Bounding Surface Modeling of Compacted Silty Sand Exhibiting Suction Dependent Post-peak Strain Softening

Ujwalkumar D Patil, Laureano R Hoyos, Mathilde Morvan, Anand J Puppala

▶ To cite this version:

HAL Id: hal-03036514

https://uca.hal.science/hal-03036514

Submitted on 2 Dec 2020

Bounding Surface Modeling of Compacted Silty Sand Exhibiting

Suction Dependent Post-peak Strain Softening

Ujwalkumar D. Patil[1], Laureano R. Hoyos[2], Mathilde Morvan[3] and Anand J. Puppala[4]

SUMMARY

This article focuses on modeling the strain hardening-softening response of statically compacted

silty sand as observed from a comprehensive series of suction-controlled, consolidated-drained

triaxial tests accomplished in a fully-automated, double-walled triaxial test system via the axis-

translation technique. The constitutive model used in this work is based on the theory of

Bounding Surface (BS) plasticity, and is formulated within a critical state framework. The

essential BS model parameters are calibrated using the full set of triaxial test results and then

used for predictions of compacted silty sand response at matric suction states varying from 50 to

750 kPa. Complementary simulations using the Barcelona Basic Model (BBM) have also been

included, alongside BS model predictions, in order to get further enlightening insights into some

of the main limitations and challenges facing both frameworks within the context of the

experimental evidence resulting from the present research effort. In general, irrespective of the

value of matric suction applied, the BBM performs relatively well in predicting response at peak

[1] Faculty Research Associate (<u>Corresponding Author</u>), Department of Civil Engineering, University of Texas at Arlington, Arlington, Texas 76019. E-mail: ujwalkumar.patil@uta.edu

[2] Professor, Department of Civil Engineering, University of Texas at Arlington, Arlington, Texas 76019. E-mail: lhoyos@uta.edu

[3] Assistant Professor, Clermont Université, Université Blaise Pascal, Institut Pascal, BP 10448, F-63000 Clermont-Ferrand , CNRS, UMR 6602, Institut Pascal, F-63171 Aubière, France. E-mail: mathilde.morvan@univ-bpclermont.fr

[4] Professor, Department of Civil Engineering, University of Texas at Arlington, Arlington, Texas 76019. E-mail: anand@uta.edu

20 and critical state failure under low net confining pressure while the BSM performs relatively well

21 under high net confining pressures.

22 **Keywords:** unsaturated soil, matric suction, triaxial testing, dilatancy, strain-softening,

23 elastoplasticity

24

25 <div align="center">1. BACKGROUND AND SCOPE</div>

26   Over-consolidated and compacted silty sand type soil upon shearing are most likely to

27 manifest the strain-induced post-peak softening along with the suction-induced dilatational

28 volume change. Strain softening in saturated soil during shearing has been attributed to either

29 damage to the soil structure, i.e. localization, or to the dilation, i.e. increase in volume [1].

30 However, hydro-mechanical response of unsaturated soil becomes complicated due to multi-

31 phase interaction between air-solids, water-solids and air-water interface within void space of the

32 soil skeleton. The variation of pressure difference across air-water interface at grain point contact

33 varies with degree of saturation and contributes largely to hydro-mechanical response of soil,

34 provided the water phase is continuous.

35   However, at high suction, i.e. beyond residual suction, the water phase is no longer

36 continuous and hence the same cannot hold true. Key data set on stress-strain response,

37 especially the one obtained via suction-controlled triaxial testing, not only plays a crucial role in

38 accurate assessments of unsaturated soil strength-deformation but is also essential in subsequent

39 validation of predictive models. The experimentally validated model could further be

40 incorporated within finite element codes to study complex soil-structure interaction of man-made

41 structures.

Alonso *et al*. [2] extended the classic Cam-Clay model to unsaturated soils based on critical state framework and could explain the wetting induced collapse phenomenon along with being able to reasonably simulate the compressive volume change and strain-hardening type triaxial response. Since then, despite efforts by many researchers in fine tuning and improving this seminal model, none of them have been able to successfully reproduce the strain-induced softening and the stress-induced dilatancy observed in heavily overconsolidated soils when compacted and tested under unsaturated state.

Bounding Surface (BS) plasticity theory was initially developed to simulate plasticity in metals [3] and was soon extended to different geomaterials such as cohesive soils [4–6]; sands [7, 8]; pavement base materials [9]; geosynthetic reinforced material [10, 11]; concrete [12]; and rockfill materials [13, 14]. The BS theory underwent further simplification and refinement by several researchers [5–7, 15–19].

Bardet [7] introduced a comprehensive BS framework for soils, including nine material constants, and was quite successful in reproducing the complex post-peak strain softening and dilatational response of dry and saturated sands under conventional triaxial stress paths. However, unsaturated soil behavior differs from that of saturated soil because of the presence of a contractile air-water interphase across which a pressure balance is created between the pore-air and pore-water phases within the soil matrix, also known as matric suction [20]. Extensive experimental evidence has clearly demonstrated the paramount influence of matric suction on the stress-strain-strength response of unsaturated soils.

Russell and Khalili [21] were among the first to extend the BS plasticity theory to unsaturated soils within a critical state framework using concept of effective stress and taking into account the particle crushing associated with shearing. The model was calibrated using

65   results from triaxial compression tests on speswhite kaolin reported by Wheeler and Sivakumar

66   [22], as well as triaxial compression and oedometric tests on kurnell sand with no fines reported

67   by Russell [23].

68   More recently, Morvan et al. [24] presented an extension of Bardet's [7] original BS

69   framework with the aim of simulating post-peak softening of unsaturated soils using 12

70   constitutive parameters. The extended BS framework allowed them to reproduce, with

71   reasonable accuracy, all the following: 1) increase in shear strength with increasing suction; 2)

72   volumetric collapse or slight rebound upon wetting, depending on mean stress level; and 3) post-

73   peak softening and gradual transition from contractant to dilatant nature of volume change

74   during continuous shearing. However, the calibration and validation of the extended BS

75   framework have been based on a limited set of experimental data obtained from suction-

76   controlled triaxial tests on dense Kurnell sand [23], Jossigny silt [25], and Speswhite Kaolin [26,

77   27]. These limited studies suggest a need for additional experimental evidence on different types

78   of soils, particularly heavily overconsolidated soils, for a more thorough and conclusive

79   validation of the extended BS framework, a chief motivation for the present work.

80

81   2.   EXPERIMENTAL PROGRAM

82   *2.1. Test Material*

83   The soil used in the present work is an intermediate geomaterial, hence neither a perfect sand, silt

84   or clay, and tested in a very dense state, thus making it a suitable material to assess the suitability

85   of the extended BS framework introduced by Morvan et al. [24] for overconsolidated, cohesive-

86   frictional soils.

