Bis_MinNo	2.01171	0.393351	2.4051	2.405	16537	2.37861e-05	44.3608
	Bis_ApproxMinVo	2.1812	0.463583	2.6448	2.645	20169	2.29849e-05	44.3671
	Bis_MinVo	1.89541	0.447547	2.34299	2.343	20147	2.22141e-05	44.367
	Bis_RecursiveBSplines	1.80978	0.334163	2.14397	2.144	15657	2.13427e-05	44.3583
Bis_RecursiveBSplines2	1.80819	0.322732	2.13094	2.131	15263	2.11447e-05	44.3583	
4	BissectionInterval	1.90735e-05	107.874	107.874	01:47	12487643	8.63847e-06	33.2034
	Bis_Bernstein	3.75943	157.974	161.733	02:41	2849919	5.54311e-05	32.069
	Bis_MinVariance	4.8809	171.963	176.844	02:56	2849919	6.03397e-05	32.069
	Bis_BSplines	3.81144	971.472	975.283	16:15	17213671	5.64361e-05	32.1699
	Bis_MinNo	4.55665	187.383	191.938	03:11	3138605	5.97025e-05	32.0741
	Bis_ApproxMinVo	5.30422	238.04	243.344	04:03	4068111	5.85136e-05	32.0855
	Bis_MinVo	3.95182	229.929	233.88	03:53	4049073	5.67855e-05	32.0854
	Bis_RecursiveBSplines	5.09866	169.468	174.566	02:54	2930803	5.78229e-05	32.069
Bis_RecursiveBSplines2	5.19393	179.245	184.439	03:04	3012149	5.95075e-05	32.069	

Table 23: problem 2D : bissection:MinFirst problem:19

5 3D robot

5.1 Presentation fo the robot

In this section we consider the same kind of problem presented in 1 but with 3D-robots. To assess the effectiveness of our method on more complex systems, we consider the following problem:

$$\begin{aligned}
 & \text{Find} && q \in \mathbb{Q} \subset \mathbb{R}^n \\
 & \text{such as} && \min_q F(q) \\
 & \text{with} && x = x_d + [-0.01; 0.01] \\
 & && y = y_d + [-0.01; 0.01] \\
 & && z = z_d + [-0.01; 0.01] \\
 & \forall j = \{1, \dots, n\} && \Gamma_j \in [-\alpha\bar{\Gamma}_j : \alpha\bar{\Gamma}_j]
 \end{aligned} \tag{12}$$

We present the results for two different 3D robots (with $n = 4$ and $n = 6$ degrees of freedom) which derive from the KUKA LWR as presented on Figure 1. The 4-dof robot is obtained by considering the joint number 3 and 5 equal to zero.

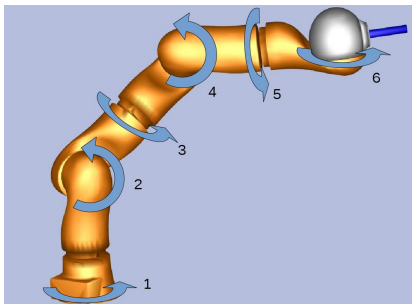


Figure 1: The 3D robot : KUKA LWR

The solution of Problem 12 is the static posture, i.e. the joint position, that minimizes the cost function $F(q)$ and ensures that the end effector position $\{x, y, z\}$ is within a 0.02 meter width box center on a desired position $\{x_d, y_d, z_d\}$ and that the joint torque Γ_i are within the joint limits $[-\bar{\Gamma}_j : \bar{\Gamma}_j]$ considering different coefficient α . Theses function can be obtained by using the forward kinematic model and the inverse dynamics model as presented in [1].

