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Impact of RSA Optimization Objectives on Optical Network State

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Abstract—This paper focuses on off-line dimensioning of flexgrid networks. As any operator wishes to satisfy all traffic demands characterized by a traffic matrix, considering blocking as an optimization objective is not appropriate. We thus consider two other objectives, one of them related to path lengths and the other related to spectrum occupancy. We also define several indicators to characterize a mapping of demands on a given network, including some which are related to the remaining capacity of the network. We apply off-line dimensioning algorithms with the two different optimization objectives on a traffic matrix to be supported by a small flexgrid network and assess the impact of the selected optimization objective on the network level metrics in order to provide insight on how such an off-line dimensioning should be performed.

Index Terms—RSA, Optical network, flexgrid, optimization

I. INTRODUCTION

The emergence of coherent transponders offering variable baud rate and modulation formats, together with the evolution of WDM from fixed to variable frequency grid, generated a great interest in the optical networking community on the Routing and Spectral Assignment (RSA) problem and related topics such as spectrum fragmentation.

Optical networks design presents two successive phases: first, in the “off-line” or dimensioning phase, which occurs before a network is deployed, a set of known traffic demands (expressed as a traffic matrix) are mapped on a (previously empty) network. Afterwards, in the “on-line” or operational phase, new traffic demands arrive, stay for sometime and eventually depart; these should be accommodated on top of the current load of the network.

The off-line network dimensioning process is addressed in the present work. A classical method for solving this problem is to derive exact or heuristic formulations of an optimization problem with a suitable objective function. The choice of optimization objective is not straightforward and there is no clear indications in literature which is the best optimization objective that yields the best network state.

Network cost seems a natural choice, but the way to assess this cost raises many questions. Should the cost only include CAPEX or should it take account of amortization and operational expenses over a given period? How to account for technology and cost evolution and traffic changes during that period? In other words, it is difficult to formulate pertinently the off-line problem without having an idea of the future

evolution of the network, which corresponds to the on-line problem. Another well-known approach considered for (optical) network dimensioning consists in computing the blocking probability in presence of statistical variations of the global set of demands. This method has the merit to account for some demand variability but it has a major drawback, as it does not yield a resource allocation that ensures that all demands are satisfied. Therefore, it cannot be used for an operational deployment.

Thus we adopted the approach of satisfying a static set of demands while optimizing a given objective function and tried to reach a better understanding of the pros and cons of different objective functions. This approach was followed e.g. in [1] for the off-line design of IP networks. Usually in literature only one metric is optimized and no comparison is done between solutions achieved with different optimization objectives. To perform such a comparison, we defined a set of optical “network health” metrics, in order to evaluate the network ability to accommodate new demands or some growth of the existing ones. Some of these metrics can be used as objective functions, other cannot, depending on the possibility of applying them to the formulation that we adopted. We will then use them with different objective functions and compare the values of the set of metrics for the different solutions corresponding to these objective functions. Network health metrics are expected to play various roles: firstly, to help analysing the characteristics of the network dimensioning obtained with different objective functions and secondly to evaluate the ability of the different designs to accept a “reasonable” amount of additional traffic demands. Ideally, some of these metrics could serve, when crossing a threshold value, as a decision tool for setting up additional resources in some parts of the network in order to prevent a possible congestion during network operation.

In the context of the RSA problem, the evaluation of how much the available spectrum has been fragmented has generated many studies. In particular, several “fragmentation metrics” have been proposed. It seems clear that increasing fragmentation contributes to reducing the optical spectrum availability and is worth to be considered when assessing network state. However, fragmentation metrics are generally measured at link level and it is not easy to aggregate them into network level indicators. This kind of problem concerns the construction of any network metric derived from elementary

one (e.g. at link or path level). In particular, an important question is whether it is more relevant to average individual values or to consider worst case values.

The rest of the paper is organized as follows: Section II identifies relevant works related to network level metrics. Section III introduces the optimization algorithms used to map a given traffic matrix on a given network. In Section IV, several network level metrics are classified into some that characterize the state of the network and others that estimate the remaining capacity while Section V applies optimization algorithms to a practical dimensioning problem. Section VI concludes the paper.

II. RELATED WORKS

A metric can be used either as an optimization criterion or as a network health indicator.