87 The test material used is classified as a poorly graded silty sand (SM) with clay, according to the

88 Unified Soil Classification System (USCS), with a standard proctor dry density of 1.87 g/cm^3,

89 optimum water content of 12.2% and specific gravity of soil solids, G_s = 2.67. It is well known

90 that the structure and fabric of a test specimen gets altered by the type of compaction method and

91 hence tends to influence the subsequent shear response. In the present work, a stress-controlled

92 approach was used to produce compacted specimen with an artificially induced over-

93 consolidated stress history. Target density of approximately 1.87g/cm^3, corresponding to +2%

94 wet of Proctor optimum, was achieved by compacting each specimen in nine equal lifts, each lift

95 being axially stressed in a split mold to a maximum vertical stress equal to 1600 kPa. The

96 specimen produced had initial voids ratio varying between 0.46-0.48 and approximate air entry

97 value between 8-10 kPa [28].

98

99 *2.2 Experimental Variables*

100 Strength tests were conducted using a fully-automated double-walled triaxial test equipment that

101 accommodates the essential modifications for unsaturated soil testing, including high-air-entry

102 (HAE) ceramics in the bottom pedestal; pore-water pressure control; pore-air pressure supply via

103 the top cap; and diffused-air flushing assembly, as shown in Fig. 1. A panoramic view of the

104 entire test set-up is shown in Figure 2.

105   A comprehensive series of saturated and unsaturated consolidated drained (CD) triaxial

106 tests on statically compacted silty sand specimens was performed. The axis-translation technique

107 was used to impose and control matric suction in the range of 0 to 750 kPa (s = 0, 50, 250, 500

108 and 750 kPa). Three net confining pressures, $(\sigma_3 - u_a)$ = 100, 200 and 300 kPa, were applied.

109 Detailed protocols on soil preconditioning, pore-fluid equalization, s-controlled isotropic loading

110 and monotonic shear loading, capability of test equipment to replicate test results and other test

111 related methodology are described by Patil [28].

112

113   3.   SILTY SAND RESPONSE FROM SUCTION-CONTROLLED TRIAXIAL TESTING

114 *3.1. Effect of Net Confining Pressure at Constant Matric Suction*

115 Test results from suction-controlled CTC (Conventional triaxial compression) tests under

116 constant matric suction, $s = 250$ kPa, and under three net confining pressures, $(\sigma_3 - u_a) = 100$,

117 200 and 300 kPa, are shown in Figure 3. A negative sign in the volumetric strain response

118 represents dilating volume change, while a positive sign indicates compressive type behavior.

119 The convention used to designate the specimen is $CD_{x\text{-}y}$ where "CD" denotes the consolidated

120 drained test; "x" represents the net confining pressure $(\sigma_3 - u_a)$, while "y" represents the imposed

121 constant suction (s).

122   As expected, Figure 3(a) clearly indicates an increase in stiffness (initial and subsequent

123 tangent modulus), as well as brittleness, with increasing confinement. Post-peak reduction in

124 deviator stress, also known as strain-softening type behavior, was also observed. Moreover, an

125 increase in net confinement pressure is expected to cause higher compression of the specimen

126 and hence suppress the amount of dilation. This is clearly manifested in Figure 3(b), where

127 dilation is significantly suppressed with an increase in confining pressure from 100 to 300 kPa.

128 Such type of stress-strain and volume change response are typical of relatively dense and

129 overconsolidated soils, as is the case of statically compacted silty sands [21, 29]. On the other

130 hand, triaxial tests performed on saturated specimens of compacted silty sand $(s = 0)$ resulted in

131 compressive type behavior. Similar response was observed by previous researchers [21, 30–33].

132

3.2. Effect of Matric Suction at Constant Net Confining Pressure

Results from suction-controlled CTC tests conducted on compacted silty sand specimens, under varying matric suction states, $s = 0$, 50, 250, 500, and 750 kPa, and constant net confining pressure, $(\sigma_3 - u_a) = 100$, are shown in Figure 4. In these figures, the change from strain-hardening type behavior, as seen in saturated specimens ($s = 0$), to strain-softening type response, due to the introduction of matric suction in all unsaturated specimens, is readily manifest. The amplitude of strain-softening increases and the specimen fails at lower axial strain, while taking less axial strain to reach critical state, with increasing matric suction, as shown in Fig. 4(a).

Figure 4(b) shows a rather drastic change in volumetric response, from compressive to dilational type behavior, with the imposition of a matric suction of magnitude as low as 50 kPa. Matric suction tends to increase the initial stiffness and the apparent cohesion between individual soil particles so that the particles all together during shearing tend to ride over one another instead of undergoing slippage, resulting in an increase in volume (i.e. dilation). However, the rate of dilation decreases as the air-water menisci gets destroyed with continued shearing beyond peak stress, and the specimens get softened back to critical state, at which point they are expected to exhibit only shear deformations (plastic flow) without further change in strength or volume. All the test specimens were observed to develop multiple shear bands while simultaneously bulging at the center as the shearing continued [28].

In addition, experimental results indicate that an increase in matric suction has more influence on peak shear stress than on critical shear stress. Continued shearing beyond peak shear stress tends to weaken and subsequently destroy the matric suction effect around soil grains due to perturbation caused to air-water menisci interface. This phenomenon can be attributed to

156 the observed experimental softening of stress, the amplitude of which depends largely on the

157 magnitude of suction imposed, ultimately resulting in peak strength being more affected by

158 matric suction than by critical or large strain strength.

159

160   4.   CONSTUTIVE MODELING OF OVERCONSOLIDATED SILTY SAND RESPONSE

161 *4.1. Bounding Surface Model: General Framework*

162 The original framework of the Bounding Surface Model (BSM), particularly the one introduced

163 by Morvan et al. [24] for unsaturated soils, has proved reasonably efficient in modeling both

164 compressive volumetric response and strain-induced hardening. These behaviors are typically

165 observed in normally and lightly overconsolidated soils. However, this BSM framework also has

166 the potential to reproduce dilational volumetric response and strain-induced softening that are

167 typically observed in heavily overconsolidated soils subjected to suction-controlled monotonic

168 shearing. Furthermore, it is possible with BSM to capture the gradual transition between elastic

169 and elasto-plastic soil response. The chief motivation for present work was to finetune and refine

170 the original BSM to predict the stress-strain and volume change response and compare it with the

171 one obtained via suction-controlled triaxial testing from present research. Since the tests to be

172 modeled are performed at constant suction and the volumetric variations of the sample are low,

173 we chose not to use the fully coupled version taking into account water retention curve hysteresis

174 and its dependency on void ratio.

