A simple metaheuristic approach to the simultaneous scheduling of machines and automated guided vehicles

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Abstract. In this paper we address the problem of simultaneous scheduling of machines and vehicles in flexible manufacturing systems. The studied problem is a job shop where the jobs have to be transported between the machines by automatic guided vehicles. In addition to the processing of jobs, we consider transportation aspect as an integral part of the optimization process. To deal with this problem, we propose a new solution representation based on vehicles rather than machines. Each solution can thus be evaluated using a discrete event approach. An efficient neighbouring system is then described and implemented into three different metaheuristics: iterated local search, simulated annealing and their hybridisation. Computational results are presented for a benchmark of 40 literature instances. New upper bounds are found for 11 of them, showing the effectiveness of the presented approach.

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1. Introduction

A Flexible Manufacturing System (FMS) can be described as follows: FMS consists of several machine tools along with part and tool handling devices such as robots, arranged so that it can handle any family of parts for which it has been designed and developed. These systems require a high investment but they have a high potential. As result, scheduling of FMS has been extensively investigated during the last three decades.

This paper is devoted to the study of the classical job shop problem where transportation times are considered. It is classified as a $JR|t_{kl},t'_{kl}|C_{\max}$ problem according to the $\alpha|\beta|\gamma$–notation introduced by Graham *et al.* (1979) and extended by Knust (1999) for transportation problems. *J* indicates a job shop, *R* indicates that we have a limited number of identical vehicles (robots) and all jobs can be transported by any of the robots. t_{kl} indicates that we have job-independent, but machine-dependent loaded travel times. t'_{kl} indicates that we have machine-dependent empty travel times. The objective function to minimize is the makespan $C_{\max}$. Several authors note the necessity to carefully plan the interactions between process and transportation activities (Raman *et al*. (1986), Ulusoy *et al.* (1993), Knust (1999)). Unfortunately, the resulting problem is quite difficult to solve. We have to deal with two NP-hard problems simultaneously: the job-shop scheduling problem (Lenstra and Rinnooy Kan, 1979) and the vehicle scheduling problem, which is similar to a pick up and delivery problem (Lenstra and Rinnooy Kan, 1981). Additionally, problems with transportation times are NP-hard since they are a generalization of classical NP-hard

shop problems. More precisely, concerning the job shop, Knust (1999) writes: "*The classical job shop problems* $J2||C_{\max}$ *and* $J3|p_{ij}=1|C_{\max}$ *are both strongly NP-hard, which implies that only very restrictive special cases with transportation times may be polynomially solved"*.

As consequence, exact methods are mainly used for the study of simple or particular FMS, with strict assumptions. Thus, Blazewicz *et al*. (1991) study a FMS in which identical parallel machines are laid out in loop. Raman *et al*. (1986) present a MIP (Mixed Integer Programming) formulation of this problem but with the unrealistic assumption that the vehicles come back to the load/unload (LU) station after each achieved transport. Bilge and Ulusoy (1995) propose a MIP formulation raising this constraint on the vehicles. According to the authors, the resulting model is intractable in practice, because of its nonlinearity and its size.

Approximated methods are well adapted to study most of the FMS, but within an offline framework. Nevertheless, many works are dedicated to simplified forms of this problem. There are essentially two kinds of simplifications. The first ones consist in defining dispatching rules for the vehicles, for example described and studied by Egbelu and Tanchoco (1984) and Le Anh and De Koster (2005). These rules allow to determine how AGVs move for a given scheduling of machines. The second ones are the restriction of the material handling system at only one AGV. As illustration, Soylu *et al.* (2000) and Hurink and Knust (2002) propose respectively neural networks and tabu search approaches. Finally, few works are undertaken on the job shop with transportation. Ülusoy and Bilge (1993) and Bilge and Ülusoy (1995) propose an iterative method based on the decomposition of the master problem into two subproblems. Ülusoy *et al.* (1997) propose a genetic algorithm for this problem. Abdelmaguid *et al.* (2004) describe a hybrid method composed of a genetic algorithm for the scheduling of machines and a heuristic for the scheduling of vehicles.

For online scheduling of such systems, simulation is often the only available tool. Discrete event simulation is used for various purposes: for example to improve the AGV design process (Gobal and Kasilingam, 1991), to compare the vehicles on-line dispatching rules (Egbelu and Tanchoco, 1984) or to test the problems of collision between the vehicles (Revielotis, 2000).

Nuhut (1999), Qiu *et al.* (2002) and Le Anh and De Koster (2006) present more complete surveys of the job shop problem with transportation.

For this problem, most of the (small) instances considered in this paper remains today unsolved. Ganesharajah *et al.* (1998) note that due to the intractability of the joint scheduling problem, the literature contains mainly heuristic

approaches. In this paper, we propose a new effective metaheuristic approach for the offline $JR|t_{kl},t'_{kl}|C_{\max}$ problem.

This paper is articulated as follows. In Section 2 we present a problem formulation and we propose a new solution representation based on vehicles. In Section 3 we show how feasible solutions can be evaluated using a discrete event approach. Section 4 is devoted to the description of an efficient neighbouring system. The metaheuristics implemented are described in Section 5 and some computational results can be found in Section 6.

2. The job-shop problem with transportation: formulation and solution representation

In this section we introduce the job shop problem with transportation using the Brucker's notations (2001), we present the chosen solution representation and we discuss its advantages and its drawbacks.

2.1. Problem formulation

In a job shop scheduling problem, we have a set $J=\{J_1,\ldots,J_n\}$ of *n* independent jobs that have to be processed without preemption on a set $M=\{M_0,M_1,\ldots,M_m\}$ of $m+1$ machines (M_0 represents the load / unload or LU station from which jobs enter and leave the system). Each job $J_i \in J$ consists of a sequence of n_i operations o_{ij}. Let us note $O_i=\{o_{ij}, j=1,\ldots,n_i\}$ the set of operations of job J_i, and $O=\bigcup_{i=1}^{n} O_i$ the set of $o=\sum_{i=1}^{n} n_i$ operations. There is a machine $\mu_{ij} \in \{M_0,...,M_m\}$ and a processing time p_{ij} associated with each operation o_{ij}. Additionally, a vehicle has to transport a job whenever it changes from one machine to another. We have a given set $V=\{V_1,\ldots V_k\}$ of *k* vehicles. We assume that transportation times are only machine-dependant. $t(M_i,M_j)$ and $t'(M_i,M_j)$ indicate respectively the loaded travel time and the empty travel time from machine M_i to machine M_j ($i,j=0,...,m$). Vehicles can handle at most one job at a time.

The objective function is the minimizing time required to complete all jobs or makespan.

2.2. Representation of the search space

It is well known that the efficiency of metaheuristics highly depends on finding a suitable representation of the search space. For that a new solution representation is introduced, based on vehicles rather than on machines. Each operation $o_{ij} \in O$ requires a transportation task we will note $\tilde{o}_{ij}$, and we propose to schedule the set $\tilde{O}$ of

transportation tasks on the material handling system. In the following of this paper, we will designate by operations the members of *O*, and by transportation tasks (or tasks) the members of $\widetilde{O}$. Let us give in more details the link between *O* and $\widetilde{O}$.

Each operation o_{ij} requires the transportation of job J_i from the previous machine that is defined in its sequence of operations (M_0 if $j=1$ or μ_{ij-1} if $j>1$) to the machine μ_{ij}. So a transportation task $\tilde{o}_{ij}$ is associated with each operation o_{ij}. Moreover, when the processing of a job is completed, a vehicle has to transport it to the unload station M_0. So we consider an additional task $\tilde{o}_{in_i+1}$ for each job. To find again a link between operations and tasks, a fictitious operation o_{in_i+1} is added to the sequence of each job. Obviously, we have $\mu_{in_i+1}=M_0$ and $p_{in_i+1}=0$. We give in table 1 an example of a feasible representation for jobset 1 (see the appendix section for the jobsets definition).

Table 1. A feasible schedule for jobset 1 with three vehicles

Vehicle	Task schedule						
AGV1	$\tilde{o}_{11}$	$\tilde{o}_{31}$	$\tilde{o}_{22}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$	
AGV2	$\tilde{o}_{21}$	$\tilde{o}_{42}$	$\tilde{o}_{13}$	$\tilde{o}_{32}$	$\tilde{o}_{33}$		
AGV3	$\tilde{o}_{41}$	$\tilde{o}_{12}$	$\tilde{o}_{51}$	$\tilde{o}_{43}$	$\tilde{o}_{24}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$

We will indicate by $v(\tilde{o}_{ij})\in V$ the vehicle that complete the task $\tilde{o}_{ij}$. $\lambda(\tilde{o}_{ij})$ represents the position of the task $\tilde{o}_{ij}$ in the schedule. For example, in the table 1 we have $v(\tilde{o}_{13})=V_2$ and $\lambda(\tilde{o}_{13})=3$.