5.2 Constraint evaluation

First, Table 24 and 25 presents the results of the constraint and cost function computation considering several inputs to compare the effectiveness of the Evaluation.

input	eval	X	Y	Z	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	$\Sigma \Gamma_i^2$	
[-0.5;1.5]	Interval	[-4.71;5.18]	[-3.53;6.62]	[-3.10;7.10]	[-4.11;4.667]	[-3.829;3.027]	[-4.13;8.558.4]	[-5.57;8.380.1]	[-7.13;8.91]	[2.82;1.86]	[-30;6;36e6]	
	Bernstein	[-5.91;6.23]	[-5.96;6.15]	[-2.75;3.93]	[-4.29;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	BSplines	[-5.910;6.227]	[-5.961;6.152]	[-2.752;3.928]	[-4.294;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	MinVo	[-5.910;6.227]	[-5.961;6.152]	[-2.752;3.928]	[-4.294;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	ApproxMinVo	[-5.910;6.227]	[-5.961;6.152]	[-2.752;3.928]	[-4.294;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	MinNo	[-5.910;6.227]	[-5.961;6.152]	[-2.752;3.928]	[-4.294;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	MinVa	[-5.910;6.227]	[-5.961;6.152]	[-2.752;3.928]	[-4.294;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	Recursive1	[-5.91;6.23]	[-5.96;6.15]	[-2.75;3.93]	[-4.29;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	Recursive2	[-5.91;6.23]	[-5.96;6.15]	[-2.75;3.93]	[-4.29;5.4845.6]	[-3.035;3.3690.0]	[-5.53;3.5888.2]	[-4.41;4.415.3]	[-6.78;6.64]	[-1.868;1.999]	[-5.3e6;3.36e6]	
	[0;1]	Interval	[-1.28;2.10]	[-0.28;3.01]	[-0.90;2.90]	[-6.10;6.657.7]	[-8.22;5.408.9]	[-6.3;36;116.3]	[-1.59;35.49]	[-1.50;2.76]	[-0.97;0.43]	[-7.53e3;1148e3]
Bernstein		[-0.978;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
BSplines		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
MinVo		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
ApproxMinVo		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
MinNo		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
MinVa		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
Recursive1		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
Recursive2		[-0.979;1.643]	[-0.597;1.686]	[-0.148;1.664]	[-2.81;8.198.4]	[-2.81;8.198.4]	[-36.93;53.12]	[-62.44;35.49]	[-0.999;1.375]	[-0.427;0.289]	[-1.39e3;151e3]	
[0.25;0.75]	Interval	[-0.217;1.286]	[0.024;1.514]	[0.061;1.50]	[-1.143;5.145.8]	[-2.48;3.110.4]	[-1.18;91;31.54]	[-61.00;14.40]	[-0.538;1.223]	[-0.492;0.098]	[-4981;58764.4]	
	Bernstein	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	BSplines	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	MinVo	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	ApproxMinVo	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	MinNo	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	MinVa	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	Recursive1	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
	Recursive2	[-0.046;0.928]	[0.076;0.960]	[0.476;1.268]	[-68.91;558.37]	[-93.1;31.34]	[-7.98;13.487]	[-27.34;1.89]	[-0.228;0.701]	[-0.254;0.021]	[-6774.1;11891.3]	
[0.45;0.55]	Interval	[0.326;0.609]	[0.350;0.635]	[0.743;1.089]	[-1.13;7.13.57]	[-5.3;3;-7.21]	[-1.12;4.94]	[-17.88;-5.98]	[0.090;0.405]	[-0.189;-0.082]	[-103.84;3376.0]	
	Bernstein	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	BSplines	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	MinVo	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	ApproxMinVo	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	MinNo	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	MinVa	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	Recursive1	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
	Recursive2	[0.376;0.551]	[0.400;0.563]	[0.839;0.986]	[-6.703;6.434]	[-38.25;-18.37]	[0.388;3.268]	[-13.91;-9.02]	[0.159;0.327]	[-0.156;-0.107]	284.57;1518.63	
[0.495;0.505]	Interval	[0.450;0.478]	[0.466;0.494]	[0.897;0.931]	[-1.258;1.247]	[-30.51;-25.97]	[-1.421;2.002]	[-12.00;-10.82]	[0.2274;0.2588]	[-0.1374;-0.1267]	[792.0;1081.0]	
	Bernstein	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	BSplines	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	MinVo	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	ApproxMinVo	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	MinNo	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	MinVa	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	Recursive1	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	
	Recursive2	[0.455;0.473]	[0.472;0.488]	[0.907;0.921]	[-0.625;0.618]	[-29.19;-27.25]	[1.569;1.851]	[-11.65;-11.17]	[0.235;0.251]	[-0.134;-0.130]	[744.59;856.05]	