175

176 *4.2. Effective Stress: Definition and Radial Mapping*

177 The effective stress, σ', is expressed in terms of the equivalent pore pressure, π, as defined by

178 Pereira et al. [34], and hence is given as follows:

$$\sigma' = \sigma + \pi(s)\mathbf{1} \qquad \text{with } \pi(s) = -u_a + sS_r + \frac{2}{3} \int_{S_r}^{1} s(S)dS \qquad (1)$$

179 As previously mentioned, the model introduced by Morvan *et al.* [24] to simulate unsaturated

180 soil response is essentially an extension of the BS model postulated by Bardet [7] for saturated

181 soils, and hence is based on the bounding surface theory within a critical state framework.

182 Therefore, due consideration is given to the existence of a limit state line (LSL) in the p', q plane

183 that defines an upper bound to the stress ratio $\eta_p = q/p'$, as illustrated in Fig. 5. The plastic

184 modulus is forced to be dependent on the distance δ between the current stress state σ' and its

185 image-point, $\bar{\sigma}$ obtained by projection on the distance δ between the current stress state σ' and its

186 image-point, obtained by projection on a surface called bounding surface, as shown in Figure 5.

187   Radial mapping technique [7], also illustrated by Yu [29], is used to define the image-

188 point $\bar{\sigma}'$ of the actual stress σ' on the bounding surface BS, via a translation vector, by extending

189 the line linking the origin to the point σ' until it cuts the bounding surface (Fig. 5). Further details

190 on the classic BS formulation have been presented by Dafalias and Herrmann [5], Crouch et al.

191 [35], Manzari and Dafalias [36], Russell and Khalili [21], and Yu [29].

192

193 *4.3. BSM Framework in Triaxial Stress Space*

194 Classic triaxial test variables are used for cylindrical symmetry and triaxial stress space to define

195 the mean net stress (p'), deviatoric stress (q), peak state line (PSL) slope (η_p), and critical state

196 line (CSL) slope (M), which are expressed as follows:

$$p' = \frac{1}{3}(\sigma'_1 + 2\,\sigma'_3), \;\; q = \sigma'_1 - \sigma'_3, \qquad \eta_p = \frac{q_p}{p'_p}, \qquad M = \frac{q_{cs}}{p'_{cs}} \qquad (2)$$

197   Different variables/parameters used are defined in the list of symbols at the end of paper. In this

198   work, peak state line (PSL) is used exchangeable to limit sate line (LSL), as shown in Fig. 5. A

199   third line, identified as the characteristic state line (CL), which marks a transition from

200   contractant to dilatant volume change behavior during initial stages of shear loading, is also

201   postulated.

202   The conjugated variables in p', q plane is volumetric and deviatoric strains, ε_p and ε_q,

203   which are respectively defined as follows:

$$\varepsilon_p = \varepsilon_1 + 2\,\varepsilon_3, \qquad \varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{3}$$

204   The total strain is assumed to be divided into elastic and plastic parts:

$$\varepsilon_p = \varepsilon_p^e + \varepsilon_p^p, \qquad \varepsilon_q = \varepsilon_q^e + \varepsilon_q^p \tag{4}$$

205   The BS yield function represents an ellipse-shape yield surface in p':q plane (Fig. 5), beyond

206   which plastic compression occurs on account of increased stress or decreased suction, and is

207   defined as follows:

$$f\left(\bar{p}', \bar{q}, \varepsilon_p^p, s\right) = \left(\frac{\bar{p}' - A_\pi}{\rho - 1}\right)^2 - \left(\frac{\bar{q}}{M}\right)^2 - A_\pi{}^2 \tag{5}$$

208   The stress image obtained by the radial mapping, as shown in Fig. 5, can be defined as follows:

$$\bar{p}' = \gamma A_\pi, \quad \bar{q} = \gamma x M A_\pi, \qquad x = \frac{q}{Mp' + q_0} \tag{6}$$

$$\gamma = \frac{(1 + (\rho - 1)\sqrt{1 + x^2\rho(\rho - 2)}}{1 + (\rho - 1)^2 x^2} \tag{7}$$

209   Employing the regular concept and the definitions of plastic multiplier ζ and plastic modulus H_b,

210   the following can be established:

$$\frac{\partial f}{\partial \varepsilon_p^p} \frac{d\varepsilon_p^p}{\left\|\frac{\partial f}{\partial \bar{\sigma}}\right\|} = -H_b d\zeta \tag{8}$$

211 The gradient of f contains 3 terms, i.e. the partial derivatives with respect to $\bar{p}'$, $\bar{q}$, and s:

$$\frac{\partial f}{\partial \bar{\sigma}'} = \left\{ \frac{\partial f}{\partial \bar{p}'}; \frac{\partial f}{\partial q}; \frac{\partial f}{\partial s} \right\}^{\mathrm{T}} \tag{9}$$

212 After few simplifications, the 3 components of the exterior normal at the image-point can be

213 expressed as follows:

$$\eta_p = \frac{\partial f}{\partial \bar{p}'} \Big/ \left\| \frac{\partial f}{\partial \bar{\sigma}} \right\| = \frac{M(\gamma - 1)}{g} \tag{10}$$

$$\eta_q = \frac{\partial f}{\partial \bar{q}} \Big/ \left\| \frac{\partial f}{\partial \bar{\sigma}'} \right\| = \frac{(\gamma x (\rho - 1))^2}{g} \tag{11}$$

$$\eta_s = \frac{\partial f}{\partial s} \Big/ \left\| \frac{\partial f}{\partial \bar{\sigma}} \right\| \tag{12}$$

with: $g = \left\| \dfrac{\partial f}{(\partial \bar{\sigma}')} \right\| \dfrac{M(\rho - 1)^2}{2A_\pi}$ $\tag{13}$

214 When the stress point is on the bounding surface, the plastic modulus is given by:

$$H_b = \frac{1 + e_0}{\lambda(s) - \kappa} \frac{A l_1(s)}{g^2} M^2 (\gamma - 1)[\gamma + \rho(\rho - 2)] \tag{14}$$

215 where κ is the volumetric deformability and e_0 is the initial void ratio. On the other hand, if the

216 stress point lie within the bounding surface, an additional term H_f is added to H_b. The plastic

217 modulus H is then expressed as follows:

$$H = H_b + H_f \tag{15}$$

218 H_f is related to the plastic modulus H by the distance δ between the current stress state σ' and its

219 projected image-point $\bar{\sigma}'$ on the bounding surface, as illustrated in Fig. 5, and is expressed as

220 follows:

$$H_f = \frac{1 + e_0}{\lambda(s) - \kappa} \frac{\delta}{\delta_{max} - \delta} h_0 p' \frac{\eta_p - \eta}{M} \tag{16}$$

221 where h_0 is a material parameter.