2.3. Advantages of the proposed solution representation

The proposed representation has major differences with those of the literature. (Ülusoy *et al.*, 1997) represent a schedule by a chromosome in which each gene has two data: the operation to be completed and the vehicle at which this operation is assigned. The solution given in table 1 is represented by the chromosome $[(\sigma_{11},V_1),(\sigma_{21},V_2),(\sigma_{41},V_3),...,(\sigma_{52},V_1),(\sigma_{34},V_3),(\sigma_{53},V_3)]$. The tasks assigned to one vehicle (for example $\sigma_{11},\sigma_{31},\sigma_{22},\sigma_{14},\sigma_{23}$, and σ_{52} for vehicle 1) are transported according to the order given by the chromosome. However, the same solution can also be represented by the chromosome $[(\sigma_{11},V_1),...,(\sigma_{52},V_1),(\sigma_{21},V_2),...,(\sigma_{33},V_2),(\sigma_{41},V_3),...,(\sigma_{53},V_3)]$, and by many others. As a consequence, one schedule can be represented by many chromosomes, which define an equivalence class. Our representation consists

in working with the set of equivalence class representatives. The immediate consequence is that the search space is significantly reduced, and thus probably easier to explore.

(Abdelmaguid *et al.*, 2004) get round this difficulty by using a chromosome in which only the order of the operations is mentioned. Thus, the chromosome $[\sigma_{11},\sigma_{21},\sigma_{41},...,\sigma_{52},\sigma_{34},\sigma_{53}]$ indicates the order in which the operations must be treated. A heuristic assigns successively each task to the 'best' vehicle (the chosen criterion is the vehicle that will carry out this task as soon as possible). The authors came back thus to the classical job-shop problem $J \| C_{\max}$, integrating dynamically the transportation times into the makespan evaluation. The advantage is that the number of solutions is considerably reduced. Nevertheless, we can notice that vehicle assignments are locally optimal, but not necessarily globally.

The use of metaheuristics implies that many solutions must be considered. Such approach requires a makespan evaluation with a computationally fast evaluation subroutine. We present it in the next section.

3. Solutions evaluation

The makespan evaluation is described in algorithm 1 using a discrete event approach. $T(\sigma_{ij})$ (respectively $T(\widetilde{\sigma}_{ij})$) indicates the completion time of the operation σ_{ij} (respectively the transportation task $\widetilde{\sigma}_{ij}$). We use the notion of *available task*, which can be defined as follows:

A transportation task is said to be available at a given time *t* if and only if it has not been carried out yet and it satisfies one of the two following conditions:

- It is associated to the first operation of the job sequence,
- The previous operation (if it exists) in the job sequence is completed at time *t*.

The available tasks correspond in fact to the transports that can be effectively accomplished at a given moment.

We can notice the following remarks about algorithm 1:

(a) At the initial state, the only available tasks $\widetilde{\sigma}_{ij}$ are those for which $j=1$ (corresponding to the first operation defined in the sequence of the job J_i).

(b) A new event is created if the following transportation task of the vehicle V_l is available (its previous corresponding operation has been already completed).

(c) The task $\widetilde{\sigma}_{ij}$ has been just completed. This means that the task $\widetilde{\sigma}_{ij+1}$ (if it exists) becomes an available one. A vehicle waiting for this task can now accomplish it, and an event is created. Notice that if tasks $\widetilde{\sigma}_{ij}$ and $\widetilde{\sigma}_{ij+1}$ are carried out by the same vehicle, this event has been already created in the case (b).

Algorithm 1: Makespan evaluation

// Initialization
For each task $\widetilde{\sigma}_{ij} \in \widetilde{O}$ **Do**
  $T\left(\widetilde{\sigma}_{ij}\right) = +\infty$
End For
// Creation of the initial events
For each vehicle $V_l \in V$ **Do**
  If the first task (a) $\widetilde{\sigma}_{ij}$ of V_l is an available one **Then**
    Compute $T\left(\widetilde{\sigma}_{ij}\right)$ and create event $\left[V_l, T\left(\widetilde{\sigma}_{ij}\right)\right]$
  End If
End For
// Main loop
While it exists some events $\left[V_l, T\left(\widetilde{\sigma}_{ij}\right)\right]$ with $T\left(\widetilde{\sigma}_{ij}\right) < +\infty$ **Do**
  Choose the next event $\left[V_l, T\left(\widetilde{\sigma}_{ij}\right)\right]$ and compute $T\left(\sigma_{ij}\right)$
  If the next task $\widetilde{\sigma}_{i'j'}$ of V_l exists and is available **Then** (b)
    Compute $T\left(\widetilde{\sigma}_{i'j'}\right)$ and create event $\left[V_l, T\left(\widetilde{\sigma}_{i'j'}\right)\right]$
  End If
  If the task $\widetilde{\sigma}_{ij+1}$ exists and is the next task of a vehicle $V_{l'} \neq V_l$ **Then** (c)
    Compute $T\left(\widetilde{\sigma}_{ij+1}\right)$ and create event $\left[V_l, T\left(\widetilde{\sigma}_{ij+1}\right)\right]$
  End If
End While

Completion times must satisfy the following constraints:

(1) $T\left(\widetilde{\sigma}_{i1}\right) \geq t\left(M_0, \mu_{i1}\right),\ \forall J_i \in J$

(2) $T\left(\widetilde{\sigma}_{ij}\right) \geq T\left(\sigma_{ij-1}\right) + t\left(\mu_{ij-1}, \mu_{ij}\right),\ \forall \widetilde{\sigma}_{ij} \in \widetilde{O},\ j > 1$

(3) $T\left(\widetilde{\sigma}_{i_2 j_2}\right) \geq T\left(\widetilde{\sigma}_{i_1 j_1}\right) + t'\left(\mu_{i_1 j_1}, \mu_{i_2 j_2 - 1}\right) + t\left(\mu_{i_2 j_2 - 1}, \mu_{i_2 j_2}\right),\quad \forall \widetilde{\sigma}_{i_1 j_1}, \widetilde{\sigma}_{i_2 j_2} \in \widetilde{O},\quad v\left(\widetilde{\sigma}_{i_1 j_1}\right) = v\left(\widetilde{\sigma}_{i_2 j_2}\right),$

$\lambda\left(\widetilde{\sigma}_{i_2 j_2}\right) = \lambda\left(\widetilde{\sigma}_{i_1 j_1}\right) + 1$ (with the convention $\mu_{i0} = M_0$)

(4) $T\left(\sigma_{i_2 j_2}\right) \geq T\left(\sigma_{i_1 j_1}\right) + p_{i_2 j_2},\ \forall \sigma_{i_1 j_1}, \sigma_{i_2 j_2} \in O,\ \mu_{i_1 j_1} = \mu_{i_2 j_2},\ T\left(\widetilde{\sigma}_{i_2 j_2}\right) > T\left(\widetilde{\sigma}_{i_1 j_1}\right)$

(5) $T\left(\sigma_{ij}\right) \geq T\left(\tilde{\sigma}_{ij}\right) + p_{ij},\ \forall \sigma_{ij} \in O$

Constraint 1 indicates that each job has to be transported from the loading station to its first machine. Constraint 2 indicates that a task cannot be completed before its previous corresponding operation has been completed plus the loaded travel time between the two machines. Constraint 3 gives the minimal time between two consecutive transportation tasks (on the same vehicle). The vehicle makes the empty travel from its last delivery destination to its next pick up destination, plus the loaded travel from here to its next delivery destination. Constraint 4 indicates that an operation can only begin after all previous operations on the same machine have been completed. Constraint 5 indicates that an operation cannot begin before the job arrives on the machine.

Also, the constraint 4 means that the order of operations on a machine is given by the order of arriving corresponding tasks on this machine. In other words, machine's input buffer follows FIFO rule. FIFO is a good performing rule (although nothing indicates that it exists an optimal solution satisfying this rule) and has some practical applications. This additional assumption allows to simplify the computation of the makespan. Only the completion time of the previous operation has to be considered.

Moreover, we assume that the task schedule follows a no-delay strategy (an event occurs as soon as possible). So, the completion times are equal to the minimal value satisfying constraints (1) to (5). The makespan value is equal to the latest completion time.

