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INFINITY AND THE SELF: ROYCE ON DEDEKIND

Sébastien Gandon

In *Die Zahlen* (1888), Dedekind defines an infinite set as a set that is isomorphic with one of its proper parts. In *The World and the Individual* (1900), the American philosopher Josiah Royce relates Dedekind's notion to Fichte's and Hegel's concept of Self defined as an entity that reflects itself into itself. The first aim of this article is to explain Royce's analysis and to put it in its proper context, that of a critique of Bradley's mystical idealism. The second aim is to urge a shift in focus in Dedekind's scholarship: instead of addressing the question of the relationship between mathematics and philosophy in Dedekind's work through the supposed intentions of its author, it is more fruitful to analyze the reception that philosophers have made of his texts.

The spoken word belongs half to him who speaks, and half to him who listens. (Michel de Montaigne, *Essay*, III, 13)

1. Introduction

In the *Supplementary Essay of The World and the Individual*, the American philosopher Josiah Royce (1855–1916) explains that he wants “to bring Dedekind's research into its proper relationship to general metaphysical inquiry” (1900, 527). This phrase is puzzling, because Richard Dedekind (1831–1916) is well known as one of the first “modern” mathematicians—modern in the sense that he was taking great care to expurgate his writing of all metaphysical considerations, and of anything that was not strictly mathematical. The whole construction of

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Dedekind's (1888) work, which Royce is referring to here, is based on rigorous definitions, theorems, and proofs at a time and on a subject where such a practice was not commonplace. If Dedekind's developments aim at self-sufficiency, why then is Royce planning to articulate them to "general metaphysical inquiry"?

Royce's target is the masterpiece of the British philosopher Francis Herbert Bradley (1893/1916). But as a matter of fact, Bradley and Royce were philosophically close: Bradley was the main leader of Absolute Idealism (the family of positions that in the nineteenth century drew inspiration from the works of Hegel, Fichte, and Schelling) in the UK, a role Royce could legitimately claim in America.¹ Absolute Idealism has a bad reputation in the history and philosophy of science. When they were not frankly anti-intellectualists and even hostile to the sciences, as Bradley was, many of the idealists considered that mathematics represents only a degraded (at best preliminary) stage of rationality. Furthermore, Absolute Idealists are often regarded as ignorant when it comes to science.

Even if this diagnosis is not unfounded, it would be wrong to extend it to all the philosophers who subscribe to this line.² Royce, in particular, was an exception. He had many interactions with his mathematician colleagues during his career, including as professor of philosophy at Harvard from 1884 until his death in 1916. In the preface of his 1900 work, he thanks Maxime Bôcher and William F. Osgood for their "remarks concerning specifically mathematical topics" (xiii), and he pursued his collaboration with them and Edward V. Huntington afterwards. Royce was interested in logic early on in his career, but Royce's attendance, in 1898, at a series of lectures by Charles S. Peirce at Harvard significantly influenced his understanding of the relation between logic and metaphysics (see Parker 2012). In 1905, he published a long and a quite technical paper generalizing Alfred Kempe's multiset theory and geometry.³ But above all, it is as a

1. It would be a mistake to give too strong a unity to post-Kantian or Absolute Idealism. Royce (1919, 2) states: "In more recent times, post-Kantian idealism, influencing thought in France, in England, and in this country, has led to a complication of opinions that it would require many courses of lectures to unravel. A list of those who, with more or less obvious justice, might be called in some sense post-Kantian idealists, would include Cousin, Strauss, Fechner, Lotze, von Hartmann, T. H. Green, Bradley, and even Martineau, despite his pronounced hostility to Hegelianism. And, in a measure, most of our own American pragmatists could be viewed as the outcome of the same movement. Where such varieties of opinion are in question, there is no longer any reason to speak of a school at all."

2. Absolute Idealism played a role in the emergence of some new mathematical ideas. Grassmann's mathematical work was, for instance, inspired by Schleiermacher. For a lesser known contribution, see the fascinating study of Timmermans (2012) on the idealist roots of modern algebra and crystallography.

3. See Royce (1905). A good summary is given in Lewis (1918, 362–72). Numerous logical and mathematical manuscripts are still kept in the archives and have not received the attention they deserve. Compared to Peirce and James, Royce has not been much studied, and his scientific and epistemological work is certainly the part that has been the most neglected.

teacher and as a PhD supervisor that Royce contributed to logic. As Grattan-Guinness (2000, 567) rightly emphasized, Royce must be regarded as an important source of logical research in America: among his students were Henry Sheffer, Clarence Irving Lewis, Norbert Wiener, Ralph M. Eaton, and Morris R. Cohen.⁴ Thus, the Absolute Idealist Royce played a key institutional role in the constitution of an American logical community and in the promotion of science more generally.⁵

Royce's main goal in the *Essay* is precisely to criticize Bradley's anti-intellectualism. Bradley considered that mathematics cannot avoid falling into insurmountable contradictions, and must be left behind to reach the Absolute. On the contrary, Royce claimed that modern mathematics is an instrument for clarifying metaphysical ideas: "the metaphysics of the future," said he, "will take fresh account of mathematical research" (1900, 527). To be more specific, in the *Supplementary Essay*, Royce explains that Dedekind's definition of an infinite system in *Die Zahlen* gives us the means to articulate Fichte's and Hegel's conception of the Absolute as a Self in a rigorous way. And he claimed, against Bradley, that this conception of the Absolute as a Self is the keystone of Absolute Idealism. Far from being an obstacle, the Dedekindian definition of an infinite system would allow us to articulate in an intelligible way the nature of the Absolute Self.

But my last sentence is precisely of the kind that Dedekind scholars cannot listen to without shuddering. Connecting the unbridled metaphysics of Fichte and Hegel with the rigorous and sober mathematics of Dedekind is like trying to mix oil with vinegar.⁶ What connection can there be between an outdated metaphysical doctrine on the nature of the Absolute and *Die Zahlen's* definition 64? Wouldn't it be more reasonable, in order to avoid a dubious mixing of genres, to separate completely (as Bradley wished) the work of Dedekind the mathematician from the musings of the metaphysicians?

In the literature, Dedekind's metaphysical position has been a topic of debates (see sec. 3, below). I do not want, however, to engage in that discussion in this paper. Indeed, I would like to suggest that focusing on Dedekind's intentions is not the best way to deal with the issue of the relations between mathematics and metaphysics in *Die Zahlen*. Historians and bibliographers have taught us that the contexts of reception, variable and changing, must, as much

4. Russell took Royce seriously: he is one of the few to whom he sent the first edition of *Principia Mathematica*. Eaton published a *General Logic* in 1931. Cohen co-authored the *Introduction to Logic and Scientific Method* with Ernst Nagel.

5. Josiah's wife, Katharine, translated Enriques's *Problems of Science* into English in 1914, and he wrote a preface for the book.

6. Dedekind's only known comment about Fichte is negative. See Dugac (1976, 157).

as the contexts of production, be taken into account when studying a text (see Chartier 1989, 2013; McKenzie 2004). And the idea that focusing on multiple appropriations and readings enriches our understanding of the original work has been particularly developed in the history of mathematics (Goldstein 1993; 1995). Why, then, exclude from the field of investigation of the historian the early reception of *Die Zahlen* by the idealist readers? It is in this spirit that the following article is written: I will focus on the process of appropriation of Dedekind's thought by Royce and his followers, bracketing any question relating to the philosophical background and motivations (if he had any) of Dedekind himself.⁷

In section 2, I give some information about Dedekind's definition of the infinite system, and on Royce's own formulation. In section 3, I focus on Dedekind's proof of Theorem 66, because this proof plays an important role in Royce's reading. In section 4, I summarize in broad outline what project Royce is pursuing in *The World and the Individual*. In section 5, I focus on the *Supplementary Essay* and explain the connection Royce made between Dedekind's notion of infinite system and his anti-Bradleyan conception of the Self. In section 6, I return to the methodological issue to argue that a genuinely historical perspective on Dedekind cannot leave aside its Roycean meta-physical posterity.