Table 24: Results of the end effector position, joint torque and sum of square joint torque for several inputs considering several basis functions.

input	eval	X	Y	Z	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	$\sum \Gamma_i^2$
[-0.5 : 1.5]	Interval	9.89	10.14	10.20	8708.6	6855.7	972.3	937.9	16.05	4.69	67e6
	Bernstein	12.14	12.11	6.68	9575.1	6125.5	1141.5	826.7	13.42	3.87	66e6
	BSplines	12.14	12.11	6.68	11982.4	6889.0	1322.9	958.3	13.42	3.867	198e6
	MinVo	12.14	12.11	6.68	9703.7	6134.7	1144.1	828.52	13.42	3.867	54e6
	ApproxMinVo	12.14	12.11	6.68	9705.3	6134.8	1144.1	828.5	13.42	3.867	54e6
	MinNo	12.14	12.11	6.68	9610.9	6128.1	1142.2	827.2	13.42	3.867	56e6
	MinVa	12.14	12.11	6.68	9575.1	6125.5	1141.5	826.7	13.42	3.867	66e6
	Recursive1	12.14	12.11	6.68	9780.9	6251.9	1173.2	850.8	13.42	3.867	121e6
	Recursive2	12.14	12.11	6.68	9767.6	6243.0	1167.7	846.1	13.42	3.867	121e6
[0 : 1]	Interval	3.38	3.29	3.80	1268.3	1231.4	179.7	322.1	4.26	1.40	1.9e6
	Bernstein	2.621	2.283	1.812	521.3	480.2	90.05	97.93	2.373	0.716	290e3
	BSplines	2.621	2.283	1.812	627.24	505.68	95.98	105.78	2.373	0.716	747e3
	MinVo	2.621	2.283	1.812	532.93	480.32	90.11	97.99	2.373	0.716	275e3
	ApproxMinVo	2.621	2.283	1.812	533.08	480.32	90.11	97.99	2.373	0.716	276e3
	MinNo	2.621	2.283	1.812	524.52	480.22	90.06	97.94	2.373	0.716	268e3
	MinVa	2.621	2.283	1.812	521.27	480.19	90.05	97.93	2.373	0.716	290e3
	Recursive1	2.621	2.283	1.812	523.13	481.94	90.33	98.40	2.373	0.716	435e3
	Recursive2	2.621	2.283	1.812	523.13	481.94	90.33	98.40	2.373	0.716	420e3
[0.25 : 0.75]	Interval	1.503	1.490	1.789	289.3	358.7	50.45	75.40	1.762	0.589	137460.3
	Bernstein	0.974	0.884	0.792	105.3	124.4	21.46	29.43	0.928	0.275	18665.4
	BSplines	0.974	0.884	0.792	127.3	127.3	21.92	30.38	0.928	0.275	44419.3
	MinVo	0.974	0.884	0.792	108.30	124.43	21.46	29.44	0.928	0.275	18127.7
	ApproxMinVo	0.974	0.884	0.792	108.33	124.43	21.46	29.44	0.928	0.275	18152.2
	MinNo	0.974	0.884	0.792	106.13	124.46	21.46	29.44	0.928	0.275	17306.9
	MinVa	0.974	0.884	0.792	105.29	124.43	21.46	29.43	0.928	0.275	18665.0
	Recursive1	0.974	0.884	0.792	105.35	124.52	21.47	29.46	0.928	0.275	30229.1
	Recursive2	0.974	0.884	0.792	105.35	124.52	21.47	29.46	0.928	0.275	24215.7
[0.45 : 0.55]	Interval	0.283	0.285	0.346	27.29	46.10	6.060	11.90	0.315	0.108	3479.8
	Bernstein	0.175	0.164	0.147	13.14	19.88	2.880	4.884	0.168	0.049	1234.1
	BSplines	0.175	0.164	0.147	13.98	19.94	2.886	4.909	0.168	0.049	2505.1
	MinVo	0.175	0.164	0.147	13.26	19.87	2.879	4.880	0.168	0.049	1385.5
	ApproxMinVo	0.175	0.164	0.147	13.26	19.87	2.879	4.880	0.168	0.049	1387.62
	MinNo	0.175	0.164	0.147	13.17	19.87	2.879	4.880	0.168	0.049	1265.49
	MinVa	0.175	0.164	0.147	13.14	19.88	2.880	4.884	0.168	0.049	1234.06
	Recursive1	0.175	0.164	0.147	13.16	19.92	2.883	4.897	0.168	0.049	1690.0
	Recursive2	0.175	0.164	0.147	13.16	19.92	2.884	4.900	0.168	0.049	1325.6
[0.495 : 0.505]	Interval	0.028	0.028	0.0345	2.505	4.549	0.581	1.180	0.031	0.011	289.05
	Bernstein	0.017	0.016	0.014	1.242	1.944	0.282	0.476	0.017	0.0048	111.46
	BSplines	0.017	0.016	0.014	1.278	1.945	0.282	0.477	0.017	0.0048	223.12
	MinVo	0.017	0.016	0.014	1.248	1.944	0.282	0.476	0.017	0.0048	128.33
	ApproxMinVo	0.017	0.016	0.014	1.248	1.943	0.282	0.476	0.017	0.0048	128.54
	MinNo	0.017	0.016	0.014	1.244	1.944	0.282	0.477	0.017	0.0048	116.07
	MinVa	0.017	0.016	0.014	1.242	1.944	0.282	0.476	0.017	0.0048	111.46
	Recursive1	0.017	0.016	0.014	1.242	1.944	0.282	0.476	0.017	0.0048	115.94
	Recursive2	0.017	0.016	0.014	1.242	1.944	0.282	0.478	0.017	0.0048	112.33