Some schedules can lead to situations of deadlock. Such situations are presented on the figures 1a (deadlock with one vehicle) and 1b (deadlock with several vehicles) for jobset 1.

The figure 1a shows a deadlock with one vehicle. The task $\tilde{o}_{33}$ cannot be accomplished before the task $\tilde{o}_{32}$. The figure 1b shows a deadlock with three vehicles. Each vehicle waits for accomplishing their second task (respectively $\tilde{o}_{32}$, $\tilde{o}_{42}$ and $\tilde{o}_{12}$), that another vehicle complete a task scheduled in third position (respectively $\tilde{o}_{31}$, $\tilde{o}_{41}$ and $\tilde{o}_{11}$).

Vehicles	**Task schedule**						
AGV1	$\tilde{o}_{11}$	$\tilde{o}_{33}$	$\tilde{o}_{32}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$	
AGV2	$\tilde{o}_{31}$	$\tilde{o}_{42}$	$\tilde{o}_{13}$	$\tilde{o}_{41}$	$\tilde{o}_{22}$		
AGV3	$\tilde{o}_{21}$	$\tilde{o}_{12}$	$\tilde{o}_{51}$	$\tilde{o}_{43}$	$\tilde{o}_{24}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$

Figure 1a. *Unfeasible solution: deadlock with one vehicle*

Vehicles	**Task schedule**						
AGV1	$\tilde{o}_{51}$	$\tilde{o}_{32}$	$\tilde{o}_{11}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$	
AGV2	$\tilde{o}_{21}$	$\tilde{o}_{42}$	$\tilde{o}_{31}$	$\tilde{o}_{13}$	$\tilde{o}_{33}$		
AGV3	$\tilde{o}_{22}$	$\tilde{o}_{12}$	$\tilde{o}_{41}$	$\tilde{o}_{43}$	$\tilde{o}_{24}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$

Figure 1b. *Unfeasible solution: deadlock with several vehicles*

Deadlock situations are easily detected by our makespan evaluation. Since some tasks cannot be completed, their completion time remains infinite, and so the makespan too.

The presence of unfeasible solutions doesn't facilitate the good exploration of the search space. However it is not the case here, because it was proved that every feasible solution remains reachable from every other feasible solution into the restricted subset of feasible solutions (Deroussi, 2002).

4. Construction of an efficient neighbouring system

Neighbouring system is an essential component of metaheuristics. The efficiency of a neighbouring system can be defined as its faculty to find near optimal solutions in the shortest possible time. In this section, we describe first a neighbouring system composed by several basic moves. It is recognized that combining several moves permits to improve the quality of the obtained local minima (Mladenovic and Hansen, 1997; Hansen and Mladenovic, 2003). Then, a technique is proposed to shrink the neighbourhood size, and so to reduce its exploration at the most promising neighbours. To conclude, summarized results are reported showing the efficiency of the proposed neighbouring system.

4.1. Description of the basic moves

Tian *et al.* (1999) present some usual basic moves for scheduling problems. Amongst them, we choose exchange move and insertion move, because of their effectiveness for shop problems has been proved.

4.1.1. Exchange move

This move consists in permuting two beforehand selected tasks. The exchange moves thus defined can be intra-vehicles (exchanged tasks are on the same vehicle) or inter-vehicles (exchanged tasks are on different vehicles). Figure 2 shows an inter-vehicles exchange move.

Vehicle	**Task scheduling**						
AGV1	$\tilde{o}_{11}$	$\tilde{o}_{31}$	$\tilde{o}_{22}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$	
AGV2	$\tilde{o}_{21}$	$\tilde{o}_{42}$	$\tilde{o}_{13}$	$\tilde{o}_{32}$	$\tilde{o}_{33}$		
AGV3	$\tilde{o}_{41}$	$\tilde{o}_{12}$	$\tilde{o}_{51}$	$\tilde{o}_{43}$	$\tilde{o}_{24}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$

Vehicle	**Task schedule**						
AGV1	$\tilde{o}_{11}$	$\tilde{o}_{31}$	$\tilde{o}_{24}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$	
AGV2	$\tilde{o}_{21}$	$\tilde{o}_{42}$	$\tilde{o}_{13}$	$\tilde{o}_{32}$	$\tilde{o}_{33}$		
AGV3	$\tilde{o}_{41}$	$\tilde{o}_{12}$	$\tilde{o}_{51}$	$\tilde{o}_{43}$	$\tilde{o}_{22}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$

Figure 2. *Illustration of the exchange move; tasks* $\tilde{o}_{22}$ *and* $\tilde{o}_{24}$ *are exchanged*

The exchange move preserves the number of tasks assigned to each vehicle. It results from the fact that local search methods based on this move are strongly dependent on the structure of the initial solution.

This move allows to generate $\frac{o(o-1)}{2}$ neighbours.

4.1.2. Insertion move

The insertion move consists in removing a randomly chosen task from the schedule, and then to insert it at a randomly chosen location (figure 3). Like the exchange moves, the insertion moves can be intra-vehicles or inter-vehicles. However, in the case of inter-vehicles moves, the number of tasks assigned to the vehicles may vary.

Vehicle	**Task scheduling**						
AGV1	$\tilde{o}_{11}$	$\tilde{o}_{31}$	$\tilde{o}_{22}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$	
AGV2	$\tilde{o}_{21}$	$\tilde{o}_{42}$	$\tilde{o}_{13}$	$\tilde{o}_{32}$	$\tilde{o}_{33}$		
AGV3	$\tilde{o}_{41}$	$\tilde{o}_{12}$	$\tilde{o}_{51}$	$\tilde{o}_{43}$	$\tilde{o}_{24}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$

Vehicle	**Task schedule**						
AGV1	$\tilde{o}_{11}$	$\tilde{o}_{31}$	$\tilde{o}_{24}$	$\tilde{o}_{22}$	$\tilde{o}_{14}$	$\tilde{o}_{23}$	$\tilde{o}_{52}$
AGV2	$\tilde{o}_{21}$	$\tilde{o}_{42}$	$\tilde{o}_{13}$	$\tilde{o}_{32}$	$\tilde{o}_{33}$		
AGV3	$\tilde{o}_{41}$	$\tilde{o}_{12}$	$\tilde{o}_{51}$	$\tilde{o}_{43}$	$\tilde{o}_{34}$	$\tilde{o}_{53}$	

Figure 3. *Illustration of the insertion move; task $\tilde{o}_{24}$ is inserted at thirst position on AGV1*

The number of solutions that may be generated starting from the same schedule is given by $o(o-2+k)$.

4.2. Neighbourhood reduction

Neighbourhood reduction consists in selecting, among the set of neighbours of a given solution and according to a given criterion, a subset of neighbours containing those that are considered as the *a priori* most promising ones. Consequently, it is expected that reduction allows to accelerate the neighbourhood exploration.

We propose to determine the most promising neighbours by considering only (exchange or insertion) moves in which the chosen tasks are completed in a close interval of time. The implemented neighbourhood reduction is described in figure 4.

1. Choose a task $\tilde{\sigma}_{i_1 j_1}$.
2. Determine $T\left(\tilde{\sigma}_{i_1 j_1 - 1}\right)$ and $T\left(\tilde{\sigma}_{i_1 j_1 + 1}\right)$, respectively the completion times of the preceding and following tasks of $\tilde{\sigma}_{i_1 j_1}$, according to the sequence of job J_i .
3. Choose the second task $\tilde{\sigma}_{i_2 j_2}$ among the tasks such as $T\left(\tilde{\sigma}_{i_1 j_1 - 1}\right) \le T\left(\tilde{\sigma}_{i_2 j_2}\right) \le T\left(\tilde{\sigma}_{i_1 j_1 + 1}\right)$

Figure 4. *Description of the neighbourhood reduction.*

Once the first operation $\widetilde{\sigma}_{i_1 j_1}$ is chosen, the only moves considered are insertions or exchanges, for which the second operation $\widetilde{\sigma}_{i_2 j_2}$ will satisfy $T\left(\widetilde{\sigma}_{i_1 j_1 - 1}\right) \le T\left(\widetilde{\sigma}_{i_2 j_2}\right) \le T\left(\widetilde{\sigma}_{i_1 j_1 + 1}\right)$ with the following convention:

- $T\left(\widetilde{\sigma}_{i_1 0}\right) = 0$ if $\sigma_{i_1 j_1}$ is the first operation in the sequence of J_{i_1},
- $T\left(\widetilde{\sigma}_{i_1 n_1 + 2}\right) = +\infty$ if $\sigma_{i_1 j_1}$ is the last operation in the sequence of J_{i_1}.