2. Dedekind's Definition of the Infinite and Royce's Self-Representative System

Dedekind (1888) defines the sequence of integers from the concepts of set (*System*) and mapping (*Abbildung*). If it has nothing extraordinary today, this project was unprecedented then, and its reception by the mathematical community would take some time (see Dugac 1976, 93–96). On the notion of “System” we will say nothing, except that a set, for Dedekind, is not necessarily finite. The concept of a mapping requires some clarification. An “Abbildung” φ of the system S is a “law” that associates, to each element s of S , an element $\varphi(s)$ of S , called the image of s . Apart from the reference to the “law,” we find here the usual notion of a map φ of S into itself. In section 3 (Def. 26), Dedekind introduces the notion of *ähnliche Abbildung* in this way: φ from S to S is similar if and only if it never maps distinct elements of S to the same image (i.e., if and only if φ is an injective mapping). As he considers that φ is a

7. In Gandon (2009), I introduced Royce's interpretation to a French-speaking readership. At the end of my paper, I raised the issue of the historical accuracy of Royce's interpretation: would Dedekind have subscribed to this reading? My perspective is different in this article.

map from S to $\varphi(S)$, Dedekind in fact works with bijective mappings. Dedekind extends the concept of similarity to sets, and says that, when φ (from S to S) is similar, then S and $\varphi(S)$ are similar. Thus, two sets S_1 and S_2 are similar when there is a similar map φ such that $S_2 = \varphi(S_1)$ —or in a more familiar way, S_1 and S_2 are similar when there is a bijective mapping that relate each element of S_1 to each element of S_2 . The relation of similarity between sets is reflexive, symmetrical and transitive, and thus, similar classes form exclusive equivalence classes.

In *Die Zahlen*, section 4, Dedekind uses these concepts to define the structure \mathbf{N} . For the sake of convenience, I will not follow Dedekind's presentation but the way he formulates his reasoning in the letter to Keferstein (see van Heijenoort 1967, 98–103). Dedekind begins by defining an infinite system, that is, a structure $\langle S, \varphi \rangle$, which satisfies the following three conditions:

1. $\varphi(S) \subset S$.
2. φ is similar.
3. There is an element in S (call it “1”) such that $1 \notin \varphi(S)$.

Condition 3 means that there is an element in S , which does not belong to $\varphi(S)$, and then that $\varphi(S)$ is a proper part of S . According to Definition 64 in Dedekind's *Zahlen*, a set S satisfying the three conditions together is infinite: “A system S is said to be infinite when it is similar to a proper part of itself.”

Now, the set of integers satisfies the three constraints: the mapping $\varphi(n) = n + 1$, which associates to each integer its successor, is similar, and there is an element in \mathbf{N} —namely, 1, which does not belong to the set of successors. As Dedekind notes, these three conditions do not suffice to characterize \mathbf{N} , because they do not guarantee the induction principle. The last condition that Dedekind introduces is based on his definition of a chain. As Reck (2020) explains, a chain is the minimal closure of a set A (contained in B) under a function φ on B (where being “minimal” is conceived of in terms of the notion of intersection). The fourth condition is the following one:

4. S is the chain of $\{1\}$ under φ .

A set S that satisfies the four conditions is called, according to Definition 71 of *Die Zahlen*, a simply infinite set.⁸ A simply infinite set is isomorphic to \mathbf{N} , and it is infinite. But the converse is, of course, not true: some infinite sets are not simply infinite, and thus not isomorphic to \mathbf{N} . Royce is only interested in

8. Dedekind's four conditions are equivalent to Peano axioms.

the notion of infinite set, but he is not always rigorous, and he sometimes refers to the structure of integers (i.e., simply infinite set) while talking about the structure of an infinite set.

Indeed, according to Royce, what he calls a “self-representative system” satisfies the three following conditions:

1. The system is such that to every ideal element in it, M , M' or, in general, $M^{(n)}$, there corresponds one and only one other element of the system, which, taken in its order, is the next element of the system. This next element may be viewed, if we choose, as derived from its predecessor by means of the recurrent process. But it may also be viewed as in a relation to its predecessor, which is the same as the relation of a map to an object mapped. We shall accordingly call it, henceforth, the Image or Representation of this former element.
2. These images are all distinct, so that various elements always have various representatives. For the recurrent process is such that, in the system that should finally express it, one and only one element would be derived from any given element, or would be the next element in order after that given element.
3. At least one element, M , of the system, although imaged by another, is itself the image or representative of no other element, so that only a portion of the system is representative. (1900, 508–9)

Royce’s verbose style is different from Dedekind’s rigorous and lapidary prose. But the conditions above are just a rephrasing of Dedekind’s three postulates. A self-representative system S is a system that is “precisely represented by a proper fraction or portion of itself” (Royce 1900, 509), that is, a Dedekind-infinite system. But, as the phrasing of condition 1 shows, Royce tends to see the mapping φ as the relation of succession in a “recurrent process,” and sometimes, Royce seems to consider that a self-representative system S is the *Kette* (chain) of its first element.⁹ This lack of terminological clarity does not seem to create any serious mistakes, however. Royce is always speaking about infinite systems, never about simply infinite systems (even when his words suggest otherwise).¹⁰

His concept of self-representativity is illustrated by a beautiful image:

9. See, e.g., the passages quoted in n. 29 and sec. 5 (where the Self is compared to the “number series”).

10. Helm (1973) claims that Royce did not understand Dedekind, but Helm’s argument seems to be grounded on terminological inaccuracies. Thus, it seems that an easy way to deal with Royce’s “inaccurate” passages about infinite and simply infinite systems is to consider that Royce is referring to an infinite system, of which the simply infinite system he is talking is a proper part.

To fix our ideas, let us suppose, if you please, that a portion of the surface of England is very perfectly levelled and smoothed, and is then devoted to the production of our precise map of England. That in general, then, should be found upon the surface of England, map constructions that more or less roughly represent the whole of England, all this has nothing puzzling about it. Any ordinary map of England spread out upon English ground would illustrate, in a way, such possession, by a part of the surface of England, of a resemblance to the whole. But now suppose that this our resemblance is to be made absolutely exact, in the sense previously defined. A map of England, contained within England, is to represent, down to the minutest detail, every contour and marking, natural or artificial, that occurs upon the surface of England. At once our imaginary case involves a new problem. This is now no longer the general problem of map making, but the nature of the internal meaning of our new purpose. (Royce 1900, 504)

Here, S is the set of points belonging to the territory of England, and φ the mapping that associates to each point of S its image in the map. The map is supposed to be drawn on a portion of the surface of England (condition 1), it is supposed to be exact (condition 2), and such that it does not recover the whole territory of England (condition 3). Then, the map will involve in itself an infinite repetition of the same pattern: “For the map, in order to be complete, according to the rule given, will have to contain, as a part of itself, a representation of its own contour and contents. In order that this representation should be constructed, the representation itself will have to contain once more, as a part of itself, a representation of its own contour and contents; and this representation, in order to be exact, will have once more to contain an image of itself; and so on without limit” (1900, 504–5).

What Royce insists on is that Dedekind has found a “positive” definition of infinity for the first time. An infinite set is not a set that has no end, a magnitude that can always be extended, like a geometrical line, or like the series of integers. The infinity of a self-representative system is merely the consequence of a “single internal purpose,” namely, of the three general conditions listed above.¹¹

11. Royce (1900, 509): “The system is, therefore, defined as endless merely by being defined as thus self-representative. But since the self-representation of any system of facts is capable of definition, as a single internal purpose, in advance of the discovery that such purpose involves an endless series of constituents, we may, with Dedekind, use the generalized conception of a self-representation of the type here in question as a means of positively defining what we mean by an infinite system or multitude of elements.”