Table 25: Diameter of the end effector position, joint torque and sum of square joint torque for several inputs considering several basis functions.

5.2.1 Results of the 3D robot problem

We assess our contribution with 16 different constraint satisfaction problems, with the two bisection processes (MinFirst and MaxFirst) such as presented in Table 4, with n the number of degrees of freedom.

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	1.90735e-05	2453.17	2453.17	40:53	45921593	5.34209e-05	386.241
Bis_Bernstein	66.1749	3665.57	3731.74	01:02:11	11993815	0.000305622	173.024
Bis_MinVariance	73.8135	3722.79	3796.61	01:03:16	11993815	0.000310393	173.024
Bis_BSplines	62.467	23350	23412.5	06:30:12	77244655	0.000302287	174.267
Bis_MinNo	64.6503	4035.9	4100.55	01:08:20	12997957	0.000310503	173.063
Bis_ApproxMinVo	71.1042	4859.22	4930.33	01:22:10	15817603	0.000307203	173.174
Bis_MinVo	61.0809	5002.66	5063.75	01:24:23	15742779	0.000317775	173.172
Bis_RecursiveBSplines	78.3969	3696.23	3774.62	01:02:54	12006073	0.000307863	173.024
Bis_RecursiveBSplines2	78.2053	3698.18	3776.39	01:02:56	12094871	0.000305765	173.024

Table 27: problem 3D : bisection:MinFirst problem:1

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	3.09944e-05	2921.78	2921.78	48:41	53357881	5.47581e-05	1672.95
Bis_Bernstein	66.0452	1152.29	1218.33	20:18	3763345	0.000306187	1454.08
Bis_MinVariance	73.9706	1178.46	1252.43	20:52	3763345	0.000313143	1454.08
Bis_BSplines	62.359	4574.38	4636.74	01:17:16	15017501	0.000304603	1456.89
Bis_MinNo	65.7517	1306.28	1372.03	22:52	4060855	0.000321676	1454.17
Bis_ApproxMinVo	70.5401	1457.91	1528.45	25:28	4752739	0.000306752	1454.42
Bis_MinVo	59.3938	1457.7	1517.09	25:17	4737025	0.000307725	1454.42
Bis_RecursiveBSplines	78.9049	1190.84	1269.75	21:09	3770341	0.000315845	1454.08
Bis_RecursiveBSplines2	77.7705	1162.92	1240.69	20:40	3771941	0.000308308	1454.08