4.3. Summary of the results

We build a neighbouring system by considering two basic moves with a reduction of the neighbourhood size. In this section, we verify the efficiency of this neighbouring system. For this, we compute the average value of 1000 executions of each of the 40 instances that compose our benchmark test. One execution consists in a local search method, starting from a randomly generated feasible solution (using the stochastic alternative of the construction heuristic described in section 5.1.). The duration of an execution is evaluated by counting the calls of the makespan evaluation. The results obtained are summarized in Table 2. The column 'Average cost' indicates the average cost of the obtained local minima. The column 'Number of evaluations' gives the average number of makespan computations necessary for obtaining a local minimum.

Table 2. Fast comparison of neighbouring systems

	Average cost	**Number of evaluations**
Exchange move	135.99	1629
Reduced exchange move	135.84	516
Insertion move	131.07	3872
Reduced Insertion move	132.76	778
Combined reduced moves	125.82	1822

First, we can notice that the neighbourhood reduction significantly improves the behaviour of local search methods. Thus, for the exchange move, a local minimum is obtained three times more quickly with the same quality of local minima. For the insertion move, time is divided by a factor of 5, while a weak loss of quality (1.2%) is noted.

Concerning the combination of reduced basic moves, the quality of the local minima is improved by 7.5% compared to the exchange move, and this in a similar duration. The improvement of quality compared to the insertion move is 4% with a time divided by 2. So, the combination of reduced moves allows to improve the quality of local minima. Our objective is now to include this neighbouring system into several metaheuristics, and to evaluate the efficiency of their behaviour.

5. Description of the resolution methods

We focus in this section on the resolution methods used. We present first the construction heuristic we have developed, and then the metaheuristics according to the Blum and Roli (2003) formulation.

5.1. Construction heuristic

This greedy heuristic use the available tasks earlier defined. The basic principle is to successively schedule each available task by assigning it to a vehicle. When a task $\widetilde{\sigma}_{ij}$ is completed, the set of the available tasks is updated, by replacing $\widetilde{\sigma}_{ij}$ by its successor $\widetilde{\sigma}_{ij+1}$, if it exists. The algorithm 2 describes the construction heuristic.

Algorithm 2: Construction heuristic
Let $D = \{\widetilde{\sigma}_{i1}, i = 1,...,n\}$ be the initial set of available tasks
While $D \neq \varnothing$ **Do**
Choose a task $\widetilde{\sigma}_{ij} \in D$,
Assign $\widetilde{\sigma}_{ij}$ to a vehicle $k \in K$,
Let $D := D / \{\widetilde{\sigma}_{ij}\}$,
If it exist let $D := D \cup \{\widetilde{\sigma}_{ij+1}\}$
End While

Two alternatives are given for the choice of the task $\widetilde{\sigma}_{ij}$ and the vehicle *k*. The first alternative (the stochastic one) builds a random feasible solution, while the second alternative (the deterministic one) tries to build a good feasible solution.

- the task is randomly chosen in D and is accomplished by a randomly chosen vehicle,
- the selected task is the one corresponding to the job having the largest remaining processing time (LPT rule) and the chosen vehicle is the one that will accomplish this task in the shortest time.

5.2. Iterated local search methods

Iterated Local Search methods (ILS) were the subject of interesting development (Lourenço *et al.*, 2003). The principle algorithm of ILS is described in algorithm 3.

The method that we have implemented is one of the simplest forms of ILS. The initial solution (*GenerateInitialSolution*) is given by the construction heuristic (deterministic alternative) described in the previous section. The local search (*LocalSearch*) is a variable neighbourhood descent (VND) composed by the reduced exchange and insertion moves. To perturb a solution (*perturbation*), three successive exchange moves are applied,

while making sure that each of them preserves the feasibility of the current solution. The acceptance criterion (*ApplyAcceptanceCriterion*) consists in accepting the new local minimum $s^{*'}$ if and only if it is better or equal than s^* (Best Walk Criterion). The stopping criterion is a maximum number of iterations arbitrarily fixed at 1000.

Algorithm 3 Iterated local search methods (ILS)

$s_0 \leftarrow$ GenerateInitialSolution()
$s^* \leftarrow$ LocalSearch(s_0)
While stopping criterion is not met **Do**
    $s' \leftarrow$ Perturbation($s^*, history$)
    $s^{*'} \leftarrow$ LocalSearch(s')
    $s^* \leftarrow$ ApplyAcceptanceCriterion($s^*, s^{*'}, history$)
End While

5.3. Simulated annealing

Simulated annealing (SA) is recognized to be one of the oldest metaheuristic (Kirkpatrick *et al.*, 1983). Its principle algorithm is given in algorithm 4.

Algorithm 4 Simulated annealing (SA)

$s \leftarrow$ GenerateInitialSolution()
$T \leftarrow T_0$
While stopping criterion is not met **Do**
    Randomly choose s' in the neighbourhood of s
    If s' is better than s **Then**
        $s \leftarrow s'$
    Else
        $s \leftarrow s'$ according to a probability $p(T, s, s')$
    End If
    Update(T)
End While

We recall that our main goal is to verify the efficiency of the proposed neighbouring system. As consequence, no study of the parameters space has been investigated, and we choose an empirical tuning of parameters. This choice is based on a basic cooling schedule; the temperature decreases at each iteration (as suggested first by Lundy and Mees, 1986) following a geometric law. This allows to determine the other parameters by considering some simple rules (Van Laarhoven and Aarts, 1987) as indicated just below.

An initial solution is randomly generated by the construction heuristic (stochastic alternative). In order that the final solution be independent from the starting point, the initial temperature must be hot enough. So, The parameter T that simulates the temperature is initialised with $T_0 = 5$ (We have then a probability of 0.5 for accepting a deterioration of the current solution of $T_0 \times \ln(2)$ at the beginning of the cooling schedule). The stopping

temperature is arbitrarily fixed at $T_a = 10^{-3}$ (cold enough to consider that the probability for accepting a worse transition is null). As mentioned above, the temperature is a geometric sequence of common ratio T_f. T_f is calculated so as to make a given number of iterations, which is calculated so that the same time has passed for ILS and SA methods. The probability of accepting a worse transition is given by the Boltzmann factor $\exp\left(-\frac{\Delta E}{T}\right)$.

Thus tuned, experimental behaviour of simulated annealing is proven to be quite good.

5.4. Hybrid method simulated annealing / local search

This hybrid method consists in applying the simulated annealing, but on the (very restricted) subspace of local minima rather than to the whole space of solutions. This technique was initially proposed by (Martin and Otto, 1996) and (Desai and Patil, 1996). By analogy with GLS methods ('Genetic Local Search'), we will use the term of SALS ('Simulated Annealing Local Search') to indicate this method. The principle algorithm of SALS is contained into algorithm 3, in which the acceptance criterion is defined by the SA rules.

The adjustments are the same ones as for simulated annealing, except for the choice of the common ratio T_f, which is calculated so as to carry out 1000 iterations (the same ones as for ILS method).

6. Experiments and computational results

After the description of the benchmark test, we present the results obtained by metaheuritics. A comparison with the literature results is provided in the third part of this section.

6.1. Description of the benchmark test

We use the benchmark test suggested by (Bilge and Ülusoy, 1995). It is composed of 40 instances. All of them are made up of one LU station, four machines and two vehicles. The loading/unloading times are neglected compared to the transportation times. Empty moves and transportation moves are supposed to spend the same amount of time.

These instances are generated according to ten jobsets, combined with four different topologies for the workshops (see appendices A and B). Topologies are assumed to be representative of existing systems, and the jobsets include between 5 and 8 jobs, and between 13 and 21 operations to be scheduled. The name given to the instances is composed by the prefix 'Ex', followed of two numbers representing respectively the jobset and the topology. Thus, Ex53 indicates the instance generated by jobset 5 and topology 3.

Moreover, we will consider two criteria for the objective function:

- C1: to minimise the exit time of the last job of the system,
- C2: to minimise the completion time of the last job (makespan).