A. Optimization metrics

In [2], [3], the proposed optimization objective metric aims to minimize the volume of unsatisfied traffic. In [4], the proposed optimization process minimizes the sum of lengths of the paths carrying the demands, which aims to route demands over shortest paths in order to minimize global spectrum occupancy and delay. In [5], the optimization process minimizes the total transponders cost necessary to satisfy all demands. Other works aim to limit fragmentation by minimizing the max slice-index of the network in [6], or to minimize the summation, over links, of the max slice-index of each link in the network [7].

B. Network State Metrics

One of the main aspects of the off-line design phase is to minimize links utilization (number of slices used) or maximize the residual spectrum (free slices) after serving the required demands of the traffic matrix. In this context, different works in literature propose metrics to characterize the state of the available spectrum. In [8], authors use the spectrum usage which represents the number of slices used such that a slice is said “used” if and only if it is used at least on one edge. In [9], the metrics used are the spectrum utilized in Hertz, and resource utilization ratio which is the spectrum utilized over the number of demands established. The available spectrum is also reduced by fragmentation resulting from RSA in the flexgrid network.

Several fragmentation metrics have been proposed in literature to monitor spectrum fragmentation quantitatively and qualitatively, including external fragmentation (EF), Shannon’s entropy (SE) and access blocking probability (ABP). EF is derived by computing the ratio of the maximum number of available contiguous slices to the number of all available slices [10]. SE is based on Shannon’s entropy formula, it is computed as follows [11]: $SE = -\sum_{i=1}^N \frac{D_i}{D}$, where D is the total number of frequency slices in the spectrum of a link or path, and D_i is the size, in slices, of the i th block of contiguous used or unused spectrum (hence, N is the number of such blocks). ABP computes the ratio between the number of

possible demands that can be placed on the available spectral voids in a certain link and the number that could be placed if all those free slices were contiguous [12].

One first notes that EF, SE and ABP are link-based metrics; therefore, the fragmentation of a given path is defined as the average of the fragmentation of each link of this path. This implies that the resulting information is quite imprecise because it fails to recognize misalignment of spectrum holes in a path. Furthermore, as the usability of a given spectrum hole depends on the granularity of the transponder (i.e. the number of slices that it used in a given configuration), a fragmentation metric should be expressed as a function of granularity. However, this is the case neither for EF nor for SE which are unable to distinguish the case when fragmented spectrum holes match the available granularities from the inappropriate fragmented cases. On the other hand, ABP, which does take into account transponders’ granularity, provides only an average measure of fragmentation over transponders granularities. Lastly, the network state metrics mentioned in this section provide qualitative assessments of a given mapping, but provide no quantitative information on the remaining capacity on the network.

C. Network’s Remaining Capacity

[1] proposes some strategies to roughly estimate networks’ capability to host future demands. However, this paper deals with IP networks only which significantly differ from flexgrid networks as they present no spectrum fragmentation issue.

The authors in [13] divide the residual spectrum resources or the “free” slices into “wasted”, “usable” and “unusable” slices. The wasted slices are those that are only free on some, but not all, of the links of a path; unusable slices are those free on all links of a path but belong to a block of slots that is too small with respect to the number of slices requested by a transponder. Lastly, the remaining slices are the usable ones.

D. Contributions

In this work, we propose a new optimization objective that aims to reduce fragmentation and consequently, increases the capability of the network to host future demands. Unlike work in literature’ where a single optimization metric is used without comparing it to other optimization objectives, we compare our proposed optimization objective to a traditional one in literature that aims to which aims to route demands over the shortest paths.

The comparison includes, but is not limited to, the remaining capacity (wasted and usable) and fragmentation metrics at the network level as a function of transponder’s granularity.

III. OFF-LINE DIMENSIONING

In this section we propose a network model and then explain how we set the number of slices ensuring a scenario without any blocking.

A. Network Model

An input of the off-line RSA problem is formally defined by

- an optical-network topology represented as an undirected, loopless, and connected graph $G = (V, E)$,
- an edge-length (in kms) vector $\ell \in \mathbb{R}_+^E$,
- an optimum spectrum $S = \{1, \dots, \bar{s}\}$ os available frequency slices, and
- a multiset K of traffic demands where each $k \in K$ is specified by an origin node $o_k \in V$, a destination node $d_k \in V \setminus \{o_k\}$, a requested number $w_k \in \mathbb{N}_+$ of slices, and a transmission reach $\bar{\ell} \in \mathbb{R}_+$ (in kms).