3. Theorem 66

Definition 64 raises a problem: how to show that the conditions 1, 2, and 3 are not mutually contradictory. How do we know that the concept of infinite system is possible? To meet the challenge, Dedekind presents in his famous Theorem 66 of *Die Zahlen* a “proof” that there is an infinite set. As we will see, Dedekind’s reasoning raises several challenges that have puzzled scholars for a long time: Dugac (1976, 87–88) says that Theorem 66 is “the only theorem of Dedekind whose “proof” does not fit with Dedekind’s mathematical thought.” I will deal briefly in this section with the way the recent literature addresses this issue, because Theorem 66 plays an important role in Royce’s reading. Here is Beman’s translation of the passage:

The world of my thoughts [*Gedankenwelt*], i.e., the totality S of all things that can be objects of my thought, is infinite. For if s denotes an element of S , then the thought s' , that s can be an object of my thought, is itself an element of S . If s' is regarded as the image $\varphi(s)$ of the element s , then the mapping φ on S determined thereby has the property that its image S' is a part of S and indeed S' is a proper part of S , because there are elements in S (e.g., my own ego), which are different from every such thought s' and are therefore not contained in S' . Finally it is clear that if a, b are different elements of S , then their images a', b' are also different, so that the mapping φ is distinct (similar). Consequently, S is infinite, q.e.d. (Dedekind 1901, 31)

Let me explain Dedekind’s idea. The mapping φ associates to each object of my thought s , the image $\varphi(s)$ “ s can be an object of my thought.” That is, $\varphi(s)$ is the reflexive thought, “ s can be an object of my thought.” This reflexive thought is itself a (second-order) object that can generate, in turn, a new reflexive thought, and so forth. Dedekind maintains that $\varphi(s)$ belongs to S (condition 1), that the identity of the reflexive thought $\varphi(s)$ depends on the identity of s (condition 2), and that there is an object x in S that is not a reflexive thought, namely (at least) my own ego (condition 3). Thus, Dedekind claims that the world of my thoughts is an infinite system: the self has the capacity to reflect itself onto a proper part of itself, and as the self exists, at least one infinite system exists. Definition 64 is then not contradictory.

Dedekind’s reasoning has been much analyzed and widely criticized.¹² The reference to metaphysical, extra-mathematical notions such as “world of thought,”

12. See Klev (2018, 262–66) for a panorama of these reactions.

“my own ego,” and so forth—unparalleled in the Dedekindian corpus—seems odd for an author who attached such importance to the issue of purity (see van Heijenoort 1967, 98–103). Moreover, Cantor showed that the set of all my thoughts, as it is described in Theorem 66, is an inconsistent multiplicity.¹³ Dedekind’s purported proof is then flawed: in standard set theory today, a postulate guarantees the existence of an infinite set. Zermelo, who first set such an axiom, explicitly referred to Dedekind’s Theorem 66 to explain its formulation.¹⁴

In a footnote to Theorem 66, Dedekind explains that a “similar consideration” can be found in Bolzano’s *Paradoxien des Unendlichen*. But in the preface to the second edition, Dedekind claims that he had the idea of an infinite system “at a time when the work of Bolzano was unknown to [him] even by name.” Even if this statement relates only to definition 64 and not to the proof of Theorem 66, some scholars took this opportunity to challenge the consensus that Dedekind would have drawn on Bolzano.¹⁵ An additional argument that goes in their sense is that certain features of Dedekind’s proof are foreign to Bolzano’s reasoning, notably the reference to the self and to the *Gedankenwelt*. Thus, Klev claims that the occurrence of this last term points to Lotze rather than to Bolzano.¹⁶ In any case, what is certain is that “the claim and proof for the existence of an infinite system occur first . . . in the fourth section of [the 1887 manuscript]” (Sieg and Schlimm 2005, 149)—that is, very late in the long process of meditation that results in the publication of *Die Zahlen* in 1888. As Dedekind explains in his preface: “[the draft dated from 1872/78] bears the same title and contains, though not arranged in the best order, all the essential fundamental ideas of my present paper, in which they are more carefully elaborated” (1901, 14). This seems to indicate that the “proof” of the Theorem 66 was a late addition, which was not regarded by Dedekind as one of “the essential fundamental ideas” of the book.¹⁷

In view of the extremely unusual character, for Dedekind, of the “proof” of Theorem 66, and of the fact that it is a late addition, could one not downplay

13. On Cantor’s objection to Dedekind, see Klev (2018, 263–65).

14. Recall that, in Zermelo (1908), the axiom of infinity asserts the existence of a set Z such that (i) $\emptyset \in Z$, and (ii) $\{a\} \in Z$ whenever $a \in Z$. As Klev (2018, 265) says: “The inspiration of Dedekind’s proof on the formulation of this axiom should be obvious: the empty set takes the place of the self and the mapping $a \rightarrow \{a\}$. . . the place of Dedekind’s mapping φ .”

15. See, e.g., Sieg and Schlimm (2005); Klev (2018, 246–49). For a presentation of the position that was once consensual, see Dugac (1976, 84, 88–89; Ferreirós 1999, 243–46).

16. Dedekind followed a course from Lotze (published as Lotze [1882]) when he was still a student. On Lotze and Dedekind, see Sieg and Morris (2018); Gandon (2009).

17. Dugac (1976, 91) remarks that the early manuscripts bear in the margin “construction of a simply infinite sequence,” which suggests that Dedekind initially wanted to “construct” a simply infinite set before deciding to prove the existence of an infinite set.

the importance of these few lines in the work? As Benis-Sinaceur and Haffner (2020, 184) note, “mathematicians quickly forgot this proposition 66 whose fate was settled for them by Zermelo’s axiom of infinity [and] historians, for the most part, restrict themselves to explaining the reasons and circumstances of its late genesis”; only philosophers and logicians “have never ceased to be interested in it, perhaps precisely because of its shortcomings.” With respect to Theorem 66, one can indeed distinguish two branches in Dedekind’s scholarship: there are those who see the flawed “proof” as an unfortunate accident, a rare and easily forgivable misstep within a body of work whose mathematical rigor meets the highest standards.¹⁸ There are also those who consider Theorem 66 to be important because it is a window on Dedekind’s philosophical background. For instance, McCarty (1995) and Klev (2018) rely heavily on Theorem 66 to try to uncover Dedekind’s metaphysical commitments (the first relates Dedekind to Kant, and the second to Lotze). And it seems that the attitude that experts adopt towards Dedekind’s flawed “proof” is a good indication of how they conceive the articulation of mathematics and metaphysics in *Die Zahlen*. Some adopt the idea that we are witnessing in Dedekind (e.g., more so than in Cantor) a complete autonomization of mathematical reflection from all metaphysical considerations, and that Theorem 66 does not reflect this “structural” turn at all.¹⁹ The others think that the Dedekindian rupture must be situated within the metaphysical environment of his time, and that Theorem 66, even if its proof is flawed, helps us to discover Dedekind’s hidden metaphysical assumptions.

In this debate, I agree with the first family of commentators: it seems to me that the traces left by Dedekind in Theorem 66 or elsewhere are too thin to allow us to reconstruct any coherent metaphysical position that would underlie his mathematical work. I share Dugac’s bold claim that the “proof” of Theorem 66 “does not fit with Dedekind’s mathematical thought.” However, I do not think that this part of *Die Zahlen* should be put in brackets and forgotten. What I want to document in this article is that Theorem 66, whatever the intention of the author who wrote it, was put to metaphysical use in nonmathematical circles soon after its publication, in particular in Royce’s work. I see Theorem 66, then, not as an embarrassing passage that is best left out to preserve the purity of the work, nor as one of the few perspectives one has on Dedekind’s metaphysical views. I see it as a thread loosely connected to the rest of the book that can, however, when placed in an appropriate environment, attach to *Die Zahlen* a new network of concepts, much as a fishing net, hanging

18. In this family I would put Dugac (1976), Benis-Sinaceur (2008), and Sieg and Schlimm (2005).

19. For more on this opposition between Dedekind and Cantor, see Dieudonné’s preface to Dugac 1976.

behind a boat, can trap many fish. Using Theorem 66 as a trigger, Royce inserted Dedekind's 1888 book into a context that Dedekind certainly didn't know about, probably didn't care about, yet that fit the German mathematician's words particularly well. It is to Royce's metaphysical appropriation of Dedekind's work that I will now turn.