The C2 criterion is used in the literature and we consider it for comparison purpose. The C1 criterion takes into account, in the optimisation process, the return of the vehicles to the unloading station at the end of the schedule. Let us notice that there is a close link between these two criteria. Indeed, they both seek to minimise the completion time of the last task carried out. C1 criterion consider all the tasks $\{\widetilde{\sigma}_{ij} \in \widetilde{O}, i = 1..n, j = 1..n_i + 1\}$, while C2 consider only the tasks corresponding to non-fictitious operations $\{\widetilde{\sigma}_{ij} \in \widetilde{O}, i = 1..n, j = 1..n_i\}$. It will be expected that C1 would be more difficult, since it considers more tasks, and it is thus of higher combinatorial.

Although these instances are of relatively low size (13-21 tasks to be scheduled for C2 and 18-27 tasks for C1), they remain very difficult to solve. To date, few instances were solved in an exact way, and the best published lower bounds are of poor quality (Ulusoy *et al*., 1997).

The best known solutions are summarised in table 3. They will be useful mainly as comparative data for the results that we present. Programs are written in language C, and tested on a Pentium 4, 3.4 GHz. CPU times is between one and four seconds per algorithm execution.

Table 3. Best known upper bound for the benchmark test

	Criterion	
Instances	**C1**	**C2**
Ex11	114	96
Ex12	90	82
Ex13	98	84
Ex14	140	103
Ex21	116	100 (102)
Ex22	82	76
Ex23	89	86
Ex24	134	108
Ex31	121	99
Ex32	89	85
Ex33	96	86
Ex34	148	111
Ex41	138	112
Ex42	100	87 (88)
Ex43	102	89
Ex44	163	121 (126)
Ex51	110	87
Ex52	81	69
Ex53	89	74
Ex54	134	96

	Criterion	
Instances	**C1**	**C2**
Ex61	129	118
Ex62	102	98
Ex63	105	103 (104)
Ex64	151*	120
Ex71	133*	111 (115)
Ex72	86	79
Ex73	93	83 (86)
Ex74	161	126 (127)
Ex81	167	161
Ex82	155	151
Ex83	155	153
Ex84	178	163
Ex91	127*	116 (117)
Ex92	106	102
Ex93	107	105
Ex94	149	120 (122)
Ex101	153	147
Ex102	139	135 (136)
Ex103	140*	138 (141)
Ex104	183	159

Values between brackets give the previously best published upper bound (C2 criterion only)
The * symbol means that these upper bounds have been obtained beside the experiments described in this paper

6.2. Comparison of the studied methods

Table 4 contains the results obtained by the deterministic construction heuristic for each instance and each criterion. Column 'res' indicates the makespan obtained, while the column '%' gives the relative gap compared to the best known upper bound (table 3).

Table 4. Results obtained with the construction heuristic

	C1		C2			C1		C2	
Instances	**res**	**%**	**res**	**%**	**Instances**	**res**	**%**	**res**	**%**
Ex11	152	33.33	114	18.75	**Ex61**	175	35.66	138	16.95
Ex12	106	17.78	90	9.76	**Ex62**	136	33.33	122	24.49
Ex13	110	12.24	88	4.76	**Ex63**	147	40.00	124	20.39
Ex14	174	24.29	124	20.39	**Ex64**	213	41.06	143	19.17
Ex21	154	32.76	136	36.00	**Ex71**	200	50.38	140	26.13
Ex22	122	48.78	118	55.26	**Ex72**	136	58.14	110	39.24
Ex23	134	50.56	128	48.84	**Ex73**	149	60.22	111	33.73
Ex24	202	50.75	160	48.15	**Ex74**	258	60.25	160	26.98
Ex31	158	30.58	120	21.21	**Ex81**	203	21.56	197	22.36
Ex32	108	21.35	94	10.59	**Ex82**	181	16.77	177	17.22
Ex33	116	20.83	96	11.63	**Ex83**	185	19.35	183	19.61
Ex34	190	28.38	134	20.72	**Ex84**	252	41.57	219	34.36
Ex41	194	40.58	156	39.29	**Ex91**	170	33.86	141	21.55
Ex42	150	50.00	127	45.98	**Ex92**	139	31.13	125	22.55
Ex43	161	57.84	131	47.19	**Ex93**	144	34.58	127	20.95
Ex44	227	39.26	175	44.63	**Ex94**	211	41.61	154	28.33
Ex51	146	32.73	112	28.74	**Ex101**	208	35.95	184	25.17
Ex52	103	27.16	92	33.33	**Ex102**	184	32.37	180	33.33
Ex53	117	31.46	100	35.14	**Ex103**	195	39.29	182	31.88
Ex54	172	28.36	134	39.58	**Ex104**	249	36.07	199	25.16

The construction heuristic provides a mean deviation from the value of the best known solution of 36% for the C1 criterion and of 28% for the C2 criterion. These results clearly show the relative weakness of the heuristic, and the necessity of improving the obtained solutions with metaheuristics.

The metaheuristics results are presented in tables 5 (columns 'ILS', 'SA' and 'SALS' gives the results of the corresponding method, while the column 'Time' indicates the CPU time in seconds for one algorithm execution). Table 5a contains results for C1 criterion and Table 5b for C2 criterion. Values in bold type indicate that the corresponding method has found the best known upper bound. Furthermore, because SA and SALS are stochastic algorithms, 10 replications are carried out for each instance so as to limit the random influence on the results. Tables 5 give the results obtained by the best replication.

In a general way, the results obtained are homogeneous. As an example, the three methods give the same results for 34 of the 40 instances tested for the C1 criterion. In addition, ILS, SA and SALS find the best known upper bound respectively for 35, 37 and all instances. More scattered results are obtained for C2 criterion (the best known upper bound is reached respectively 22, 28 and 36 times by ILS, SA and SALS). It seems that the additional tasks make effectively the instances harder to solve.

Table 5a. Comparison of the three studied metaheuristics (C1 criterion)

Instances	ILS	SA	SALS	Time (s)	Instances	ILS	SA	SALS	Time (s)
Ex11	**114**	**114**	**114**	**1.3**	**Ex61**	130	**129**	**129**	**2.7**
Ex12	**90**	**90**	**90**	**1.4**	**Ex62**	**102**	**102**	**102**	**2.5**
Ex13	**98**	**98**	**98**	**1.4**	**Ex63**	106	**105**	**105**	**2.6**
Ex14	**140**	**140**	**140**	**1.3**	**Ex64**	**151**	**151**	153	**2.6**
Ex21	118	118	**116**	**2.3**	**Ex71**	134	137	135	**3.9**
Ex22	**82**	**82**	**82**	**2.3**	**Ex72**	90	**86**	**86**	**4.0**
Ex23	90	90	**89**	**2.2**	**Ex73**	98	**93**	**93**	**3.7**
Ex24	138	136	**134**	**2.1**	**Ex74**	167	163	**161**	**3.8**
Ex31	**121**	**121**	**121**	**2.2**	**Ex81**	**167**	**167**	**167**	**3.0**
Ex32	**89**	**89**	**89**	**2.2**	**Ex82**	**155**	**155**	**155**	**2.4**
Ex33	**96**	**96**	**96**	**2.1**	**Ex83**	**155**	**155**	**155**	**2.4**
Ex34	149	**148**	**148**	**2.1**	**Ex84**	**178**	**178**	**178**	**3.3**
Ex41	**138**	**138**	**138**	**2.1**	**Ex91**	129	129	129	**1.9**
Ex42	102	**100**	**100**	**2.2**	**Ex92**	**106**	**106**	**106**	**1.9**
Ex43	106	104	**102**	**2.1**	**Ex93**	**107**	**107**	**107**	**1.9**
Ex44	164	**163**	**163**	**2.3**	**Ex94**	**149**	150	**149**	**1.9**
Ex51	**110**	**110**	**110**	**1.3**	**Ex101**	156	**153**	**153**	**3.3**
Ex52	**81**	83	**81**	**1.4**	**Ex102**	140	**139**	**139**	**3.1**
Ex53	**89**	**89**	**89**	**1.4**	**Ex103**	141	141	141	**3.0**
Ex54	**134**	136	**134**	**1.3**	**Ex104**	187	184	**183**	**3.3**

Table 5b. Comparison of the three studied metaheuristics (C2 criterion)