The off-line RSA problem aims at determining for each traffic demand $k \in K$, a lightpath composed of a route (i.e., an (o_k, d_k) -path P_k in G) whose length does not exceed the transmission reach ℓ_k and a channel (i.e., a subset $S_k \subseteq S$ of slices) such that

- the slices allocated to a traffic demand k (i.e., slices in S_k) are the same all along the route P_k (*spectrum continuity*),
- the slices allocated to a traffic demand k are contiguous (*spectrum contiguity*),
- a same slice cannot be allocated to two different traffic demands on the same link (*non-overlapping spectrum*),
- and some objective function is minimized.

A new Integer-Linear Program (ILP) model for the off-line RSA problem was introduced in [14]. This model has a polynomial number of variables as it is based on the so-called edge-node formulation for the route determination and a non-polynomial number of constraints that can be handled in a polynomial time through the well-known cutting-plane approach [14]. This tractable model is the first complete ILP model for the off-line RSA problem as it precisely encodes the set of all feasible solutions to the RSA problem and therefore can be used to optimize any objective function used as quality measure by a network operator. In this paper, we only focus on two specific objective functions

- the minimization of the sum of the maximum slice used over all the demands, i.e.,

$$\min \sum_{k \in K} \sum_{s \in S} z_s^k, \quad (\text{Objective 1})$$

where z_s^k is the binary variable that equals 1 if and only if the slice $s \in S$ is the last slice assigned to demand $k \in K$ and

- the minimization of the sum of the route lengths over all the demands, i.e.,

$$\min \sum_{k \in K} \sum_{e \in E} \ell_e x_e^k \quad (\text{Objective 2})$$

where x_e^k is the binary variable that equals 1 if and only if the route P_k of demand $k \in K$ uses edge $e \in E$.

The ILP model with (Objective 1) or (Objective 2) is solved through a branch-and cut framework (i.e., a cutting-plane algorithm embedded in a branch-and-bound approach) that was implemented using Cplex 12.8. The efficiency of a branch-and-cut approach significantly depends on the strength of both the lower bounds (i.e., best possible objective) on the optimal value that the linear relaxations of the ILP model provide

and the upper bounds (i.e., best found objective) obtained by feasible solutions encountered during the whole solution process. To improve the latter bounds, we also incorporated a greedy-based heuristic that is called frequently during the course of the branching tree. Such an approach may fail to find an optimal solution within a time limit but it usually provides a feasible solution whose quality can be assessed when put in perspective with the lower bound. This is known as the (*relative objective*) *gap* that is equal to the

$$\frac{\text{best found objective} - \text{best possible objective}}{\text{best objective found}};$$

the smaller the gap is the better is the incumbent solution.

B. Admissible topology

To run a simulation, that uses the above objective metrics, we need to set the number of slices (N_{slices}). In a fiber of 4 THz spectrum, the nominal number of slices is 320 (12.5 GHz per slice). However, it is better to decrease this number in order to decrease the number of constraints in the ILP model and consequently, simulation period. Therefore, to find the admissible topology (no blocking and average link occupation ($L_{\text{occupation}} \leq 80\%$), we do the following.

We start by $N_{\text{slices}} = 320$ and then:

- 1) We run a simulation which minimizes the number of blocked demands (N_{blocked}) (we set the objective function to minimize N_{blocked}).
- 2) If $N_{\text{blocked}} = 0$ and $L_{\text{occupation}} \leq 80\%$, we repeat the previous step with $N'_{\text{slices}} = N_{\text{slices}} - 40$. Otherwise, the number of slice is N_{slices} .

We keep on repeating these two steps until we reach the minimum number of slices that satisfies blocking and spectrum utilization conditions.

IV. NETWORK LEVEL METRICS

We propose network health metrics that will be used in analyzing the results. We classify these metrics into “network state” (qualitative) metrics and “remaining capacity” (quantitative) metrics.

A. Network State Metrics

The following metrics are considered:

Total number of used slices: the summation of the number of used slices over edges. This reflects network congestion.

Fiber’s last empty space size: the size of the hole bordered by fiber’s max used slice and the right edge of the spectrum.

Fiber’s max used slice index [6]: we plot maximum used slice index and fiber’s last empty space size in each fiber vs fiber ID.

Holes statistics: a spectrum hole is a set of contiguous free slices bordered by occupied slices. The larger the number of spectrum holes, the more the network is fragmented. Therefore, we will count their number, mean size and summation of spectrum holes over all links.

Network’s EF, SE and ABP: we find the fragmentation per link then, we calculate the average, over all links, to derive

network level fragmentation.