222   Inclusion of hardening of the unsaturated soil due to an increase in matric suction and

223   plastic volumetric strains is essential for the development of a consistent hardening rule. The

224   plastic flow is assumed to take the same direction as the onset of yielding by adopting an

225   associative flow rule relating the incremental plastic components of volumetric and shear strains,

226   which are expressed as follows:

$$d\varepsilon_p^p = \left[\frac{1}{H}(n_p dp' + n_q dq + n_s ds)\right]n_q \tag{17}$$

$$d\varepsilon_q^p = \left[\frac{1}{H}(n_p dp' + n_q dq + n_s ds)\right]n_q \tag{18}$$

227   while the elastic strains are defined by the classic relations:

$$d\varepsilon_p^e = \frac{\kappa}{(1+e_0)}\frac{dp'}{p'}, \qquad d\varepsilon_q^e = \frac{2(1+v)}{9(1-2v)}\frac{\kappa}{(1+e_0)}\frac{dq}{p'} \tag{19}$$

228   where v is the poisson's ratio.

229   Considering the present experimental data, the suction-dependence of A_π can be

230   expressed as follows:

$$A_\pi = A_\pi(\varepsilon_p^p, s) = l_1(s)A(\varepsilon_p^p) + l_2(s) \tag{20}$$

$$l_1(s) = 1 + k_1(S_r s - s_e) \, for \, s > s_e, \qquad l_2(s) = 0 \tag{21}$$

231   where, S_r is the degree of saturation. In order to ensure continuity at full saturation state and be

232   able to reproduce wetting induced collapse curve, the following conditions must be satisfied:

$$\forall s \leq s_e: l_1(s) = 1, and \, l_2(s) = 0 \tag{22}$$

$$\forall s \geq s_e: l_1'(s) \geq 0, and \, l_2'(s) \geq 0, and \quad \rho(A(\varepsilon_p^p)l_1'(s) + l_2'(s)) > \frac{d\pi(s)}{ds} \tag{23}$$

233   The slope of CSL in the e-log p' plane is assumed to be identical to that of the normal

234   compression line (NCL) at constant suction, and is defined as follows:

$$\lambda(s) = \lambda_0 - k_3(S_r s - s_e) \quad for \, s > s_e \tag{24}$$

235

4.4. Essential BSM Constitutive Parameters

The modified BS model postulated in the present work requires 12 material parameters, 8 of which are necessary to define the behavior at full saturation: (1) Two elastic constants, v and κ; (2) Six constants for plastic behavior, out of which three are required to determine the position and the shape of the bounding surface ρ, M and A_0, two for the plastic modulus, η_p and h_0, and one for the volumetric compressibility, λ_0. Four other constants are needed to account for suction: (1) Two constants, s_e and α, to define the water retention curve, information required to determine the equivalent pore pressure so that the effective stress can be obtained; (2) One constant, k_1, to account for suction effects on the hardening parameter A_π, and (3) the last one, k_3, to define the function $\lambda(s)$.

Systematic calibration procedure was followed to extract BS model parameters for compacted silty sand from conducted experiments. First, the parameters s_e and α are needed to be able to obtain effective stress. To achieve this, water retention curve in term of degree of saturation is used. These parameters are those given by Brooks and Corey [37]. M and η_p that define critical state and limit state are determined by plotting results of saturated triaxial tests in p',q plane. Volumetric stiffness $\lambda(s)$ was obtained by plotting isotropic tests results in e-log p' plane, which is necessary to obtain the initial value λ_0, and its variation with suction (k_3). The variation of A_π with suction is determined using loading collapse curves.

Only a few types of compacted geomaterials have been reported as being modeled with the BS framework as summarized in the above sections. Table 1 summarizes the values of all essential BS model parameters calibrated by previous researchers, along with those from present research. In the present work, results were thoroughly analyzed to gain critical insight into some

258   of the most essential elastoplastic features of compacted intermediate geomaterials under

259   controlled suction states, including the effect of suction on yield stress, apparent cohesion, tensile

260   strength, critical state line, post-peak softening and strain-induced dilatancy under suction-

261   controlled monotonic shearing.

262   It is also worth noting, however, that the mechanical properties of unsaturated soils can

263   be greatly influenced by repeated wetting and drying cycles, commonly termed as hydraulic

264   hysteresis. This hydraulic hysteresis could be taken into account by including few modifications

265   to the BS model, as proposed by Morvan et al. [38]. Nevertheless, in case of sandy soils, such as

266   the test soil used in the present work, hydraulic hysteresis effects could be reasonably neglected.

267

268   *4.5. Loading Collapse Locus using BS Model*

269   Unsaturated soil could undergo volumetric collapse or slight rebound upon wetting, depending

270   on the mean stress level. The functions $l_1(s)$ and $l_2(s)$, along with $A_\pi\left(\varepsilon_p^p, s\right)$, define the classic

271   loading collapse (LC) curve in the p', s plane, as shown in Figure 6, including the limiting values

272   of p' (p'_{lim}) on the bounding surface. The BSM predicted LC curve ensures continuity at

273   saturation and satisfies conditions in Eq. (24).

274   In the BSM framework, and in contrast with the original BBM presented by Alonso et al.

275   [2], the stress state, while moving forward along an imposed triaxial stress path, can cross the

276   CSL and hence access the domain $\eta > M$. However, the plastic modulus "H" changes sign,

277   somewhere between the CSL and the LSL, to satisfy the condition $M < \eta < \eta_p$. This means that

278   the stress state can never reach LSL. The parameter "η" is the lower limit of η_{peak} and, if properly

279   chosen, imparts the ability to reproduce the post-peak decrease in stress (softening). The value of

280   η can then be obtained by plotting the experimental values of peak deviator stress and

281 corresponding mean net pressure at $s = 0$, 50, 250, 500, and 750 kPa under three net confining

282 pressures, $(\sigma_3 - u_a) = 100$, 200 and 300 kPa, as shown in Figure 7. Thus, the best-fit line in Figure

283 7 defines the lower value of limit state line slope. A value of $\eta = 1.77$ was obtained and it was

284 rounded off to $\eta_p = 1.8$. This value represents an upper bound to the physically possible domain

285 for stress.

286

287 *4.6. Parametric Investigation of BS Model Predictions*

288   Figure 8 shows the results of a parametric investigation of BS model predictions with five

289 different values of slope η_p for triaxial testing at constant suction, $s = 50$ kPa, and net confining

290 pressure, $(\sigma_3 - u_a) = 300$ kPa. It can be clearly observed that the BS model is capable to simulate

291 a rather smooth transition of the nature of soil volume change, from contractant to dilatant, as

292 well as the corresponding transition of soil response from initial hardening to post-peak strain

293 softening. The parametric study also identifies a limiting case with $\eta_p = M = 1.3$, beyond which

294 any further increase in η_p will initiate dilatancy and the stress-strain curve will exhibit a distinct

295 peak stress.