Instances	ILS	SA	SALS	Time (s)	Instances	ILS	SA	SALS	Time (s)
Ex11	**96**	**96**	**96**	**1.3**	**Ex61**	**118**	**118**	**118**	**2.5**
Ex12	**82**	**82**	**82**	**1.2**	**Ex62**	**98**	**98**	**98**	**2.3**
Ex13	**84**	**84**	**84**	**1.3**	**Ex63**	**103***	**103***	**103***	**2.4**
Ex14	**103**	**103**	**103**	**1.3**	**Ex64**	**120**	**120**	**120**	**2.5**
Ex21	**100***	**100***	**100***	**2.1**	**Ex71**	**111***	**111***	**111***	**3.9**
Ex22	**76**	**76**	**76**	**2.0**	**Ex72**	**79**	**79**	**79**	**3.8**
Ex23	**86**	**86**	**86**	**2.0**	**Ex73**	84*	**83***	**83***	**3.6**
Ex24	**108**	**108**	**108**	**2.1**	**Ex74**	**126***	**126***	**126***	**3.9**
Ex31	**99**	**99**	**99**	**2.0**	**Ex81**	**161**	**161**	**161**	**2.6**
Ex32	**85**	**85**	**85**	**1.9**	**Ex82**	**151**	**151**	**151**	**2.3**
Ex33	**86**	**86**	**86**	**1.9**	**Ex83**	**153**	**153**	**153**	**2.3**
Ex34	**111**	**111**	**111**	**2.1**	**Ex84**	**163**	**163**	**163**	**2.9**
Ex41	**112**	**112**	**112**	**2.1**	**Ex91**	**116***	**116***	**116***	**1.8**
Ex42	88	88	**87***	**2.1**	**Ex92**	**102**	**102**	**102**	**1.8**
Ex43	**89**	**89**	**89**	**2.1**	**Ex93**	**105**	**105**	**105**	**1.8**
Ex44	**121***	125*	**121***	**2.0**	**Ex94**	**120***	**120***	**120***	**1.8**
Ex51	**87**	**87**	**87**	**1.3**	**Ex101**	148	**147**	**147**	**3.0**
Ex52	**69**	**69**	**69**	**1.3**	**Ex102**	**135***	**135***	**135***	**2.9**
Ex53	**74**	**74**	**74**	**1.3**	**Ex103**	139*	139*	**138***	**2.9**
Ex54	**96**	**96**	**96**	**1.3**	**Ex104**	160	**159**	**159**	**3.1**

The * symbol indicates new upper bounds.

In order to have more information about the robustness of SA and SALS, we give in table 6 the best, the worst and the average of all the replications for the two criteria.

We can notice that most of the execution of SALS leads to very good solutions (on average, a replication is at 0.34% of the best obtained replication for the C2 criterion, and 0.96% for the C1 criterion), while the results of SA are more scattered (we obtain respectively 1.47% for C2 and 2.67% for C1). If the best replications of SALS and SA are close (respectively 108.48 against 108.63 for C2 criterion, 124.00 against 124.38 for C1), there is a significant difference about their average replication (108.85 against 110.23 for C2 criterion, 125.19 against 127.70 for C1)

So, we can conclude that SALS is an effective method for the studied problem. In the next section, we compare it with methods proposed in the literature.

Table 6. Robustness of SA and SALS

	C1 criterion						C2 criterion					
	SALS			SA			SALS			SA		
Instances	**Min**	**Max**	**Avg**	**Min**	**Max**	**Avg**	**Min**	**Max**	**Avg**	**Min**	**Max**	**Avg**
Ex11	114	116	114.6	114	126	118.6	96	96	96.0	96	97	96.2
Ex12	90	92	90.2	90	94	91.5	82	82	82.0	82	82	82.0
Ex13	98	98	98	98	102	99.9	84	84	84.0	84	86	84.8
Ex14	140	142	141.4	140	148	144.9	103	103	103.0	103	106	104.3
Ex21	116	120	117.8	118	125	120.7	100	102	100.2	100	107	103.2
Ex22	82	84	82.2	82	90	83.7	76	77	76.1	76	77	76.1
Ex23	89	92	89.9	90	94	91.8	86	86	86.0	86	86	86.0
Ex24	134	140	138.4	136	148	142	108	108	108.0	108	113	108.9
Ex31	121	128	123.2	121	129	125.8	99	99	99.0	99	106	101.8
Ex32	89	89	89	89	97	92.6	85	85	85.0	85	86	85.1
Ex33	96	100	97	96	102	99.5	86	86	86.0	86	86	86.0
Ex34	148	153	149.9	148	162	155.3	111	112	111.8	111	116	112.8
Ex41	138	140	138.6	138	144	140.8	112	115	113.2	112	123	117.8
Ex42	100	104	101.4	100	112	103.3	87	90	88.6	88	95	90.7
Ex43	102	107	104.7	104	112	107.7	89	93	89.5	89	95	91.5
Ex44	163	166	164.3	163	175	169.9	121	126	122.7	125	134	128.7
Ex51	110	110	110	110	118	112.5	87	87	87.0	87	91	88.6
Ex52	81	81	81	83	86	84.0	69	69	69.0	69	69	69.0
Ex53	89	91	89.6	89	97	92.2	74	75	74.2	74	76	75.4
Ex54	134	138	136.2	136	151	141.4	96	97	96.2	96	105	98.6
Ex61	129	133	131.1	129	137	133.8	118	118	118.0	118	124	120.1
Ex62	102	105	103.2	102	108	103.9	98	100	98.2	98	98	98.0
Ex63	105	107	105.6	105	109	107	103	104	103.3	103	105	104.5
Ex64	153	156	155	151	164	156.8	120	123	120.6	120	130	124.8
Ex71	135	140	136.4	137	144	140.0	111	114	111.8	111	118	114.4
Ex72	86	91	89.1	86	94	89.1	79	80	79.3	79	82	80.0
Ex73	93	98	94.5	93	107	96.9	83	85	83.4	83	90	85.8
Ex74	161	167	163.7	163	173	168.6	126	128	126.6	126	135	128.9
Ex81	167	167	167	167	169	167.4	161	161	161.0	161	161	161.0
Ex82	155	155	155	155	155	155	151	151	151.0	151	151	151.0
Ex83	155	155	155	155	155	155	153	153	153.0	153	153	153.0
Ex84	178	179	178.1	178	188	182.6	163	163	163.0	163	163	163.0
Ex91	129	130	129.4	129	136	132.6	116	117	116.4	116	122	118.0
Ex92	106	106	106	106	109	107.3	102	102	102.0	102	104	103.0
Ex93	107	110	107.3	107	110	108.4	105	105	105.0	105	108	106.2
Ex94	149	151	149.7	150	159	152.8	120	123	121.8	120	126	123.0
Ex101	153	159	155.7	153	163	158.1	147	151	147.8	147	154	148.5
Ex102	139	141	139.6	139	142	140.1	135	137	135.2	135	138	135.9
Ex103	141	143	141.6	141	145	142.4	138	141	139.5	139	144	140.9
Ex104	183	190	187.0	184	197	192	159	161	159.6	159	167	161.8
Average	**124.00**	**126.85**	**125.19**	**124.38**	**131.90**	**127.70**	**108.48**	**109.73**	**108.85**	**108.63**	**112.73**	**110.23**

6.3. Comparison with the literature

Table 7 summarizes the best results obtained for C2 criterion (the only one studied in the literature):

(i) by a synthesis of three papers ((Ülusoy and Bilge, 1993), (Bilge and Ülusoy, 1995) and (Ülusoy *et al.*, 1997)) (B&U),

(ii) by the genetic algorithm of (Abdelmaguid *et al.*, 2004) (GAA),

(iii) by our hybrid metaheuristic SALS.

A performance criterion indicates, for each method, the relative gap compared to the best published upper bound (indicated in bold type). The new upper bounds we have obtained are marked with the * symbol.