4. Royce against Bradley: The Absolute as an Individual

The philosophy of religion is certainly the heart of Royce's thought: "What is God or the Absolute?" is clearly Royce's fundamental issue.²⁰ With Bradley and the German Idealists, Royce shares the idealist assumption that philosophy must start with the Absolute, not with the finite Self. But Royce's Absolute is not Bradley's undifferentiated whole; it is an individual that is composed of finite individuals (it is an "Individual of Individuals," a "Self of many Selves"). Indeed, for Royce, the absolute self needs the infinite diversity of finite individualities to be itself. In this respect, Royce does not endorse Bradley's monism but is close to the pluralist positions of his friend William James: Royce's God can be seen as the reification of the "something larger than ourselves" that, for James, we, as finite selves, feel union with in our religious experiences.²¹ Royce's "half-pluralism," wedged between Bradley's monism and James's pluralism, raises many questions that are outside the scope of this paper (see Marcel 1945, 79; chaps. II, III; see also Parker 2014). But keeping this in mind will help us to understand in which direction the developments in *The World and the Individual*, to which I will now turn, are heading.

Royce's (1900) main issue is "the problem of Being," whose epistemological side can be summarized in this way: how to articulate thought, and the world,

20. Gabriel Marcel (1945) presents a good general introduction to Royce's philosophy; Marcel's work was translated in English in 1956 as *Royce's Metaphysics* by V. and G. Ringer, and published by Regnery.

21. At the end of *The Varieties of Religious Experience*, one finds a description that applies perfectly to Royce: "I feel bound to say that religious experience, as we have studied it, cannot be cited as unequivocally supporting the infinitist belief. The only thing that it unequivocally testifies to is that we can experience union with *something* larger than ourselves and in that union find our greatest peace. Philosophy, with its passion for unity . . . 'pass to the limit' and identify the something with a unique God who is the all-inclusive soul of the world. . . . Meanwhile the practical needs and experiences of religion seem to me sufficiently met by the belief that beyond each man and in a fashion continuous with him there exists a larger power that is friendly to him and to his ideals. All that the facts require is that the power should be both other and larger than our conscious selves. Anything larger will do, if only it be large enough to trust for the next step. It need not be infinite, it need not be solitary. It might conceivably even be only a larger and more godlike self, of which the present self would then be but the mutilated expression, and the universe might conceivably be a collection of such selves, of different degrees of inclusiveness, with no absolute unity realized in it at all" (James 1902, 593).

in such a way as to make rational inquiry possible? Thought can comprehend the world, but it can also fail in this task, and the four conceptions of Being that Royce (1900) presents all aim at relating thought and world so as to encompass both possibilities. Lectures 2–4 are devoted to Realism (the first conception) and Mysticism (the second one); lectures 5–6 are dedicated to modern Critical Rationalism (Kantism, the third conception); and lectures 7–10 explain the fourth conception, Royce’s own version of idealism. For Royce, the two first conceptions fail to account for the possibility of enquiry; only Critical Realism succeeds to explain both how the thought can reach its goal (truth)—or fail to do it. But this comes with the price: Being is, in Kantianism, neither an individual, nor composed of individuals. Let me explain Royce’s analysis a bit more.

The first conception of Being, Realism, considers that the world is what it is, independently of the mere ideas. Mysticism, on the contrary, considers that Being is the immediately given, which, when felt, “ends any effort at ideal definition” (Royce 1900, 61). For Royce, the two approaches fail to account for the “disquietude” that characterizes the rational inquiry.²² The gap that Realism sets between thought and being is so large that it cannot be bridged. In Mysticism, since Being is the immediate fusion occurring in the sensible feeling, there is no longer a gap between the thought and the world—but then, there is no longer a starting point for rational enquiry, either.²³ Kantianism (Critical Realism) represents a major advance for Royce. The key idea of the third conception is to consider Being as what makes ideal judgment true. That is, contrary to Realism, Being is not viewed as an independent domain, inaccessible to thought, but as something that is selected, determined by a conceptual content. This claim does not lead to Mysticism, however, since the dependence of Being with regard to thought is also an independence: enquiry can reveal thought to be false, and there is then, in Critical Rationalism, a gap to be bridged. In the third conception, thought and Being are no longer juxtaposed next to each other; distinct, neither of the two poles can be conceived independently of the other. To explain this relation, Royce (1900) borrows from Bradley the terminology of the *what*, or internal meaning, and the *that*, or

22. Royce (1900, 57): “These two aspects of our lives, the immediate aspect and the ideal aspect, then show themselves in sharp contrast. Ideal meditation and brute immediacy stand in opposition to each other. We then know our finitude, and we are inwardly disquieted thereby. Such disquietude is our almost normal experience as finite wanderers.”

23. Following Schopenhauer, Royce (1900), in his explanation of Mysticism in lecture 3, gives a central role to Hinduism. For a discussion of the Hinduist view of the Self and its dissolution in the Being, see Royce (1900, 165–68).

external meaning: the general concept (the *what*) determines, selects the portion of the Being that correspond to a fact (the *that*), when the concept is satisfied.²⁴

In the long lecture 7 (80 pages) titled “The Internal and External Meaning of Ideas,” Royce criticizes the third conception of Being. As Royce explains, Kantianism is an “instable equilibrium” (1919, 35–36): Kant bases his whole development on a theory of knowledge, which depends on a certain “ontological situation”; but this theory, once accepted, forbids us to define any “ontological situation” whatever, except as the purely negative one according to which the thing-in-itself (the real Being) is unknowable. There is then a circle in Kant, and according to Royce, the refusal to enter into this circle is at the root of all the various tendencies that compose post-Kantian idealism.²⁵ Royce’s own way to cut the Kantian knot is based on the idea of individuality. In Kant’s thought, the *that* and the *what*, which it corresponds to (when it is satisfied), are always general, and there never is an individual or an individual determination.²⁶ It is difficult to understand what Royce exactly meant by the term “individual.” Its meaning includes various theological, moral, ontological, and epistemological elements that do not fit easily together.²⁷ But what is important for us is the epistemological consequence that Royce draws from his wish to resist Kant’s view of Being, namely, the abandon of the distinction between the external and the internal meaning. Indeed, Royce (1900, 327) claims that Being is nothing else than the full development of the thought: “What the idea always aims to find in its object is *nothing whatever but the idea’s own conscious purpose or will, embodied in some more determinate form* than the idea by itself alone at this instant consciously possesses. When I have an idea of the world, my idea is a will, and *the world of my idea is simply my own will itself determinately embodied.*” Being is then not an external *that* which corresponds to a general internal *what*; Being is the complete determination of the conscious purpose that an idea is. Royce is not endorsing Mysticism: there is still for him a genuine distinction between the present state of my vague and incomplete thought and its

24. I cannot do justice to Bradley’s analysis here, nor to Royce’s use of it. Bradley was a good logician; on this, see Allard (2005) and Marion (2009). For instance, in my too brief presentation, I mix the level of propositions (and truths) with those of concepts (and extensions). Bradley did not make this mistake: his distinction between the *what* and the *that* is grounded on a theory that construes all categorical judgments as existential judgments.

25. Royce (1919, 61): “In order to reach his [Kant’s] epistemology, . . . one has to accept his ontology, while after one has once accepted the epistemology, anything but a wholly problematic ontology is excluded.”

26. Royce (1900, 293): “Neither do our internal meanings ever present to us, nor yet do our external experiences ever produce before us, for our inspection, an object whose individuality we ever really know as such.”

27. See Marcel (1945, 61–76). Several papers in Parker and Bell (2014) deal with the ethical aspects of this issue.

possible future complete “determination” (or “embodiment”). But Being is not the correspondence with something external to the idea: it is the realization of something that is already implicitly present in it.²⁸ The distinction between thought and Being is not abolished, but it is “posited by” the thought, and internal to it.

This view may seem to us today, at best, surprising, at worst, absolutely untenable.²⁹ But it was deeply linked to certain major trends in philosophy that Royce knew and liked. Royce’s criticism of the correspondence theory of truth is thus akin, in some respects, to Peirce’s and James’ pragmatism.³⁰ The most important connection is, however, with Fichte’s and Hegel’s theory of Being (or, as they call it, Absolute) as Self. Since the reader may not have this conception in mind, let me say a few words about it. Hegel explains, in a notorious passage of the introduction to his *Phenomenology of Spirit*:

In my view, which can be justified only by the exposition of the system itself, everything turns on grasping and expressing the True, not only as Substance, but equally as Subject. . . . If the conception of God as the one Substance shocked the age in which it was proclaimed, the reason for this was on the one hand an instinctive awareness that, in this definition, self-consciousness was only submerged and not preserved. On the other hand, the opposite view, which clings to thought as thought, to universality as such, is the very same simplicity, is undifferentiated, unmoved substanti-ality. (1977, 9–10)³¹

Hegel is here contrasting his own view from Spinoza and Kant. For Spinoza, God or the Absolute is the one Substance, and the goal is, for the Subject, to

28. Royce (1900, 329): “In seeking its object, any idea whatever seeks absolutely nothing but its own explicit, and, in the end, complete, determination as this conscious purpose, embodied in this one way. The complete content of the idea’s own purpose is the only object of which the idea can ever take note. This alone is the Other that is sought.”