296

297 *4.7. Barcelona Basic Model: Essential Features*

298 Alonso et al. [2] postulated a unified, critical state based, constitutive framework for unsaturated

299 soils by extending the modified Cam-Clay model [39], from saturated to unsaturated form, using

300 suction as an independent stress variable, while introducing the concept of the loading-collapse

301 (LC) yield surface. Consequently, constitutive parameters postulated by both the BBM and BSM

302 frameworks were experimentally calibrated from the series of suction-controlled triaxial tests

303　(Tables 1 and 2), and then used for predicting compacted silty sand suction-controlled

304　axisymmetric shearing response at constant matric suction states varying from 50 to 750 kPa.

305　　　　Further details regarding the explicit, step by step integration of BBM constitutive

306　relations are given by Macari et al. [40] and Hoyos et al. [41]. Also, implementation of BBM

307　theory to calibrate the essential constitutive model parameter via experimental tests can be

308　obtained from Patil [28]. Complementary simulations using the Barcelona Basic Model (BBM)

309　have also been included, alongside BS model predictions, in order to get further enlightening

310　insights into some of the main limitations and challenges facing both frameworks within the

311　context of the experimental evidence resulting from the present research effort.

312

313　　　　　　　　5. IMPLICIT INTEGRATION OF BSM CONSTITUTIVE RELATIONS

314　The present section illustrates the adopted protocol for implicitly integrating all the BSM

315　constitutive relations as summarized by equations (1)-(24) presented in the earlier sections.

316　Calibrated BSM parameters from Table 1 are used in order to simulate silty sand behavior for

317　matric suction, $s = 250$ kPa, and net mean pressure, $(\sigma_3 - u_a) = 300$ kPa. Figures 9(a) and 9(b)

318　show the corresponding stress-strain and volume change simulations along with observed

319　experimental response. Figure 9(c) clearly depicts the suction-induced growth experienced by the

320　initial yield surface of an unsaturated soil, as compared to the saturated case. Further, it

321　illustrates the progression of yield surfaces [i.e. (1)-(6)] as the specimen is loaded monotonically

322　under consolidated drained condition along an 1H:3V stress path, i.e. conventional triaxial

323　compression (CTC) stress path, in p-q space. The evolution of yield surface is largely governed

324　by change in plastic volumetric strains.

325   Point "1" corresponds to the initial state of a fictitious BS surface prior to shear loading,

326   with its equation calculated from the post-consolidation void ratio and mean net stress using

327   Bardet [7] equation, as expressed below:

328    $$A_\pi = [1 + k_1(sSr - s_e)]A_0 e^{\left(\frac{\Gamma - e - \kappa ln(p)}{\lambda - \kappa}\right)}$$ (25)

329   where e = current voids ratio, p = mean net pressure, and A_0 = unit pressure. The evolution of the

330   variable A_π in Eq. (5) is described by the classic Cam-Clay hardening rule [38]. Bounding

331   surface equation (Eq. 5) depends on "A" as well as "A_π", so it permits to obtain mechanical

332   hardening as well as suction hardening,

333    $$dA = \frac{1+e_0}{\lambda_0 - \kappa} A d\varepsilon_p^p \quad and \quad A_\pi = [1 + k_1(sSr - s_e)]A(\varepsilon_p^p)$$ (26)

334   For the sake of better understanding, the typical stress-strain curve is embedded to the

335   right of the BS yield curves in Fig. 9(c). As the specimen is loaded, the induced plastic strains

336   cause the bounding surface to move during plastic flow, but the BS must always envelope the

337   current state. The bounding surface expands in size isotropically as specimen is loaded between

338   stress state 1-2-3, inducing compressive volumetric strains. Point 3 on the stress-strain curve

339   identifies the onset of dilation and is also known as the characteristic state (i.e. CL). It marks a

340   transition state of volumetric behavior from contracting to dilating. The yield surface (BS) attains

341   its maximum size at characteristic state (point 3). As the loading continues further beyond

342   characteristic point 3, the stress state continues to move upwards towards peak failure line.

343   However, it cannot cross LSL (point 4), as explained earlier. Concomitantly, the test soil starts

344   dilating, and the BS starts to shrink in size as the plastic volumetric strains starts to decrease.

345   It should be noted that the observed peak dilatancy is attained far before reaching the

346   peak shear strength. Upon reaching the peak stress state, the specimen undergoes strain-softening

347   with further increase in deviator stress and the yield surface continues to shrink in size between

348 points 4 and 5, at a rate faster than corresponding decrease in stress state. According to Bardet

349 [7] theory for saturated soil, the stress state continues to decrease while the yield surface

350 continues to shrink further, until they both reach a common point, and thereafter they both move

351 together towards and stop at a point (point 6) that represents critical state, which correspond to a

352 state of no volume change, no stress change, and hence zero plastic volumetric strains. In these

353 simulations, critical state gives an asymptote to the stress path but is not reached. The same goes

354 for the bounding surface (BS), which keeps shrinking slowly but does not reach the stress state

355 along stress path, even with imposed axial strains of up to 40%.

356

357   6. BSM AND BBM PREDICTIONS OF COMPACTED SILTY SAND RESPONSE

358 BSM and BBM simulations were compared with the experimental results obtained from the

359 series of suction-controlled CD-triaxial shear tests. All specimens were sheared along CTC stress

360 paths. Figs. 10-14 show comparisons between BSM and BBM predictions and experimentally

361 observed deviator stress vs. axial strain response of compacted silty sand from fully drained

362 (constant suction) CTC tests conducted at four different values of matric suction, $s = 0, 50, 250,$

363 500 and 750 kPa, and for initial values of net mean stress, $p = (\sigma_3 - u_a) = 100, 200,$ and 300 kPa,

364 respectively.

365   In general, no close agreement is observed between experimental behavior and BBM

366 predictions for overconsolidated silty sand, primarily given the largely brittle and dilatant nature

367 of the test soil before it reaches critical state. For hardening materials, continued shearing along a

368 suction-controlled CTC stress path (Fig. 5) is expected to cause the elliptical yield surface in $p{:}q$

369 plane to move outward (or increase in size) from the current point, while it would move inward

370 (or shrink in size) for softening materials. The relative position of the current stress state with

371 respect to the CSL governs whether the material sustains plastic dilatancy or contractancy.