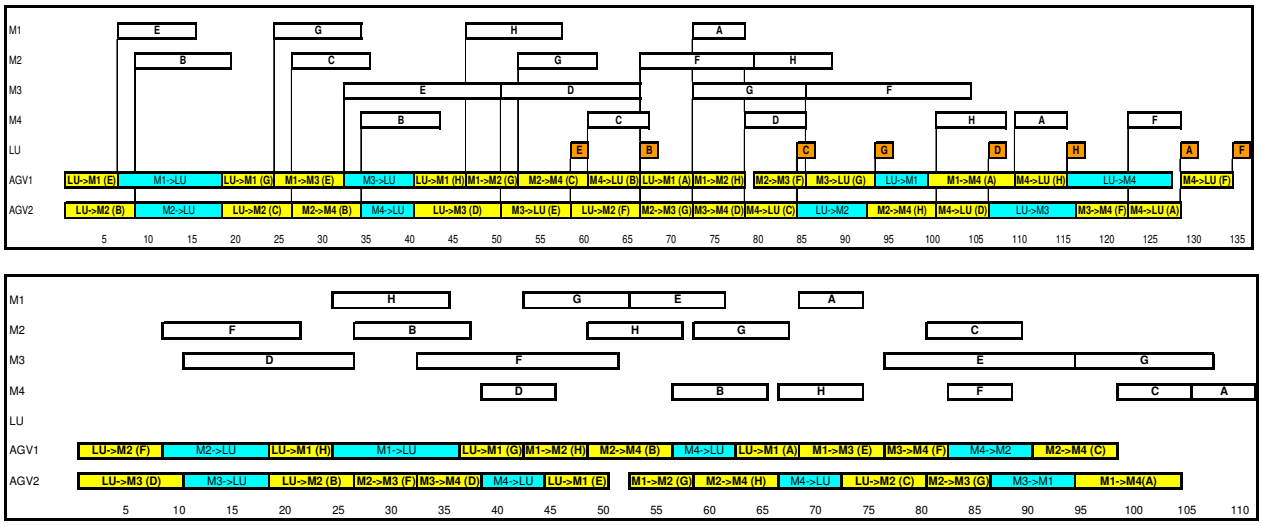
Table 7. Summary of the literature results (C2 criterion)

Inst.	B&U		GAA		SALS	
Ex11	**96**	0.00	**96**	0.00	**96**	0.00
Ex12	**82**	0.00	**82**	0.00	**82**	0.00
Ex13	**84**	0.00	**84**	0.00	**84**	0.00
Ex14	**103**	0.00	**103**	0.00	**103**	0.00
Ex21	104	1.96	**102**	0.00	**100***	-1.96
Ex22	**76**	0.00	**76**	0.00	**76**	0.00
Ex23	**86**	0.00	**86**	0.00	**86**	0.00
Ex24	113	4.63	**108**	0.00	**108**	0.00
Ex31	105	6.06	**99**	0.00	**99**	0.00
Ex32	**85**	0.00	**85**	0.00	**85**	0.00
Ex33	**86**	0.00	**86**	0.00	**86**	0.00
Ex34	113	1.80	**111**	0.00	**111**	0.00
Ex41	116	3.57	**112**	0.00	**112**	0.00
Ex42	**88**	0.00	**88**	0.00	**87***	-1.14
Ex43	91	2.25	**89**	0.00	**89**	0.00
Ex44	**126**	0.00	**126**	0.00	**121***	-3.97
Ex51	**87**	0.00	**87**	0.00	**87**	0.00
Ex52	**69**	0.00	**69**	0.00	**69**	0.00
Ex53	75	1.35	**74**	0.00	**74**	0.00
Ex54	97	1.04	**96**	0.00	**96**	0.00

Inst.	B&U		GAA		SALS	
Ex61	121	2.54	**118**	0.00	**118**	0.00
Ex62	**98**	0.00	**98**	0.00	**98**	0.00
Ex63	**104**	0.00	**104**	0.00	**103***	-0.96
Ex64	123	2.50	**120**	0.00	**120**	0.00
Ex71	118	2.61	**115**	0.00	**111***	-3.48
Ex72	85	7.59	**79**	0.00	**79**	0.00
Ex73	88	2.33	**86**	0.00	**83***	-3.49
Ex74	128	0.79	**127**	0.00	**126***	-0.79
Ex81	**161**	0.00	**161**	0.00	**161**	0.00
Ex82	**151**	0.00	**151**	0.00	**151**	0.00
Ex83	**153**	0.00	**153**	0.00	**153**	0.00
Ex84	**163**	0.00	**163**	0.00	**163**	0.00
Ex91	**117**	0.00	118	0.85	**116***	-0.85
Ex92	**102**	0.00	104	1.96	**102**	0.00
Ex93	**105**	0.00	106	0.95	**105**	0.00
Ex94	123	0.82	**122**	0.00	**120***	-1.64
Ex101	150	2.04	**147**	0.00	**147**	0.00
Ex102	137	0.74	**136**	0.00	**135***	-0.74
Ex103	143	1.42	**141**	0.00	**138***	-2.13
Ex104	164	3.14	**159**	0.00	**159**	0.00

These results clearly show the superiority of SALS. Our method always finds a result at least as good as those so far published. Moreover, new upper bounds are found for 11 of the 40 instances. SALS finds better solutions than B&U for 23 instances, and than GAA for 13 instances. The improvements are sometimes significant; a solution of cost 111 is obtained against 115 for Ex71 instance, that is to say a relative gain of 3.6 % (figure 5 shows, using a Gantt chart representation, the solutions obtained for Ex71 with each of the two criteria).

Still more than quality of the solutions obtained, it is the frequency of obtaining the best results which is most impressive, and this in extremely reasonable computing times for an optimisation problem (about a few seconds). The behaviour of the method lets suggest that the proposed solutions must be for many the optimal ones. This remark still increases the interest of the C1 criterion, which makes the instances more difficult to solve. Even if there are only few additional transports to schedule, we notice that SALS loses in robustness for these 'new' instances (see table 6), and does not systematically find the best solutions (for at least four instances). These results might constitute a new reference for future studies, more able to measure the quality of the methods.



Figures 5a and 5b. *Gantt charts of the new best known solutions for Ex71 instance (C1 and C2 criteria). The two Gantt charts show the machining of the 8 jobs (numbered from A to H) on the machines (numbered from M1 to M4). On the figure 5a, the unloading station LU allows to point out, for each job, its exit date of the system. The schedule of the vehicles (AGV1 and AGV2) gives the origin machine, the destination machine, and, if non-empty, the transported job.*

7. Conclusion and further works

In this paper, we propose a new effective representation solution for the job-shop problem with transportation. A powerful neighbouring system is described and implemented with three different metaheuristics. The results obtained clearly show the relevance of this approach. Whatever the method used, the presented results are always better or at least equal to those of the literature. one of these methods, SALS, combines robustness with quality, and is a particularly efficient method for this problem.

Also, we propose a new criterion for the solution evaluation, which allows to extend the literature benchmark test with more challenging instances.

Many perspectives can be drawn from this work.

Concerning the studied problem, the benchmark test proposed by (Bilge and Ülusoy, 1995) seems out-of-date, and it would have to be extended by more difficult instances. Concurrently to this extension, and paradoxically, the exact resolution of most of these instances is still today out of reach, and constitutes an always topical orientation research.

The consideration of online dispatching rules leads to define a more difficult makespan evaluation, which can require the use of discrete event simulation. Combining simulation with optimization methods can be a very interesting perspective to our work.

Lastly, the methods used in this paper are trajectory metaheuristics. The fact of carrying out several replications, and of keeping in memory the solution obtained by the best of them, can be seen like the use of a population method, in which the individuals are private of any possibility of communicating. Restoring this possibility (by using a population method, like a genetic algorithm) may allow to improve the results proposed in this paper.

8. Appendixes

8.1. Appendix A: The jobsets

For each job, the sequence of machines to be visited is given. The processing time is indicated in brackets.

Jobset 1:
Job 1 : M1(8); M2(16); M4(12)
Job 2 : M1(20); M3(10); M2(18)
Job 3: M3(12); M4(8); M1(15)
Job 4: M4(14); M2(18)
Job 5: M3(10); M1(15)

Jobset 2:
Job 1 : M1(10); M4(18)
Job 2: M2(10); M4(18)
Job 3: M1(10); M3(20)
Job 4: M2(10); M3(15); M4(12)
Job 5: M1(10); M2(15); M4(12)
Job 6: M1(10); M2(15); M3(12)

Jobset 3:
Job 1: M1(16); M3(15)
Job 2: M2(18); M4(15)
Job 3: M1(20); M2(10)
Job 4: M3(15); M4(10)
Job 5: M1(8); M2(10); M3(15); M4(17)
Job 6: M2(10); M3(15); M4(8); M1(15)

Jobset 4:
Job 1: M4(11); M1(10); M2(7)
Job 2: M3(12); M2(10); M4(8)
Job 3: M2(7); M3(10); M1(9); M3(8)
Job 4: M2(7); M4(8); M1(12); M2(6)
Job 5: M1(9);M2(7);M4(8);M2(10);M3(8)

Jobset 5:
Job 1: M1(6); M2(12); M4(9)
Job 2: M1(18); M3(6); M2(15)
Job 3: M3(9); M4(3); M1(12)
Job 4: M4(6); M2(15)
Job 5: M3(3); M1(9);