29. It amounts to saying that discovery is nothing but a better articulation of an already present thought. As a matter of fact, Royce considers the mathematical enquiry as the model of all enquiries (see, e.g., 1900, 329–30): “*The complete content of the idea’s own purpose is the only object of which the idea can ever take note. This alone is the Other that is sought.* That such a search as this is a genuine search for an object, that while sought appears as another and as a beyond, the experience of the mathematical sciences will at once illustrate. As we saw, . . . the mathematician deals with a world which his own present ideas, as far as they go, explicitly attempt to predetermine; yet what these ideas do not at present completely and consciously predetermine for the mathematician’s private judgment, in advance of proof, is precisely that further determination of their own meaning which they imply and seek. This further determination the mathematician wins through his process of inquiry.”

30. I cannot develop here this topic. See Royce (1900, 300–321) and Misak (2013, 81–90).

31. For a perspective on this passage and a detailed comment on Hegel’s theory of reflection, see Longuenesse (2007).

reach it (this corresponds to Royce's Realism).³² On the contrary, for Kant, the notion of a substance is a mere category of the thought—the Subject itself, so to speak, projects its conceptual framework, beyond itself, into the Object. For Hegel, Kant's Copernican revolution is a step in the right direction, but, since thought and reality are still separated from each other, Critical Rationalism (as Royce would have called it) remains trapped in "undifferentiated, unmoved substantiality." For Hegel, who follows Fichte:

The living Substance is being which is in truth Subject, or, what is the same, is in truth actual only in so far as it is the movement of positing itself, or is the mediation of its self-othering with itself. This Substance is, as Subject, pure, simple negativity, and is for this very reason the bifurcation of the simple; it is the doubling which sets up opposition, and then again the negation of this indifferent diversity and of its anti-thesis [the immediate simplicity]. Only this self-restoring Sameness, or this reflection in otherness within itself—not an original or immediate unity as such—is the True. It is the process of its own becoming, the circle that presupposes its end as its goal, having its end also as its beginning; and only by being worked out to its end, is it actual. (1977, 10)

Hegel is famous for his difficult prose, and the passage quoted is no exception. But, even if some parts remain difficult to interpret, the author's intention is clear: against Kant, the opposition between the Subject and the Object should not be considered as something given, but rather as something that results from a more fundamental process of reflection of the Subject into itself. To claim that the Absolute is a Subject is to claim that the division of thought and Being, of the *what* and the *that*, of the representation and the thing represented, is an outcome of the process of division of the Absolute Self. Note that the belief that the Object is something given (belief, which, at the beginning of the process, the Self has) is not an illusion. To be opposed to a not-Self is an essential feature of what a Self (even the Absolute Self) is: Hegel's Absolutism is not a version of Mysticism, where the Subject fuses into the Object. To understand this view, one has both to grasp that the Object is posited by the Self, and finally coincides with it, and that the Object is given to the Self from outside, and thus does not, at first, coincide with it. The temporal gap (the distinction finally/at first) indicates the place of the ineliminable reflexive process, and marks that this conception of the Absolute is distinct from any form of Mysticism, defined as a kind of static fusion of the Subject and the Object. In figure 1,

32. I follow Hegel in capitalizing "Subject" and "Object."

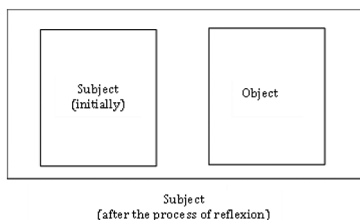


Figure 1. Absolute as Subject.

I represent this structure by stressing the fact that the difference Subject-Object is immanent to the Subject.

Royce (1900) does not take the trouble to describe this conception in detail, and contents himself with referring vaguely to Hegel's and Fichte's views, leaving his reader to fill the gap.³³ But in his lectures on post-Kantism idealism, published in 1919, he is more precise. Insisting on the crucial role that the construction of a truly philosophical notion of the Self played in the overcoming of Kantianism, he explains:

For [Fichte], the self is the principle of philosophy. . . . Whatever is asserted at any stage of the inquiry, one must forthwith add, "*The self asserts this*"; in other words, "This is known as true in so far as *I posit this*." The fact *I posit this* is thus logically prior to the fact *This is*. But hereupon one observes that the very problem of philosophy . . . may be summed up in the law that I always inevitably posit data . . . so that I view them as facts found by me, but *not* posited by me. This then is my original nature, viz., to acknowledge what I still stubbornly view *not* as my acknowledgment, but as something *not* myself, and as given, from without, to myself. . . . The first thesis of Fichte's philosophy is: The self posits just the self, and herewith posits whatever it can acknowledge as known or as knowable to the self. The equally inevitable antithesis is: The self posits a not-self; that is, defines its own object as not its own, but as another, opposed in nature to its own nature. The thesis and antithesis need to be united through a synthesis—a principle just to both these aspects of self-consciousness. (Royce 1919, 96–97)

Fichte's and Hegel's definition of the Self as an entity that posits its opposite and recovers its unity by a process of reflection is the keystone of Royce's fourth

33. For Royce's (1900) only explicit reference to Fichte's and Hegel's conception of the Self, see n. 42 below, but see also Royce (1900, 553): "The Universe, as Subject-Object, contains a complete and perfect image, or view of itself. Hence it is, in structure, at once One, as a single system, and also an endless *Kette*. Its form is that of a Self."

conception of Being, in 1900.³⁴ It is this ideal (nonpsychological) concept of the Self that Royce, as I will now show, compares in the *Supplementary Essay* to Dedekind's notion of self-representative system.³⁵

5. Royce with Dedekind against Bradley

As I said in section 1, the *Supplementary Essay* is a long criticism of Bradley's own monistic version of Absolute Idealism.³⁶ Bradley claims that all the finite substances and all the finite selves one usually distinguishes in the world have no genuine independence and self-subsistence. They are only nonsubsisting fragments of the single one-encompassing Absolute. As in Spinoza, there is for Bradley only one substance, and what the common sense views as an independent thing is a nonindependent part (a mode) of one single totality. More precisely, in his masterpiece, Bradley (1893/1916) shows, in the first book, that all the most important categories of thought (relation, quality, motion, causation, thing, activity, self, etc.), which allow us to articulate plurality and unity, are defective. They all lead to infinite regressions and contradictions, and belong, then, to the appearances. In the second book of *Appearance and Reality*, Bradley attempts to characterize reality in a more positive way; and one of his claims is that we have access to the Absolute only through sensible feeling. The Absolute is "a single and all-inclusive experience, which embraces every partial diversity in concord" (1893/1916, 147). To resume Royce's terminology, Bradley endorses then a version of Mysticism—but a strange form of mysticism, since Bradley's appeal to immediacy (Book II) is the result of long and rather complicated rational enquiry (Book I).³⁷

34. Once again, it is congenial to Fichte's conception that the Self does *not* coincide with its object. As Royce's insists on in the passage, the fact that the object is first found, and not posited by the Self (Fichte's second thesis), is not an eliminable foreplay.

35. Following Royce, I will capitalize the nonpsychological, ideal, notion of Self. As Royce emphasized, Hegel and Fichte's notion is distinctively philosophical: the new kind of ontological structure they uncover is first and foremost applied to the Absolute, and only in a derived sense to the finite human self. See Royce (1919, 97): "[The] self of philosophy is not the individual man of ordinary life, appears from the very outset of Fichte's discussion. The individual man of ordinary life is one of the beings to be defined by philosophy, and is certainly not the principle of philosophy. The self, appearing at the outset as the abstract principle of philosophy, is to be transformed, by the philosophical process, into the true self, the self rightly defined and embodied."