Table 28: problem 3D : bisection:MinFirst problem:2

problem	n	center of the target	α	criteria
1	4	{0.6; 0.6; 0.6}	1.0	$\sum_i q_i^2$
2	4	{0.4; 0.4; 0.7}	1.0	$\sum_i q_i^2$
3	4	{0.6; 0.6; 0.6}	0.1	$\sum_i q_i^2$
4	4	{0.4; 0.4; 0.7}	0.1	$\sum_i q_i^2$
5	6	{0.6; 0.6; 0.6}	1.0	$\sum_i q_i^2$
6	6	{0.4; 0.4; 0.7}	1.0	$\sum_i q_i^2$
7	6	{0.6; 0.6; 0.6}	0.1	$\sum_i q_i^2$
8	6	{0.4; 0.4; 0.7}	0.1	$\sum_i q_i^2$
11	4	{0.6; 0.6; 0.6}	1.0	$\sum_i \Gamma_i^2$
12	4	{0.4; 0.4; 0.7}	1.0	$\sum_i \Gamma_i^2$
13	4	{0.6; 0.6; 0.6}	0.1	$\sum_i \Gamma_i^2$
14	4	{0.4; 0.4; 0.7}	0.1	$\sum_i \Gamma_i^2$
15	6	{0.6; 0.6; 0.6}	1.0	$\sum_i \Gamma_i^2$
16	6	{0.4; 0.4; 0.7}	1.0	$\sum_i \Gamma_i^2$
17	6	{0.6; 0.6; 0.6}	0.1	$\sum_i \Gamma_i^2$
18	6	{0.4; 0.4; 0.7}	0.1	$\sum_i \Gamma_i^2$

Table 26: 3D Problem list and their specificities.

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	1.90735e-05	2415.29	2415.29	40:15	45920069	5.25977e-05	386.241
Bis_Bernstein	66.1389	3655.99	3722.13	01:02:02	11933629	0.000304828	173.024
Bis_MinVariance	73.8341	3732.19	3806.02	01:03:26	11933629	0.000311181	173.024
Bis_BSplines	63.705	24084.8	24148.5	06:42:28	77240097	0.000311817	174.267
Bis_MinNo	63.7855	3966.32	4030.1	01:07:10	12997669	0.000305156	173.063
Bis_ApproxMinVo	71.2538	4959.31	5030.57	01:23:50	15817015	0.000313543	173.174
Bis_MinVo	60.0333	4838.52	4898.56	01:21:38	15742189	0.00030736	173.172
Bis_RecursiveBSplines	78.1443	3708.46	3786.6	01:03:06	12005185	0.000308905	173.024
Bis_RecursiveBSplines2	78.0199	3719.12	3797.14	01:03:17	12093847	0.000307522	173.024

Table 29: problem 3D : bisection:MinFirst problem:3

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	2.09808e-05	1.19146	1.19146	1:191	22761	5.23463e-05	-1
Bis_Bernstein	66.3459	0.811293	67.1572	01:07	2565	0.000316294	-1
Bis_MinVariance	73.7091	0.834249	74.5434	01:14	2565	0.000325243	-1
Bis_BSplines	62.5166	1.06273	63.5794	01:03	3361	0.000316194	-1
Bis_MinNo	63.8336	0.832965	64.6666	01:04	2635	0.000324996	-1
Bis_ApproxMinVo	70.4124	0.840615	71.2531	01:11	2635	0.000319019	-1
Bis_MinVo	58.9792	0.850288	59.8295	59:830	2635	0.00032269	-1
Bis_RecursiveBSplines	79.0131	0.902163	79.9153	01:19	2819	0.000320029	-1
Bis_RecursiveBSplines2	77.1458	0.905783	78.0516	01:18	2823	0.000320858	-1

Table 30: problem 3D : bisection:MinFirst problem:4

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	4.1008e-05	235.058	235.058	03:55	4274953	5.49848e-05	2.612
Bis_Bernstein	65.2279	379.346	444.574	07:24	1268835	0.000298972	2.60843
Bis_MinVariance	71.5167	377.748	449.265	07:29	1268841	0.000297711	2.60843
Bis_BSplines	60.0738	927.637	987.71	16:27	3197661	0.000290098	2.61001
Bis_MinNo	61.8669	395.381	457.248	07:37	1327941	0.00029774	2.6085
Bis_ApproxMinVo	69.2628	445.225	514.488	08:34	1493577	0.000298093	2.60867
Bis_MinVo	57.8982	444.268	502.167	08:22	1490695	0.000298028	2.60867
Bis_RecursiveBSplines	75.5686	376.723	452.292	07:32	1271137	0.000296367	2.60843
Bis_RecursiveBSplines2	75.237	378.421	453.658	07:33	1270867	0.000297766	2.60843