Jobset 6:
Job 1: M1(9); M2(11); M4(7)
Job 2: M1(19); M2(20); M4(13)
Job 3: M2(14); M3(20); M4(9)
Job 4: M2(14); M3(20); M4(9)
Job 5: M1(11); M3(16); M4(8)
Job 6: M1(10); M3(12); M4(10)

Jobset 7:
Job 1: M1(6); M4(6)
Job 2: M2(11); M4(9)
Job 3: M2(9); M4(7)
Job 4: M3(16); M4(7)
Job 5: M1(9); M3(18)
Job 6: M2(13); M3(19); M4(6)
Job 7: M1(10); M2(9); M3(13)
Job 8: M1(11); M2(9); M4(8)

Jobset 8:
Job 1: M2(12); M3(21); M4(11)
Job 2: M2(12); M3(21); M4(11)
Job 3: M2(12); M3(21); M4(11)
Job 4: M2(12); M3(21); M4(11)
Job 5: M1(10); M2(14); M3(18); M4(9)
Job 6: M1(10); M2(14); M3(18); M4(9)

Jobset 9:
Job 1: M3(9); M1(12); M2(9); M4(6)
Job 2: M3(16); M2(11); M4(9)
Job 3: M1(21); M2(18); M4(7)
Job 4: M2(20); M3(22); M4(11)
Job 5: M3(14); M1(16); M2(13); M4(9)

Jobset 10:
Job 1: M1(11); M3(19); M2(16); M4(13)
Job 2: M2(21); M3(16); M4(14)
Job 3: M3(8); M2(10); M1(14); M4(9)
Job 4: M2(13); M3(20); M4(10)
Job 5: M1(9); M3(16); M4(18)
Job 6: M2(19); M1(21); M3(11); M4(15)

8.2. Appendix B: The topologies

The transport times to go from any machine to another one is given in the following matrix:

FMS1	LU	M1	M2	M3	M4
LU	0	6	8	10	12
M1	12	0	6	8	10
M2	10	6	0	6	8
M3	8	8	6	0	6
M4	6	10	8	6	0

FMS2	LU	M1	M2	M3	M4
LU	0	4	6	8	6
M1	6	0	2	4	2
M2	8	12	0	2	4
M3	6	10	12	0	2
M4	4	8	10	12	0

FMS3	LU	M1	M2	M3	M4
LU	0	2	4	10	12
M1	12	0	2	8	10
M2	10	12	0	6	8
M3	4	6	8	0	2
M4	2	4	6	12	0

FMS4	LU	M1	M2	M3	M4
LU	0	4	8	10	14
M1	18	0	4	6	10
M2	20	14	0	8	6
M3	12	8	6	0	6
M4	14	14	12	6	0

9. REFERENCES

Abdelmaguid, T.F., Nassef, O.N., Kamal, B.A., Hassan, M.F., 2004, A hybrid GA/heuristic approach to the simultaneous scheduling of machines and automated guided vehicles. *International Journal of Production Research*, 42, 267-281.

Bilge, Ü., and Ülusoy, G., 1995 A time window approach to simultaneous scheduling of machines and material handling system in a FMS. *Operation Research*, 43, 1058-1070.

Blazewicz, J., Eiselt, H.A., Finke, G., Laporte, G., Weglartz, J., 1991, Scheduling tasks and vehicles in a flexible manufacturing system. *International Journal of Flexible Manufacturing System*, 4, 5-16.

Blum, C., and Roli, A., 2003, Metaheuristics in combinatorial optimisation: overview and conceptual comparison. *ACM Computing Surveys*, 35, 268-308.

Brucker, P., 2001, Scheduling Algorithms, Third edition, Springer-Verlag, Berlin.

Deroussi, L., 2002, Heuristiques, métaheuristiques et systèmes de voisinage : application à des problèmes théoriques et industriels de type TSP et ordonnancement. Ph. D. Thesis (in French), University of Clermont-Ferrand II, France.

Desai, R., and Patil, R., 1996, SALO: Combining simulated annealing and local optimisation for efficient global optimisation. Technical Report LA-UR, available at http://citeseer.nj.nec.com/desai96salo.html .

Egbelu, P.J., and Tanchoco, J.M.A., 1984, Characterisation of automated guided vehicle dispatching rules. *International Journal of Production Research*, 22, 359-374.

Ganesharajah, T., Hall, N.G., Sriskandarajah, C., 1998, Design and Operational Issues in AGV-Served Manufacturing Systems, *Annals of Operations Research*, 76, 109-154.

Gobal, S.L., and Kasilingam, R.G., 1991, A simulation model for estimating vehicle requirements in automated guided vehicle systems, *Computers Industrial Engg*, 21, 623-627.

Graham, R.L., Lawler, E.L., Lenstra J.K., Rinnooy Kan., A.H.G., 1979, Optimisation and approximation in deterministic sequencing and scheduling: a survey. *Annals of Discrete Mathematics*, 5, 236-287.

Hansen, P., and Mladenovic, N., 2003, Variable Neighborhood search. In F. Glover and G. Kochenberger (eds.), *Handbook of Metaheuristics*, Kluwers Academic Publishers, 145-184.

Hurink, J.L., and Knust, S., 2002, Tabu A tabu search algorithm for scheduling a single robot in a job-shop environment. *Discrete Applied Mathematics*, 119, 181-203.

Kirkpatrick, S., Velatt, C.D. Vecchi, M.P., 1983, Optimisation by simulated annealing. *Sciences*, 222, 671-680.

Knust, S., 1999, Shop-Scheduling Problems with Transportation, Ph. D. Thesis, Fachbereich Mathematik/Informatik Universität Osnabrück.

Le-Anh, T., and De Koster, M.B.M., 2005, On-line dispatching rules for vehicle-based internal transport systems, *International Journal of Production Research*, 43, 1711-1728.

Le-Anh, T., and De Koster, M.B.M., 2006, A review of design and control of automated guided vehicle systems, *European Journal of Operational Research*, 121, 1-23.

Lenstra, J.K., and Rinnooy Kan, A.H.G., 1979, Computational complexity of discrete optimization problems. *Annals of Discrete Mathematics* 4, 121-140.

Lenstra, J.K., and Rinnooy Kan, A.H.G., 1981, Complexity of vehicle routing and scheduling problems, *Networks* 11, 221-227.

Lourenço, H.R., Martin, O.C., Stützle, T., 2003, Iterated local search. In F. Glover and G. Kochenberger (eds.), *Handbook of Metaheuristics*, Kluwers Academic Publishers, 321-353.

Lundy, M., and Mees, A., 1986, Convergence of an annealing algorithm, *Mathematical Programming*, 34, 111-124.

Martin, O.C., and Otto, S.W., 1996, Combining simulated annealing with local search heuristics. *Annals of Operations Research*, 63, 57-75.

Mladenovic, N., and Hansen, P., 1997; Variable Neighborhood Search. *Computers & Operations Research,* 24, 1097-1100.

Nuhut, O., 1999, Scheduling of Automated Guided Vehicles. Available at http://citeseer.nj.nec.com/nuhut99scheduling.html

Qiu, L., Hsu, W., Huang, S.Y., Wang, H., 2002, Scheduling and routing algorithms for AGVs: a survey, *International Journal of Production Research*, 40, 745-760.

Raman, N., Talbot, F.B., Rachamadgu, R.V., 1986, Simultaneous scheduling of machines and material handling devices in automated manufacturing. In Proceedings of the 2nd ORSA/TIMS Conference on Flexible Manufacturing Systems, pp. 455-466.

Revielotis, S.A., 2000, Conflict resolution in AGV systems. *IIE Transactions*, 647-659.

Soylu, M., Özdemirel, N.E., Kayaligil, S., 2000, A self-organising neural network approach for the single AGV routing problem. *European Journal of Operational Research*, 121, 124-137.

Tian, P., Ma, J., Zhang, D.M., 1999, Application of the simulated annealing algorithm to the combinatorial optimisation problem with permutation property: an investigation of generation mechanism. *European Journal of Operational Research*, 118, 81-94.

Ulusoy, G., and Bilge, Ü., 1993, Simultaneous scheduling of machines and automated guided vehicles. *International Journal of Production Research*, 31, 2857-2873.

Ülusoy, G., Sivrikaya-Serifoglu, F., Bilge, Ü., 1997, A genetic algorithm approach to the simultaneous scheduling of stations and automated guided vehicles. *Computers Operations Research*, 24, 335-351.

Van Laarhoven, P.J.M., and Aarts, E.H.L., 1987, Simulated Annealing: Theory and Applications, *Kluwer Academic Publishers*, Berlin.