Metric 1: The sum of the maximum slice used over all the demands, which corresponds to the objective 1 of the previous section.

Metric 2: The sum of the route lengths over all the demands (corresponding to objective 2).

B. Remaining Capacity Metrics

Network’s remaining capacity: the following is inspired from [13], however, here we compute the number of usable and non-usable slices as function of each transponder’s granularity. For each $(s - d)$ pair m , we have $I_m(n)$ feasible paths (a path that satisfies transponder’s transmission reach and has at least n free slices). On each path $p_{m,i}$, there are occupied and un-occupied (free) slices. Free slices are divided into two categories:

- Wasted slices $W_{m,i}$ which is the summation of all slices that are only free on some, but not all, of the links of a path $p_{m,i}$.
- Accessible $A_{m,i}$ slices: a set of slices, in which each slice of the hole satisfies continuity over the path $p_{m,i}$ and could be used to allocate traffic if it satisfies the transponder’s granularity. For more ease, we can this kind of holes as path-hole.

If each path-hole $h_{m,i,j}$ can accommodate $x_{m,i,j}(n) = |h_{m,i,j}| \text{DIV}(n)$ and $\text{DIV}(n)$ is the integer division, and n is transponder’s granularity. The total number of transponders of granularity n , that can be accommodated on path $p_{m,i}$ is:

$$X_{m,i}(n) = \sum_{j=1}^J x_{m,i,j}(n) \quad (1)$$

where J is the number of path-holes in $p_{m,i}$. At network level, the average number of transponders, of granularity n , that can be accommodated is:

$$X(n) = \frac{\sum_{m=1}^M \left(\frac{\sum_{i=1}^{I_{m,n}} X_{m,i}(n)}{I_{m,n}} \right)}{M} \quad (2)$$

Usable-to-Accessible-ratio (UAR): is the path $p_{m,i}$ fragmentation defined as follows:

$$UAR_{m,i}(n) = \frac{U_{m,i}(n)}{\text{accessible}_{m,i}} \quad (3)$$

To find network’s UAR , we use the cumulative distribution function (CDF) instead of the average value over all paths. Indeed, as $UAR_{m,i}$ is a ratio, averaging its value over all paths would be hard to analyze. Hence, for each pair, we find the least fragmented path’s UAR . Then, all these values of all the pairs are saved in an array whose CDF is plotted.

V. SMALL TOPOLOGY RESULTS

A. Problem Specification

The transponder used has a capacity that varies between 100 Gb/s (with a 3000 km reach), 200 Gb/s (1500 km reach), and 400 Gb/s (600 km reach) which respectively requires 3, 5 and 6 slices. A set of 15 demands (total demand of 68 slices) is randomly generated and the highest possible rate format is

TABLE I
DIFFERENT NETWORK HEALTH METRICS FROM OBJECTIVE 1 AND 2.

Metric	Objective 1	Objective 2
Metric 1	117	260
Metric 2	7040	6604
Number used slices	80	77
Number holes	2	12
$\Sigma holes $	6	111
EF	0.003	0.051
SE	0.050	0.244
ABP	0.001	0.006

allocated. The topology used is the 5 Node Spain topology which has 7 links of average length 367 km, average node degree 2.8 [15].

The number of slices/fiber is 30 as explained in Section III-B. We compare the network health metrics resulting of two optimization objectives mentioned in Section III-A using metrics detailed in Section IV. The simulation results shown here are optimal as Gap=0.

B. Network state

Using objective 2 aims at minimizing the sum of demands’ route lengths, i.e. demands are routed over the shortest paths possible. Consequently, they traverse fewer links (which limits the total number of used slices), as shown in Tab. I. On the other hand, using objective 1 aims at minimizing the sum of demands’ highest index, therefore, it forces demands to get closer to the left hand side of the spectrum. To achieve this goal, demands may be routed over longer paths (which increases metric 2) as they traverse more links (higher total number of used slices). However result, demands are “squeezed” and the spectrum load is distributed in a balanced way among links as illustrated in Fig.1.

To better explain demands “squeezing” and load distribution, we plot fibers’ max used-slice-index and last empty space size Vs. fibers IDs in Figs. 1 and 2 resulting of using objectives 1 and 2, respectively. These two plots show the distribution of slices in the fibers. The size of the empty space, from the last-slice used to left edge of the spectrum in most of the fibers (blue area), is up to 17 to 20 slices using objective 1, while it ranges between 1 to 5 slices for objective 2. This significant difference between the two plots clearly shows the advantage of squeezing obtained by using objective 1. It explains also the huge difference in the number of spectrum holes, between the two objectives as shown in Tab. I) where objective 1 yields 2 holes ($\Sigma |holes| = 6$ slices), and objective 2 yields 12 holes ($\Sigma |holes| = 111$ slices).