Table 31: problem 3D : bisection:MinFirst problem:11

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	2.00272e-05	485.34	485.34	08:05	9329531	5.20219e-05	1.70478
Bis_Bernstein	63.5205	731.934	795.455	13:15	2505349	0.000292149	1.70291
Bis_MinVariance	71.6537	745.622	817.276	13:37	2505385	0.000297608	1.70291
Bis_BSplines	62.4694	2087.75	2150.22	35:50	6934689	0.000301016	1.70433
Bis_MinNo	61.6404	783.424	845.064	14:05	2632737	0.00029757	1.70297
Bis_ApproxMinVo	70.5762	918.946	989.522	16:29	3006751	0.000305627	1.70313
Bis_MinVo	58.115	885.339	943.454	15:43	3000011	0.000295112	1.70313
Bis_RecursiveBSplines	76.1085	744.322	820.431	13:40	2508047	0.000296774	1.70291
Bis_RecursiveBSplines2	74.6647	759.027	833.691	13:53	2509483	0.000302463	1.70291

Table 32: problem 3D : bisection:MinFirst problem:12

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	3.60012e-05	240.518	240.518	04:00	4462989	5.38916e-05	2.612
Bis_Bernstein	63.4891	369.828	433.317	07:13	1268625	0.000291519	2.60843
Bis_MinVariance	71.5096	378.43	449.94	07:29	1268631	0.000298298	2.60843
Bis_BSplines	60.9946	950.716	1011.71	16:51	3266255	0.000291072	2.61001
Bis_MinNo	61.6341	395.885	457.519	07:37	1329487	0.000297773	2.6085
Bis_ApproxMinVo	68.3627	445.288	513.651	08:33	1499533	0.000296951	2.60867
Bis_MinVo	57.9794	441.445	499.425	08:19	1496661	0.000294953	2.60867
Bis_RecursiveBSplines	76.1437	377.01	453.153	07:33	1270953	0.000296635	2.60843
Bis_RecursiveBSplines2	74.2824	377.174	451.456	07:31	1270731	0.000296817	2.60843

Table 33: problem 3D : bisection:MinFirst problem:13

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
Bis_MinVariance	73.7124	2386.41	2460.12	41:00	7697123	0.000310039	173.024
Bis_BSplines	62.3734	19304.5	19366.9	05:22:46	63879675	0.000302201	174.267
Bis_ApproxMinVo	71.8038	3394.93	3466.73	57:46	11057339	0.00030703	173.174
Bis_MinVo	59.6567	3369.41	3429.07	57:09	10983071	0.000306783	173.172
Bis_RecursiveBSplines	77.712	2394.19	2471.9	41:11	7747365	0.000309033	173.024
Bis_RecursiveBSplines2	78.8786	2401.6	2480.48	41:20	7710273	0.00031148	173.024

Table 35: problem 3D : bissection:MaxFirst problem:1

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	2.00272e-05	3479.55	3479.55	57:59	65152871	5.34059e-05	1672.95
Bis_Bernstein	66.4963	1404.73	1471.23	24:31	4526351	0.000310345	1454.08

Table 36: problem 3D : bissection:MaxFirst problem:2

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	2.09808e-05	2074.35	2074.35	34:34	36393735	5.69974e-05	386.241
Bis_Bernstein	66.1736	2372.74	2438.92	40:38	7674961	0.000309154	173.024
Bis_MinVariance	75.5404	2458.49	2534.03	42:14	7674961	0.000320326	173.024
Bis_BSplines	62.3066	19322	19384.3	05:23:04	63832325	0.000302699	174.267
Bis_MinNo	63.8714	2591.84	2655.71	44:15	8396017	0.000308698	173.063
Bis_ApproxMinVo	71.153	3392.31	3463.46	57:43	11033453	0.000307457	173.174
Bis_MinVo	59.4465	3343.39	3402.84	56:42	10959203	0.000305076	173.172