372 The original BBM framework, however, does not contemplate a stress state that lies

373 beyond the CSL at which all induced deformation is plastic, since the peak stress is always

374 assumed to be reached at critical state. Therefore, it is not expected to be suitable for reproducing

375 the transition from initial contractancy to dilatancy, and hence the post-peak strain softening

376 commonly observed in dense or overconsolidated geomaterials (Figs. 11-14). For this reason,

377 only BBM predictions of stress-strain response are shown in Figures 10-14, obviating the

378 corresponding volume change response.

379 In effect, although predictions of deviatoric stress (end values) at critical state still proved

380 to be reasonably close to those experimentally observed (Figs. 10-14), BBM predictions

381 considerably deviate from the stress-strain response of compacted silty sand, particularly at

382 higher matric suctions and net confining pressures, which are precisely the stress state for which

383 the test soil exhibits largest post-peak softening, accompanied by significant dilation. From a

384 qualitative standpoint, however, reasonably good predictions are generally observed for most of

385 the initial shearing stage, up to about 1-2% shear strain, and at considerably large values of shear

386 strain, i.e., critical state.

387 On the contrary, the BSM is able to reproduce the stress-strain and volume change

388 response of compacted silty sand, under both saturated and unsaturated conditions, with

389 reasonable accuracy. It is worth noting that saturated silty sand samples showed compressive

390 type volume change response, which turned into dilating type as soon as the soil was subjected to

391 a matric suction as low as 50 kPa. The BSM was able to capture this drastic transition in volume

392 change quite smoothly. Furthermore, the BSM is able to simulate the increase in dilation with

393   increasing matric suction, rather smoothly as well, and with reasonable accuracy, as shown in

394   Figs. 10(c)-14(c). This transition is possible because of the term added to the plastic modulus

395   when the stress point lies inside the surface (Eqs.15 and 16).

396   On the other hand, saturated silty sand samples showed hardening type stress-strain

397   response, which quickly changed to post-peak softening type behavior under suction-controlled

398   conditions (s = 50, 250, 500 and 750 kPa). Likewise, the BSM was able to capture this transition

399   rather smoothly and successfully. The increase in magnitude of post-peak softening with

400   increasing matric suction is also closely captured by the BSM, as shown in Figs. 11(a)-14(a).

401   This close prediction of post-peak strain softening can be attributed to the incorporation of two

402   state lines into the BSM framework: the commonly used CSL (critical state line) and the LSL

403   (limit state line), as shown in Fig. 5. The stress path can cross the CSL, but the LSL limits the

404   accessible domain and whether it is crossed or not, the CSL will be the asymptote of the stress

405   path.

406   Figs. 11(a & b)-14(a & b) indicate that although the BBM is not able to closely capture

407   the typical stress-strain response, it performs relatively well in predicting deviator stress at peak

408   and critical state under low net confining pressure irrespective of magnitude of matric suction

409   applied. On the other hand, BSM performs relatively well in predicting peak and critical deviator

410   stress under high net confining pressures irrespective of magnitude of matric suction applied.

411   Finally, good correlations were obtained, for the tested suction and confining pressure

412   range, between experimental and the extended BSM predicted deviator stress values, both at

413   critical state failure (R^2 = 0.91), and peak state failure (R^2 = 0.85), as shown in Figure 15(a) and

414   (c). Although, relatively better correlations were obtained with BBM as compared to BSM at

415   critical state failure (R^2 = 0.96), and peak state failure (R^2 = 0.93) as shown in Fig. 15(b) and (d),

416 it cannot closely reproduce the continuous stress-strain and volume change response (Figs. 10-

417 14). The slope of the critical state line in p'-q plane did not significantly changed with suction,

418 and thus can be considered to remain virtually constant and non-sensitive to changes in suction.

419 Figure 16 shows the variation of the slopes of critical state lines in p'-q plane for different soils

420 as reported from current and previous research works [22, 42–44].

421   During suction-controlled experiments, the practical range of applying matric suction

422 inside soil specimen is only up to 1500 kPa, due to limitations of ceramic disk in using axis-

423 translation technique [20]. In addition, increase in magnitude of induced suction, well beyond

424 residual suction, will dramatically escalate the magnitude of drop in post-peak strength and

425 hence softening upon monotonic shearing as well as reduce the overall amount of dilational

426 volume change [28].

427   Incorporating such a dramatic variation in stress-strain response into modeling, that alters

428 from strain-hardening for saturated soil to strain-softening with introduction of matric suction

429 and which further tends to show increase in amplitude of softening with further increase in total

430 suction for test soil (i.e. silty sand), poses a great challenge. Currently, the authors are

431 investigating into experimental response at high suction to improvise the existing model,

432 especially to incorporate additional parameters to reflect high suction impact on response, to

433 enable possible better predictions, particularly above residual suction, where any change in

434 moisture content is merely due to vapor-phase exchange.

435

7. SUMMARY AND CONCLUSIONS

437 12 consolidated drained (CD), conventional triaxial compression (CTC) tests were conducted on

438 statically compacted specimens of unsaturated silty sand (SM) under four suction-controlled

439   conditions and three net confining pressures. Experiments were performed in a fully-automated

440   double-walled triaxial test system and target matric suction states varying from 50 to 750 kPa

441   was induced via the axis-translation technique. Experimental results from present research

442   showed persuasive evidence of augmentation in shear strength, stiffness, yield strength, tensile

443   strength, post-peak strain-softening and dilatancy with increase in matric suction. A distinct,

444   experimental compaction induced loading-collapse (LC) locus was readily identified in the *p:s*

445   plane and comparison was made with BS model predicted LC locus.

446   Essential constitutive parameters postulated by the BBM and the extended BS model

447   framework were then calibrated and used for prediction of peak deviator stress at matric suction

448   states varying from 50 to 750 kPa. There were no good agreements between the observed

449   experimental and predicted BBM stress-strain responses, especially the post-peak softening

450   behavior. BBM predictions, however, hold reasonably well mostly during the early shearing

451   stage (i.e. 1-2% axial strain) and at higher values of shear strain, i.e. critical state.

452   On the other hand, the extended BS model was able to simulate the stress-strain and

453   volume change response reasonably well. The BS model was able to capture smoothly the

454   transition of stress-strain response from strain-hardening to post-peak strain softening along with

455   the contemporaneous transition of volume change from compressive to dilatant type on account

456   of suction with reasonably well captured simulations. The major advantage of BS model is its

457   flexibility to allow for shrinking of yield surface based on value of ($\eta_{\text{peak}} - \eta_{\text{critical}}$), to

458   accommodate the post-peak softening behavior, which otherwise, was not possible with classical

459   model such as BBM. In general, irrespective of the value of matric suction applied, the BBM

460   performs relatively well in predicting response at peak and critical state failure under low net

461 confining pressure while the extended BSM performs relatively well under high net confining

462 pressures.