36. There is not much in the literature about this debate. But see Marcel (1945, 77–92) and Helm (1973).

37. Royce (1900, 548–49): "In brief, mysticism turns upon a recognition of the failure of all thinking to grasp Reality. But this recognition is itself thought's own work. . . . Mr. Bradley's account of the Absolute often comes near to the use of mystical formulations, but Mr. Bradley is of course no mystic; and nobody knows better than he the self-contradictions inherent in the effort to view the real as a simple unity, without real internal multiplicity. As we have seen, Mr. Bradley's Absolute is One, and yet does possess, as its own, all the manifoldness of the world of Appearance. The central difficulty of metaphysics, for Mr. Bradley, lies in the fact that we do not know how, in the Absolute, the One and the Many are reconciled."

In the *Supplementary Essay*, Royce's reference to Dedekind is used as a negative weapon against Bradley. Contrary to what Marcel suggests, Dedekind's mathematics is not supposed to be the basis for Royce's individualistic metaphysics.³⁸ Thus, the concept of self-representative system is not meant to explain the relationship between the individuality of God and that of finite minds. Royce's goal is more modest. It is to show that what Bradley considers as an unintelligible and inconsistent category—namely, the category of the ideal Self—is free from contradiction. Royce's philosophical use of mathematics is thus the same as that claimed by Russell in 1912. Speaking of Hegel's and Bradley's metaphysics, Russell says:

Most of the great ambitious attempts of metaphysicians have proceeded by the attempt to prove that such and such apparent features of the actual world were self-contradictory, and therefore could not be real. The whole tendency of modern thought, however, is more and more in the direction of showing that the supposed contradictions were illusory, and that very little can be proved *a priori* from considerations of what *must* be. . . . The attempt to prescribe to the universe by means of *a priori* principles has broken down; logic instead of being, as formerly, the bar to possibilities, has become the great liberator of the imagination, presenting innumerable alternatives which are closed to unreflective common sense. (1912, 227–28)

For instance, Russell drew on Weierstrass's theory to show that the alleged antinomies relating to movement and its continuity are not contradictions in the logical sense of the term (see Russell 1903, chap. 42). In the same way, Royce drew on Dedekind's definition of an infinite system to show that the concept of the Self is devoid of any contradiction.³⁹ Let me explain Royce's move more precisely.

As I said, Bradley considers that the concept of the Self belongs to the sphere of appearance.⁴⁰ It is in chapter 9 (1893/1916, 88–96) that Bradley deals with

38. Marcel (1945, 77–80). Helm (1973) is even more radical than Marcel.

39. Royce (1900, 513): “[The concept of the Self], so elaborately studied by Mr. Bradley, [has been] condemned by him as Appearance. And, indeed, if the Self is anything final at all, it is certainly in its complete expression (although of course not in our own psychological life from instant to instant) a self-representative system; and its metaphysical fate stands or falls with the possibility of such systems. Dedekind's really very profound use of *meine Gedankenwelt* as his typical instance of the infinite, also suggests the interesting relation between the concept of the Self and that of the mere mathematical form called the number series.”

40. Bradley (1893/1916, 119): “We cannot reach any defensible thought, any intellectual principle, by which it is possible to understand how [in the Self] diversity can be comprehended in unity. But, if we cannot understand this, and if whatever way we have of thinking about the self proves full of inconsistency,

what he considers as the “most important way of understanding the Self” (88), namely Fichte’s and Hegel’s view of the Self as containing its opposite not-Self. According to Bradley, this view leads to contradictions, because one needs to identify and distinguish the Self and the not-Self as two different terms, while no clear boundary between the two can be drawn. Each pole (the Subject as the Object) endlessly becomes the other, without any way to stop the infinite regression. As Royce says, for Bradley, “the Self does . . . unite diversity and unity in a profoundly important way; but the mere fact that this is somehow done does not show us how it is done” (1900, 482). Royce based his rejection of Kant’s “sundering” between the internal (the *what*) and the external (the *that*) meaning on the idea that the thought, as a thought of a Self, posits the reality. This solution is not available to Bradley, since the category of the reflective Self is “full of inconsistencies” (1893/1916, 119). The only possibility for Bradley to overcome the opposition between the *what* and the *that* is to return to the mystical fusion we experience in feeling. For Royce then, Bradley’s anti-intellectualist Mysticism and his rejection of Fichte’s and Hegel’s conception of the Self are two sides of the same coin.⁴¹

The clearest statement of Dedekind’s inspired argument that Royce opposes to Bradley is not found in the *Supplementary Essay* but in an article on infinity published two years later.⁴² The quotation is worth quoting in its entirety:

Whatever our view of the psychology of self-consciousness, or of the mental limitations under which we now are forced to live in this world, we must all of us recognise that one characteristic function of the Self is the *effort* reflectively to know itself. Self-consciousness we never fully get,

we should then accept what must follow. The self is no doubt the highest form of experience which we have, but, for all that, is not a true form.”

41. Royce (1900, 484): “The nature of relational thought, its inevitable sundering of the what and the that, and its inevitably infinite process in trying to unite them again, are two topics discussed, with the result, as Mr. Bradley states the case, that ‘Thought desires a consummation in which it is lost,’ as ‘the river’ runs ‘into the sea,’ and ‘the self’ loses itself ‘in love.’”

42. Note as well this passage from Royce (1900, 526–27): “The precise logical source of the good order of the number-system . . . is simply due to the fact that the number-series is a purely abstract image, a bare, dried skeleton, as it were, of the relational system that must characterize an ideally completed Self. This observation, in the present form, cannot be said to be due to Hegel, although both his analysis and Fichte’s account of the Self, imply a theory that apparently needs to be developed into this more modern form. . . . The foregoing observation as to the parallelism between the structure of the number-series and the bare skeleton of the ideal Self, is due, then, in its present form, rather to Dedekind than to the idealistic philosophers proper. It shall be briefly expounded in the form in which he has suggested it to me, although his discussion seems to have been written wholly without regard to any general philosophical consequences.”

but we aim at it; it is our ethical as well as our metaphysical goal. Now what would be the conscious state of a being who had attained complete self-consciousness, who reflectively knew precisely what he meant, and did, and was? To such a being we easily ascribe godlike characters. God Himself we often conceive as such a completed Self. . . . But what our observation of the self-representative systems has shown us is, that in their form, however trivial their content, these . . . systems possess *a structure correspondent to the one that we must ascribe to any ideally complete Self, in so far as it is conceived as self-conscious*. A completely self-conscious being would contain within himself, as a part of his whole consciousness, . . . a complete rational representation of his own nature, and of the whole of this nature. In consequence, as we have now seen, he would be, *ipso facto*, an infinite being. *To define the ideally or formally complete Self, is thus to define the infinite.* (Royce 1902, 33–34)

The notion of Self that Royce is dealing with here is the Fichtean and Hegelian concept we talked about in the previous section. Indeed, a Self is characterized by “the *effort* reflectively to know itself,” and the Absolute (God) is nothing else than an ideal Self in which this process of self-consciousness has been completed. The notion of a completion can create a confusion. As I explained in the previous section, in this view there is no fusion between Subject and Object in the ideal completed Self: the reflexive “effort” does not disappear and is not replaced by a peaceful merging of the two poles. On the contrary, the ideality of the Absolute Self comes from the fact that, in it, self-reflection is perfect, and never stops. The reference to Dedekind can explain this point.

What Royce claims in the passage is that the ideal Self is a self-representative system: it satisfies the three conditions set by Dedekind in his definition of an infinite system. Let me suggest the translation Royce has in mind. S is the Self, the set of ideas or thoughts; among these thoughts, some are regarded as coming from outside, as given and not posited by the Self. Thus, within the Self, one can draw a distinction between what belongs to the not-Self (the ideas that are not posited by the Self), and what belongs properly to the Self (what Royce calls “reflexive ideas”). How to express the notion of reflexivity? By the mapping φ . Here, Royce follows Dedekind’s Theorem 66: to each idea $s \in S$ corresponds one and only one idea $\varphi(s)$, the idea that “ s can be an object of my thought.” That the reflexive effort of the finite self is imperfect means that the psychological mapping φ is, in this case, not injective. But in the ideal Self, the mapping is injective, and $\varphi(S)$ is thus an exact image of S . The complete Self would be then a system $\langle S, \varphi \rangle$ that satisfies the three following conditions:

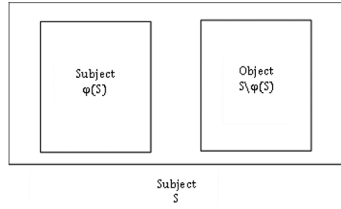


Figure 2. Absolute as Subject and Dedekind's infinite.