Table 37: problem 3D : bissection:MaxFirst problem:3

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	2.31266e-05	1.19815	1.19815	1:198	22761	5.26404e-05	-1
Bis_MinVariance	76.0539	0.880963	76.9349	01:16	2565	0.000343455	-1
Bis_RecursiveBSplines	79.0002	0.938215	79.9384	01:19	2819	0.000332818	-1

Table 38: problem 3D : bissection:MaxFirst problem:4

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	2.21729e-05	69.0065	69.0065	01:09	1316949	5.23987e-05	2.612
Bis_Bernstein	63.593	112.334	175.927	02:55	385333	0.000291525	2.60843
Bis_MinVariance	71.4541	115.033	186.488	03:06	385333	0.00029853	2.60843
Bis_MinNo	62.5039	121.34	183.844	03:03	407965	0.000297428	2.6085
Bis_ApproxMinVo	70.3861	144.65	215.036	03:35	473155	0.000305713	2.60867
Bis_MinVo	57.7641	137.992	195.756	03:15	472011	0.000292349	2.60867
Bis_RecursiveBSplines	75.6969	115.238	190.935	03:10	386043	0.000298511	2.60843
Bis_RecursiveBSplines2	74.9781	114.591	189.569	03:09	387207	0.000295943	2.60843

Table 39: problem 3D : bissection:MaxFirst problem:11

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	1.90735e-05	200.832	200.832	03:20	3742187	5.3667e-05	1.70478
Bis_Bernstein	63.45	315.149	378.599	06:18	1085139	0.000290423	1.70291
Bis_BSplines	59.9882	991.575	1051.56	17:31	3433329	0.000288809	1.70433
Bis_MinNo	61.6672	342.26	403.927	06:43	1152989	0.000296846	1.70297
Bis_ApproxMinVo	68.4858	398.768	467.253	07:47	1342405	0.000297055	1.70313
Bis_RecursiveBSplines	76.1471	319.104	395.251	06:35	1085835	0.000293878	1.70291
Bis_RecursiveBSplines2	75.0553	324.652	399.707	06:39	1089149	0.000298078	1.70291

Table 40: problem 3D : bissection:MaxFirst problem:12

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	2.19345e-05	97.9807	97.9807	01:37	1859429	5.2694e-05	2.612
Bis_Bernstein	64.1402	128.805	192.945	03:12	442429	0.000291132	2.60843
Bis_MinVariance	71.3428	130.779	202.122	03:22	442435	0.000295589	2.60843
Bis_BSplines	61.707	393.005	454.712	07:34	1318089	0.000298163	2.61001
Bis_ApproxMinVo	68.6212	160.827	229.448	03:49	546743	0.000294154	2.60867
Bis_MinVo	59.1707	163.841	223.012	03:43	545443	0.000300381	2.60867
Bis_RecursiveBSplines2	74.5728	131.419	205.991	03:25	444349	0.000295755	2.60843

Table 41: problem 3D : bissection:MaxFirst problem:13

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BissectionInterval	1.90735e-05	1.19152	1.19152	1:192	22761	5.23492e-05	-1
Bis_MinVariance	71.5355	0.797958	72.3335	01:12	2565	0.000311095	-1
Bis_BSplines	60.0389	1.02575	61.0647	01:01	3361	0.000305192	-1
Bis_MinNo	61.5572	0.795066	62.3523	01:02	2563	0.000310209	-1
Bis_ApproxMinVo	68.7098	0.843484	69.5533	01:09	2635	0.000320108	-1
Bis_MinVo	57.8922	0.805592	58.6978	58.698	2635	0.000305728	-1
Bis_RecursiveBSplines	75.5932	0.873919	76.4672	01:16	2819	0.00031001	-1
Bis_RecursiveBSplines2	75.9706	0.894856	76.8655	01:16	2823	0.000316988	-1

Table 42: problem 3D : bissection:MaxFirst problem:14

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	2.09808e-05						

Table 43: problem 3D : bisection:MinFirst problem:5

solver	prep_time	comput_time	total_time	(D-H:M:S.ms)	nb_iter	time_per_iter	criteria
BisectionInterval	2.09808e-05						
Bis_Bernstein	50.7193						
Bis_ApproxMinVo	54.479						

Table 44: problem 3D : bisection:MinFirst problem:6

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