C. Network Fragmentation

Tab. I shows fragmentation metrics EF, SE, and ABP results for objectives 1 and 2. Results obtained by using the latter are higher, by a magnitude of 4 to 5, than that of objective 1. Therefore, it again shows how objective 1 significantly reduces fragmentation compared to objective 2.

The previous metrics are good in terms of comparison between

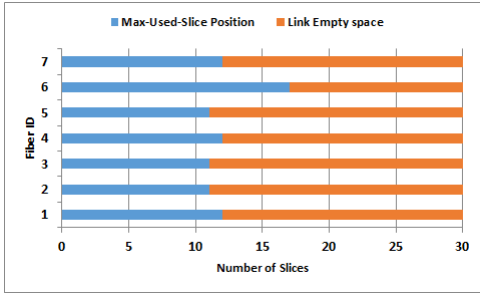


Fig. 1. Last empty space size and max used slice index vs fiber Ids (objective 1).

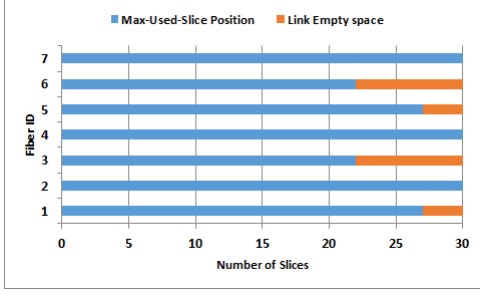


Fig. 2. Last empty space size and max used slice index vs fiber Ids (objective 2).

network states. However, to better understand the global network fragmentation, we use the CDF(n) of the paths' UAR of each objective, which are plotted in Figs. 3 and 4 for objectives 1 and 2, respectively. We observe that:

- For $n = 2$, using objective 1, all paths have UAR $< 5\%$, while using objective 2, only 50% of the paths have UAR $< 5\%$ and the remaining paths have UAR=5-12%.
- For $n = 3$, using objective 1, all paths have UAR = 0%, while using objective 2, only 60% of the paths have UAR = 0% and the remaining paths have UAR=5-30%.
- For $n = 4$, using objective 1, all paths have UAR = 10-15%, while using objective 2, only 60% of the paths have UAR $< 15\%$ and the remaining paths have UAR=15-38%.
- For $n = 5$, using objective 1, all paths have UAR = 15-20%, while using objective 2, only 50% of the paths have UAR $< 20\%$ and the remaining paths have UAR=20-60%.
- Generally, using objective 1, all the paths have UAR $< 20\%$, while using objective 2, paths have UAR = 0-60%.

Using CDF (as function of n) for paths' UAR presents some advantages. Firstly, considering only average fragmentation over paths might hide relevant information about highly fragmented paths; on the other hand, using CDF of paths' UAR provides us quantitative information about the percentage of fragmented paths at a specific fragmentation level. Hence, this could be used as an alert by the operator: when a certain number of paths (say Z) reach a fragmentation level (α), the operator could decide that an action should be taken. Secondly, the CDF(n) figures show how fragmentation of

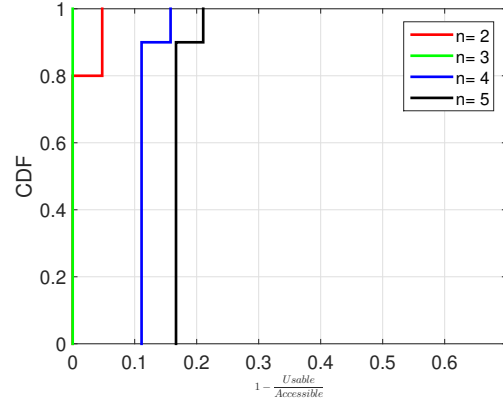


Fig. 3. Fragmentation CDF from objective 1.