- 1'. The reflexive part of the Self is included in the Self: $\varphi(S) \subset S$.
- 2'. The reflection of the ideal Self into itself is perfectly exact: the mapping φ is injective.
- 3'. Some ideas of the ideal Self are given from outside (the distinction between the Self and the not-Self is internal to the Self): $S \setminus \varphi(S) \neq \emptyset$.

One could then propose the representation of Royce's metaphysical translation of Dedekind's conditions shown in figure 2.

Condition 3' is perhaps more contentious than the two others, since Royce does not follow here the pattern given by Dedekind in Theorem 66.⁴³ But it follows from what we have said about the difference between Royce's (Fichte's and Hegel's) conception and Bradley's Mysticism. In the fourth conception of Being, there is always a gap between the idea and its full "embodiment": there is always a place for the "effort" and the process of reflection. The completed Self is no exception. Its completeness comes from the fact that φ is injective, not from the mystical coincidence between the ideas and their objects (which would correspond to the equality $S = \varphi(S)$). In the *Essay*, Royce never stops stressing the dynamical character of the Absolute: the ideal Self comes with its own *proprius motus*, the endless regression created by its own internal reflection, and to be complete does not exclude to be never-ending. Thus:

Dedekind has shown that [the self-representative system] is adequate to the logical development of the various properties of the number-system. What we here observe is that the consequent constitution of the number-system is explicitly defined as, of course in the barest and most abstract

43. Thus it would be very strange, in Royce's perspective, to give *my own ego* as an example of an idea *s* that does not belong to the set of reflexive ideas. Indeed, a nonreflexive idea is, in this view, an idea coming *from outside my own ego*. It is no coincidence then if, in his reconstruction of Dedekind's Theorem 66 proof, Royce (1902, 39) replaces *my own ego* by *my country*. Note also that in the quotation of Theorem 66, Royce (1900, 511) adds, after the mention "my own ego," a question mark "(?)." For a different interpretation of the insertion of this question mark, see Helm (1973).

outline, the form of a completed Self. Here, then, the Intellect, “of its own movement,” “itself by itself,” defines what, in our temporal experience, . . . it of course nowhere finds given, namely, a self-representative system of objects, parallel in structure to what the structure of a *Gedankenwelt* would be if it were the Welt of a completely self-conscious Thought, none of whose acts failed to be its own intellectual objects. (Royce 1900, 534)⁴⁴

The noncoincidence between the Self (S) and its reflective exact image ($\varphi(S)$), translated in condition 3', is then the true basis of Royce's opposition to Bradley's Mysticism.

Even if, as we have just seen, Royce does not slavishly follow Dedekind, Theorem 66 plays an instrumental role in his reading of *Die Zahlen*.⁴⁵ The fact that Dedekind himself appeals to the notion of *Gedankenwelt*, Self, and reflexive ideas is blessed bread for him: it shows that the metaphysical interpretation he develops, and that I have just presented, is not off target. Note, however, that the American philosopher did not attribute to Dedekind his metaphysical view.⁴⁶ He would not necessarily have disagreed with Dugac's claim that the proof of Theorem 66 “does not fit with Dedekind's mathematical thought.” But the emphasis Royce put on this section of *Die Zahlen* is very different from that of Dugac: far from being a passage that should be put in brackets to recover the purity of Dedekind's thought, it is viewed as the interface that allows to connect Dedekind's mathematics to Fichte's and Hegel's Absolute Idealism.

With further investigation, however, things become a little more complicated. Indeed, at this stage, the issue of consistency resurfaces. Recall that Bradley argued that the notion of Self is contradictory. Now, even if Dedekind's definition articulates more precisely the conceptual content of Fichte's and Hegel's notion of a Self, it did not, by itself, show that the self-representative system is free

44. Royce is responding to Bradley, who imagines the conditions under which a multiplicity grasped by the intellect could contain within itself its unity, and thus be considered as an Absolute. See Royce (1900, 488–89): “If the diversities were complementary aspects of a process of connection and distinction, the process not being external to the elements, or, again, a foreign compulsion of the intellect, but itself the intellect's own *proprius motus*, the case would be altered. Each aspect would of itself be a transition to the other aspect, a transition intrinsic and natural at once to itself, and to the intellect. . . . If 'all that we find were in the end such a self-evident and complete whole,' the end of the intellect, and so of philosophy, would have been won. But Mr. Bradley is (p. 569) 'unable to verify a solution of this kind.'”

45. Royce goes back in several places to Dedekind's “proof”; see Royce (1900, 511; 532–33; 1902, 38–39).

46. Royce (1900, 527): “[Dedekind's] discussion seems to have been written wholly without regard to any general philosophical consequences.”

from contradiction: “Mere definitions do not of themselves ensure the possibility of their objects” (Royce 1902, 38). Only an existence proof could give Royce what he wanted, and Theorem 66 was supposed to provide just this. But there are two problems here for Royce. Since “the conception of the realm of all my possible thoughts” (39) proves to be a contradictory conception (see Cantor’s objection, sec. 3, above), Dedekind’s move turns out to be a dead end. This is a severe blow for Royce, but one that comes from outside. The other difficulty is more annoying because it comes from within: there is a circle in Royce’s argument. Against Bradley, Royce wants to show that the Self is not a contradictory concept and he resorts to Dedekind’s notion of infinite system to do so; but to show that Dedekind’s notion is not contradictory, Theorem 66 returns to the notion of the Self. To my knowledge, Royce did not mention this problem, but it can explain why Bradley (1910, 175–78), commenting on Royce’s *Supplementary Essay* and Russell’s *Principles*, maintained his view.

Let me summarize what we have seen so far. Royce intends to go beyond Bradley’s anti-intellectualist Mysticism by showing how Dedekind’s definition of a self-representative system makes it possible to harmonize, within the absolute Self, unity and diversity. For Royce, Bradley is doubly wrong: when he claims that the notion of Self is not “a true form of reality,” and when he affirms the unknowable nature of the Absolute. Dedekind’s work is thus used as a means to show that what Bradley considers as unintelligible can in fact be rationally articulated.

6. Conclusion: Metaphysics, Mathematics, and Dedekind’s Intention

Is Royce “right” to establish such an unexpected link between Dedekind’s characterization of infinity and the Fichteo-Hegelian doctrine of self-reflection? As I said, Royce did not claim that Dedekind drew the connection he himself made: “to be right” did not mean, for him, “to conform to Dedekind’s intention.” But if we give up this “conformity criterion,” then what can we rely on? On the other hand, Royce’s reading can hardly be dismissed as a mere misunderstanding of Dedekind’s *Zahlen*. Royce’s interpretation is not mathematically unsound, and, more importantly, owing to the context he was dealing with, referring to Dedekind’s definition was a natural, if not ingenious, move. Recall that Royce wanted to defend his version of Idealism against Bradley’s charge that the notion of Self is full of inconsistencies. In Hegel and Fichte, the distinction between the Self and the not-Self is presented as being internal to the Self: there is a proper part of the Self that is identical, in some respects, to the

whole. This is the reason why Hegel's and Fichte's view seemed paradoxical to Bradley. Now, in *Die Zahlen*, Dedekind attempts to show that, thanks to the notions of system and mapping, one can give a precise meaning of this idea. Better, Dedekind illustrates his notion of infinite system by the nonmathematical concept of *Gedankenworld*, Self, and reflection. When reinserted in the context of his metaphysical project, how to resist Royce's move?