463   With additional testing in future, the current version of generalized BS model framework

464 that seems to have promising applications to unsaturated soil mechanics and shear strength

465 applications could be further generalized and made more flexible to accommodate experimental

466 response from more soil types applied over broader suction range.

472

473   REFERENCES

474 1. Guo R, Li G. Elasto-plastic constitutive model for geotechnical materials with strain-softening

475 behavior. *Computers and Geosciences* 2008; 34: 14-23.

476 2. Alonso EE, Gens A, Josa AA. constitutive model for partially saturated soils. *Géotechnique*

477 1990; 40(3): 405-430.

478 3. Dafalias YF, Popov E. (1976). Plastic internal variables formalism of cyclic plasticity. *J. Appl.*

479 *Mech.* 1976; 43(4): 645-651.

480 4. Anandarajah A, Dafalias YF. Bounding surface plasticity. III: Application to anisotropic

481 cohesive soils. *J. Eng. Mech* 1986; 112(12): 1292-1318.

482 5. Dafalias YF, Herrmann LR. Bounding surface plasticity: II. Application to isotropic cohesive

483 soils." *J. Eng. Mech.* 1986; 112(12): 1263-1291.

484 6. Ling HI, Yue D, Kaliakin,VN, Themelis NJ. Anisotropic elastoplastic bounding surface model

485 for cohesive soils. *J. Eng. Mech.* 2002; 128(7): 748-758, http://dx.doi.org/10.1061/(ASCE)0733-

486 9399(2002)128:7(748).

487 7. Bardet JP. Bounding surface plasticity model for sands. *J. Eng. Mech* 1986; 112(11): 1198-
488 1217.

489 8. Wang ZL, Dafalias YF, Shen CK. Bounding surface hypoplasticity model for sand. *J. Eng.*
490 *Mech.* 1990; 116(5): 983-1001.

491 9. Mc Vay M, Taesiri Y. Cyclic behavior of pavement base materials." *J. Geotech. Eng.* 1985;
492 111(1): 1-17.

493 10. Ling HI, Liu HB, Mohri Y, Kawabata T. Bounding surface model for geosynthetic
494 reinforcements. *J. Eng. Mech.* 2001; 127(9): 963-967, DOI: http://dx.doi.org/10.1061/(ASCE)-
495 0733-9399-(2001)127:9(963).

496 11. Liu H, Ling HI. Unified elastoplastic-viscoplastic bounding surface model of geosynthetics
497 and its applications to geosynthetic reinforced soil-retaining wall analysis. *J. Eng. Mech.* 2007;
498 133(7): 801-815, DOI: http://dx.doi.org/10.1061/(ASCE)0733-9399(2007)133:7(801).

499 12. Yang BL, Dafalias YF, Herrmann LR. A bounding surface plasticity model for concrete. *J.*
500 *Eng. Mech.* 1985; 111(3): 359-380.

501 13. Luan MT, Wu XG. Li XS. Bounding-surface hypoplasticity model for rockfill materials and
502 its verification (in Chinese). *Chin. J. Rock Mech. Eng.* 2001; 20(2): 164-170.

503 14. Xiao Y, Liu HL, Zhu JG, Shi WC, Liu MC. A 3-D bounding surface model for rockfill
504 materials. *Sci. China Tech. Sci.* 2011; 54(11): 2904-2915.

505 15. Hashiguchi K, Ueno M. Elasto-plastic constitutive laws of granular materials. Speciality
506 session 9 – Constitutive Equations of Soils. *9th Int. Conf. Soil mech. Found. Eng.* 1977; Tokyo,
507 Japan, 73-82.

508 16. Aboim CA, Roth WH. Bounding surface plasticity applied to cyclic loading of sand.
509 *International symposium on Numerical Models in Geo-mechanics* 1982; Zurich, Switzerland, 65-
510 72.

511 17. Dafalias YF, Herrmann LR. Bounding surface formulation of soil plasticity. Soil Mechanics
512 – Transient and Cyclic Loads 1982; G. Pande and O. C. Zienkiewicz, Eds., John Wiley and Sons,
513 Inc., London, U.K.: 253-282.

514 18. Kaliakin VN, Dafalias YF. Simplifications to the bounding surface model for cohesive soils.
515 *International Journal for Numerical and Analytical Methods in Geomechanics* 1989; 13(1): 91-
516 100.

19. Jiang J, Ling HI, Kaliakin VN. An associative and non-associative anisotropic bounding surface model for clay. *J. Applied Mech.* 2012; 79(3): 031010, DOI: 10.1115/1.4005958-1-131010-10.

20. Fredlund DG, Rahardjo H. Soil mechanics for unsaturated soils. Wiley: New York, 1993.

21. Russel AR, Khalili N. A unified bounding surface plasticity model for unsaturated soils. *International Journal for Numerical and Analytical Methods in Geomechanics* 2006; 30(3): 181-212.

22. Wheeler SJ, Sivakumar V. An elasto plastic critical state framework for unsaturated soil. *Geotechnique* 1995; 45(1): 35-53.

23. Russell AR. Cavity expansion in unsaturated soils. Ph.D. dissertation, The University of New South Wales, Australia, 2004.

24. Morvan M, Wong H, Branque D. An unsaturated soil model with minimal number of parameters based on bounding surface plasticity. *International Journal for Numerical and Analytical Methods in Geomechanics* 2010; 34: 1512-1537.

25. Cui YJ, Delage P. Yielding and plastic behaviour of an unsaturated compacted silt. *Géotechnique* 1996; 46(2): 291-311.

26. Gallipoli D, Wheeler SJ, Karstunen M. Modelling the variation of degree of saturation in a deformable unsaturated soil. *Geotechnique* 2003; 53(1): 105-112.

27. Raveendiraraj A. Coupling of mechanical behavior and water retention behavior in unsaturated soils. Ph.D. dissertation 2009; Department of Civil Engineering, University of Glasgow, Scotland.

28. Patil UD. Response of unsaturated silty sand over a wider range of suction states using a novel double-walled triaxial testing system. Doctoral dissertation, University of Texas at Arlington, Arlington, TX, 2014.

29. Yu HS. *Plasticity and geotechnics*. Springer: New York, 2006.

30. Cattoni E, Cecconi M, Pane V. An experimental study on a partially saturated pyroclastic soil: the Pozzolana Nera from Roma. *Proceedings of The Second Workshop on Unsaturated Soils*. Tarantino and Mancuso (eds), Capri: Italy, 29-42.

31.Hossain MA, Yin JH. Shear strength and dilative characteristics of an unsaturated compacted completely decomposed granite soil. *Canadian Geotechnical Journal* 2010; 47: 1112-1126.

547 32. Ng CWW, Sadeghi H, Jafarzadeh, F. (2016). Compression and shear strength characteristics
548 of compacted loess at high suctions. *Canadian Geotechnical Journal* 2017; 54: 690-699,
549 dx.doi.org/10.1139/cg.