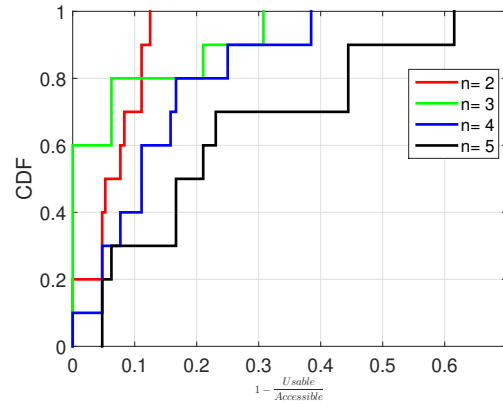


Fig. 4. Fragmentation CDF from objective 2.

the residual capacity depends on transponder's granularity (n). Moreover, it is clear how fragmentation increases as the granularity increases. This is why, considering fragmentation per granularity is better than considering only a fragmentation as average of fragmentation over different granularities, which might hide a lot of information especially if there is a large range of transponders granularities.

D. Network's Remaining capacity

After demonstrating the importance of the objective function on the network's fragmentation, we show, in this section, how it affects the usability of the residual capacity. Fig. 5 presents the network average wasted slices and accessible slice. It shows that the percentage of wasted slice decreases from 24% (using objective 2) to 5% (using objective 1) out of 30 slices per link. Consequently, the percentage of accessible slices increases from 39% (using objective 2) to 56% (using objective 1).

To show how the usability of the remaining capacity is enhanced by using objective 1, we plot, in Fig. 6, $X(n)$ (the average number of transponders of granularity (n) that could be allocated) versus transponders' granularity (n), for both objectives. The result are as follows:

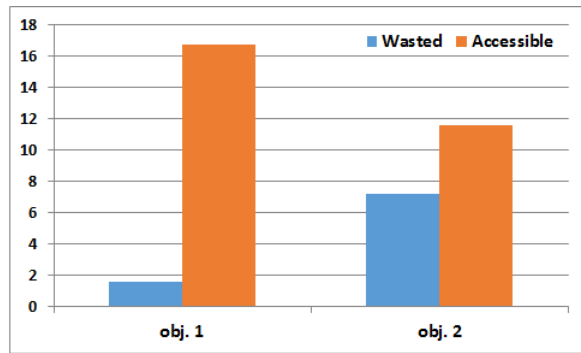


Fig. 5. Network's average wasted and accessible slices using objective 1 and 2.

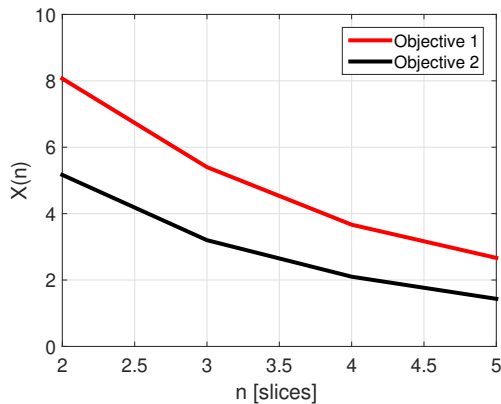


Fig. 6. Network average X Vs. n resulting from objective 1 and 2.

- For $n=2$, using objective 2, $X(n)=5$, while using objective 1, $X(n)=8$. Thus, the percentage of increase is 60 %.
- For $n=5$, using objective 2, $X(n)=1.8$, while using objective 1, $X(n)=3$. This is a 65 % increase

Therefore, using objective 1, the number of transponders that could be allocated, generally, increases by 60 % w.r.t objective 2. As a conclusion, optimizing objective 1, which decreases the number of holes by squeezing demands, leads to a lower average wasted slices and consequently, increases the accessible and usable slices compared to objective 2.

VI. CONCLUSION

In this work we have designed a flexible-grid optical network assuming a static RSA. We have compared, using the remaining capacity concept, two different optimization objective: 1) our newly proposed metric minimizes the sum of the maximum slice used over all the demands (obj. 1). 2) the second metric minimizes the sum of the route lengths over all the demands (obj. 2).

The comparison was done qualitatively using spectrum occupancy, number of spectral holes, and fragmentation metrics EF, SE and ABP. The quantitative comparison was done using the wasted slices, usable slices and UAR path fragmentation ratio. obj. 1 was able to squeeze demands, thus, it dramatically

decreases fragmentation. As a result, using obj. 1, the no. of transponders that could be allocated, increases by 60 % w.r.t obj. 2. Moreover, using obj. 1, all the paths had UAR < 20 %, while using obj. 2, paths have UAR between 0 % to 60 %.

As a future work, we plan to consider the on-line RSA problem and we will consider how to best use some of these network health metrics to decide when the network need to be partly or fully re-dimensioned in order to avoid blocking.

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