At the end of section 3, I distinguished two attitudes in the literature on Dedekind's Theorem 66: some scholars tend to downplay this theorem, and, more generally, downplay the role of philosophy in Dedekind's thought, while others consider Theorem 66 as a window that lets us glimpse Dedekind's metaphysical views. As I said in section 3, I embrace the first attitude: the traces left by Dedekind seem to me too sparse and uncertain to be able to conclude anything about his intentions. But I do not mean that we should ignore Theorem 66 and give up trying to relate *Die Zahlen* to metaphysics. My suggestion is rather that this focus put on Dedekind's philosophical intent is an obstacle to the articulation between metaphysics and mathematics, because it diverts attention from the philosophical reception of Dedekind (1888). As Dedekind's theories have been deeply incorporated into modern mathematics, there is a tendency among scholars to restrict the historical posterity of *Die Zahlen* to mathematics. My goal in this paper is precisely to emphasize that Dedekind 1888 was read and used extensively by metaphysicians very early on, and that Royce's reading should be treated as an integral part of the history of Dedekind's reception, as are those of Hilbert, Emmy Noether, or Bourbaki.

In other words, instead of applying the term "metaphysics" to Dedekind's ideas and opinions, I have interpreted it as an adjective characterizing a certain community of writers and readers at the end of the nineteenth century; and instead of looking at the relationship between mathematics and metaphysics in terms of authorial intention, I have studied how Dedekind's *Zahlen* was read and appropriated in that community (more precisely, by Royce). This shift from intention to reception is not original and has been carried out in various fields and from various perspectives (including hermeneutic, phenomenological, and aesthetic). Here, I took inspiration from McKenzie's sociology of the text, and Chartier's history of books and reading (see McKenzie 2004; Chartier 1989; 2013). In particular, McKenzie and Chartier insist on the fact that if "new readers . . . make new texts," then "their new meanings are a function of their new forms" (McKenzie 2004, 29). Moving from intention to reception means that the question of faithfulness to the original is no longer the norm: a misreading is as worthy of interest as a faithful reception. Unqualified, too strong of a focus put on reception is then dangerous, in that it seems to lead to relativism: if misreadings are allowed, then every interpretation is worth another. To close

this door, McKenzie and Chartier insist that readers' appropriations are dependent on objective (material, institutional, sociological) forms, and that the task of the historians is precisely to account for the various contextual patterns that govern the various readings or misreadings.⁴⁷ This is what I tried to do for Royce's (mis)reading of *Die Zahlen*. One could summarize my argument by saying that, unlike the mathematical readings of Dedekind 1888, which downplay Theorem 66 (the only one that should be printed in light gray, let's say), in Royce's (mis)interpretation, Theorem 66 occupies center stage.

What I have tried to do here is just to explain this difference of emphasis. Owing to the importance Royce gave to Fichte's and Hegel's view of the Absolute (see my sec. 4), and to Bradley's objection against the notion of Self (see my sec. 5), it is quite natural for him to read or misread Dedekind the way he did. That is, the problems, interests, and associations that govern Royce's (mis)reading were not the product of an arbitrary individual whim; they were the result of a set of objective historical transformations within metaphysics: Royce's (mis)interpretation is difficult to resist, owing to his endorsement of Absolute Idealism and his opposition to Bradley's Mysticism. I do not deny that Royce projected into *Die Zahlen* patterns that were not present in Dedekind, but Hilbert, Noether, and Bourbaki did the same. Yet their readings occupy a place in Dedekind's scholarship that Royce's (mis)reading does not. My suggestion is that Royce's metaphysical (mis)reading is an integral part of what makes Dedekind's 1888 work what it is today, and that Dedekind's scholars should then consider it. Seen in this light, Royce is even particularly interesting, because it played an instrumental role in the dissemination of Dedekind's definition of the infinite outside the mathematical community, and, to conclude, I would like to evoke two of these Royce's related nonmathematical posterities.⁴⁸

As surprising as it may seem, Royce's analysis of *Die Zahlen* has had a literary legacy. In "Partial Magic in the Quixote," Borges grounds his reflections on the *mise en abyme* on Royce's passage about the map of England:

The inventions of philosophy are no less fantastic than those of art: Josiah Royce, in the first volume of his work *The World and the Individual* . . . ,

47. On the difference between the sociology of texts and (Gadamer inspired) hermeneutics, see, e.g., Chartier (1989, 1510–11).

48. Royce's longevity at a key institution like Harvard, which received many foreign (especially Asian) visitors, his already genuine multidisciplinary teaching (as I said, Norbert Wiener did his PhD thesis with him, but we also find T. S. Eliot, George Santayana, and W. E. B. Du Bois among his students), and his true talent for writing (cf. the map of England that represents itself) is undoubtedly at the origin of the wide dissemination of his thought in the first half of the twentieth century.

has formulated the following: “Let us imagine that a portion of the soil of England has been levelled off perfectly and that on it a cartographer traces a map of England. The job is perfect; there is no detail of the soil of England, no matter how minute, that is not registered on the map; everything has there its correspondence. This map, in such a case, should contain a map of the map, which should contain a map of the map of the map, and so on to infinity.” Why does it disturb us that the map be included in the map and the thousand and one nights in the book of the *Thousand and One Nights*? Why does it disturb us that Don Quixote be a reader of the *Quixote* and Hamlet a spectator of *Hamlet*? I believe I have found the reason: these inversions suggest that if the characters of a fictional work can be readers or spectators, we, its readers or spectators, can be fictitious. In 1833, Carlyle observed that the history of the universe is an infinite sacred book that all men write and read and try to understand, and in which they are also written. (Borges 1962/2007, 196)

The reflection in itself of the self-representative system is connected, in Borges, with the techniques of self-reflexive embedding one finds in various literary works, like *A Thousand and One Nights*, *Don Quixote*, and *Hamlet*. Self-representation is not directly linked to the conception of the Self, as in Royce, but to a meditation on the relationship between the author and his work. Note that one also finds a connection between Dedekind’s definition of infinity and the formal technique of mise en abyme in Russell (1903), but the way Russell makes the connection has not the precision, simplicity, and elegance of Royce’s simile.⁴⁹

Royce’s interpretation also had a philosophical legacy outside the Western world, in the so-called Kyoto School. One of the central concepts of Nishida Kitarō’s thought is that of self-awareness (*jikaku* 自覚). In an early article, Nishida uses Royce’s notion of self-representative system to analyze the structure of *jikaku*. Let me quote Nishida:

What is [Royce’s] self-representative system? In the same way that Hegel takes the ego as an example of the *Für-sich-sein*, Dedekind says that “my thought-world, which can become the object of my own thought, is infinite,” that is, the thought by which a thing can become the object of my

49. Russell (1903, 363): “Tristram Shandy, as we know, took two years writing the history of the first two days of his life, and lamented that, at this rate, material would accumulate faster than he could deal with it, so that he could never come to an end. Now I maintain that, if he had lived for ever, and not wearied of his task, then, even if his life had continued as eventfully as it began, no part of his biography would have remained unwritten.”

own thought belongs also to my thought-world. We can, in our reflective consciousness, objectify the act of making the self the object of our thought. Just as in the case of the image reflected between two polished mirrors, and also in the example given by Royce of the perfect map of England [drawn in England], we are in the presence of an infinite progression. Therein lies the true meaning of infinity, the infinity of space and time comes from such an infinity of thought. (1912/2003, sec. 19)

I cannot explain the role that the notion of self-representativity plays in Nishida's theory of *jikaku*.⁵⁰ But the passage quoted above shows that Nishida's interest lies precisely in the connection between Fichte's and Hegel's doctrine of the subject and Dedekind's definition of the infinite. Thus, Royce's intuition was unexpectedly reactivated after him, in a context, that of Japanese Buddhism, that he would not have suspected. The surprises do not stop there, and they bring us back to our starting point: logic and mathematics. Akiyoshi and Arana (2019) showed how the insights underlying Takeuti's proof theory come from the Kyoto school, and especially from Nishida's theory of self-reflection (Takeuti 1975). Of course, there is no direct link between Dedekind (who was one of the sources, via Royce, of Nishida's *jikaku*) and Takeuti. In mentioning this, I just want to emphasize how convoluted history is: Royce's purely metaphysical reading of Dedekind's mathematics inspired the most important Japanese philosopher of the twentieth century, Nishida, whom Takeuti read and used to develop his own mathematical and logical program. Interpretation and misinterpretation go thus both ways: metaphysical (mis)readings of mathematics can generate mathematical (mis)readings of metaphysics. Better to rejoice than to complain, isn't it?

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50. On this, see Maraldo (2006).

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