550 33. Houston SL, Parez-Garcia N, Houston WN. Shear strength and shear-induced volume change
551 behavior of unsaturated soils from a triaxial test program. *Journal of Geotechnical and*
552 *Geoenvironmental Engineering* 2008; 134(11): 1619-1632.

553 34. Pereira JM, Wong H, Dubujet P, Dangla, P. Adaptation of existing models to unsaturated
554 states: application to CJS model. *Int. J. Numer. Anal. Meth. Geomech.* 2005; 29(11): 1127-1155.

555 35. Crouch RS, Wolf JP, Dafalias YF. (1994). Unified critical-state bounding-surface plasticity
556 model for soil. *J. Eng. Mech* 1994; 120(11): 2251-2270.

557 36. Manzari MT, Dafalias YF. A critical state two-surface plasticity model for sands."
558 *Geotechnique* 1997; 47(2): 255-272.

559 37. Brooks RH, Corey AT. Hydraulic properties of porous media. *Hydrology Papers 3* 1964;
560 Colorado State University, Fort Collins, Colorado, USA, 27 p.

561 38. Morvan M, Wong H, Branque D. Incorporating porosity-dependent hysteretic water retention
562 behavior into a new constitutive model of unsaturated soils. *Can. Geotech. J.* 2011; 48(12):
563 1855-1869.

564 39. Schofield A, Wroth CP. *Critical state soil mechanics*. McGraw-Hill: London, U.K, 1968.

565 40. Macari EJ, Hoyos LR, Arduino P. Constitutive modeling of unsaturated soil behavior under
566 axisymmetric stress states using a stress/suction-controlled cubical test cell. *Int. J. Plast*. 2003;
567 19(10): 1481-1515.

568 41. Hoyos LR, Perez-Ruiz DD, Puppala AJ. Modelling unsaturated soil response under suction-
569 controlled true triaxial stress paths. *Int. J. Geomech.* 2012; ASCE, 12(3): 292-308.

570 42. Hoyos LR. Experimental and computational modeling of unsaturated soil behavior under true
571 triaxial stress states. *Ph.D. dissertation*, Georgia Institute of Technology, Atlanta, USA, 1998.

572 43. Laikram A. Modeling unsaturated soil response under suction-controlled multi-axial stress
573 states. *Ph.D. dissertation*, University of Texas at Arlington, Arlington, TX, 2007.

574 44. Perez-Ruiz DD. A refined true triaxial apparatus for testing unsaturated soils under suction-
575 controlled stress paths. *Ph.D. dissertation*, University of Texas at Arlington, Arlington, TX,
576 2009.

List of Tables

Table 1. Essential BSM parameters calibrated from past and present works

Material constants	κ	v	ρ	M	Γ	η_p	h_0	s_e	α	k_1	k_3	λ_0
Units	-	-	-	-	-	-	-	kPa	-	-	-	-
	0.01	0.125	2.2	1.2	0.73	1.2	2	15	2.1	2	0.05	0.1425
**	0.006	0.3	1.7	1.6	0.95	1.9	5	6	0.2	5	0.05	0.155
***	0.05	0.2	2.2	0.72	1.1	0.74	5	-	0.066	0.08	0.1907	0.19
****	0.008	0.3	1.7	1.3	0.7	1.8	5	8	0.3	2.23	0.003	0.04

Notes: * – Cui and Delage [25]; Triaxial compression test on compacted jossigny silt and BS Model parameters extracted by Morvan et al. [24].

** – Russell [23]; Drained triaxial test on Kurnell sand and BS Model parameters extracted by Morvan et al. (2010).

*** – Raveendiraraj [27] and Gallipoli et al. [26]; Triaxial test on speswhite kaolin and BS Model parameters extracted by Morvan et al. (2010).

**** – CD triaxial test on poorly graded silty sand (SM) with clay; Present research.

Table 2. Essential BBM Parameters for current test material

Parameter	Current work	Units
$\lambda(0)$	0.020	-
κ	0.0015	-
β	2.000	MPa^{-1}
r	0.260	-
p^c	0.046	MPa
G	25.00	MPa
M	1.420	-
k	0.223	-
$p_o(0)$	0.070	MPa

Figure 13. Experimental and BSM/BBM predictions of unsaturated silty sand response at matric suction, $s = 500$ kPa, and different net confining pressures(a), (b) stress-strain response, and (c) volume change response

Figure 14. Experimental and BSM/BBM predictions of unsaturated silty sand response at matric suction, $s = 750$ kPa, and different net confining pressures: (a), (b) stress-strain response, and (c) volume change response

Figure 15. Comparison of predicted and measured values of deviator stress at peak and critical state failure as postulated by the BSM and BBM framework

Figure 16. Current and previously reported values of critical state line slope variation with matric suction

Following is the list of symbols used in this paper:

π	equivalent pore pressure
η	stress ratio q/p
η_p, η_{cr}	maximum stress ratio at peak state and critical state failure
$\bar{\sigma}$	projection of stress state i.e., image point on bounding surface
δ	distance between stress state and image stress
p, q, p'	mean pressure, deviator stress, and net mean pressure
$\bar{p}, \bar{q}$	mean pressure and deviator stress at image point
p'_{cr}, q_{cr}	mean net pressure and deviator stress at critical state
$\varepsilon_i \ i = 1, 2, 3$	components of strain in triaxial space
$\varepsilon_p, \varepsilon_q$	volumetric and deviator strain
$\varepsilon_p^e, \varepsilon_q^e$	elastic volumetric and deviator strain
$\varepsilon_p^p, \varepsilon_q^p$	plastic volumetric and deviator strain
A, A_π	position of bounding surface summit on the p-axis
K_p	elastic bulk modulus
e, e_c, e_0	present, critical, and initial voids ratio
G, K_q	elastic shear moduli
g	amplitude of gradient of bounding surface at image stress
H, H_b, H_δ	plastic moduli
K_1, k_2, K_3	material constants to account for unsaturated state
l_1, l_2	material constants to account for unsaturated state
M, M_π	position of bounding surface summit on the p-axis
n_p, n_q, n_s	components of unit vector normal to bounding surface at image point
s	suction
S_1	degree of saturation
x, y	normalized stress ratio, variable defining image position
ζ	plastic multiplier
α, s_e	constants defining water retention curve
κ	elastic volumetric deformability
$\lambda_0, \lambda, \Gamma, q_0$	values determining position of the critical state
v	elastic poisson ratio
ρ	aspect ratio of ellipse
$\sigma_i \ i = 1, 2, 3$	components of stress in triaxial space

$\sigma_i' \; i = 1, 2, 3$ components of effective stress in triaxial space